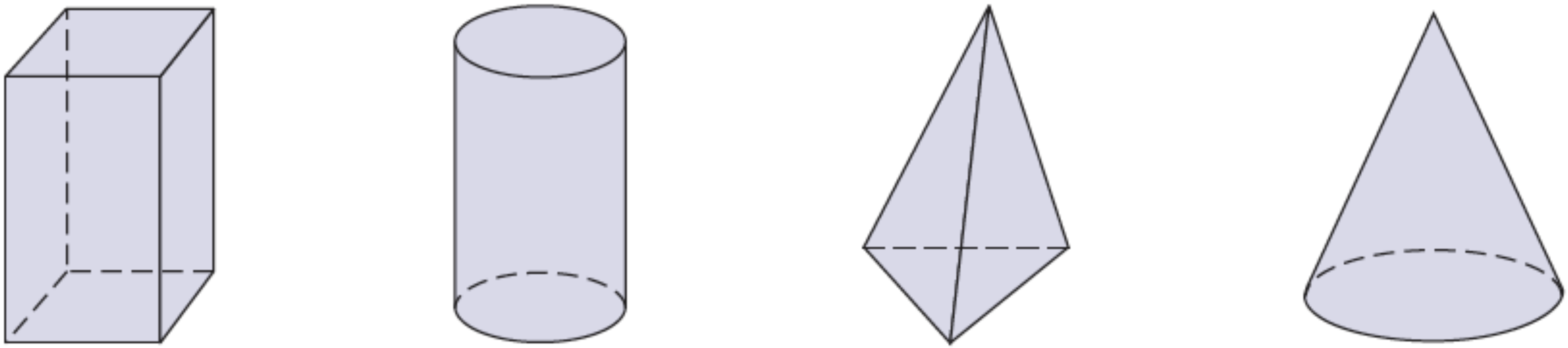
CHAPTER 12: SURFACE AREA AND VOLUME

INTRO TO SOLIDS

SOLIDS

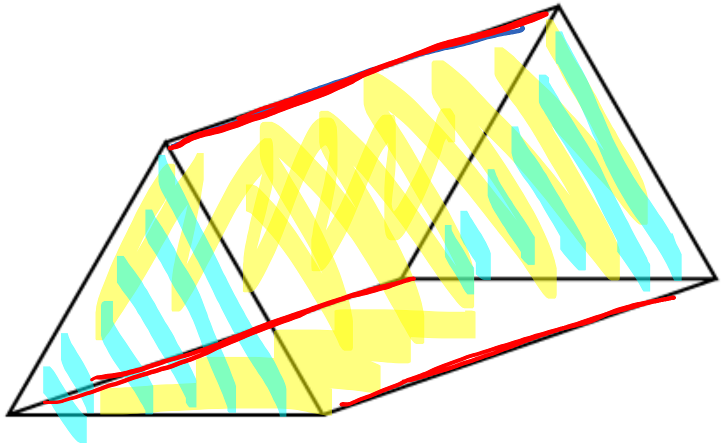
All figures above are examples of **solid figures** or **solids.**

Solids with flat surface that are polygons are called **polyhedrons** or **polyhedra.**



Cylinders Cones

PRISMS AND PYRAMIDS

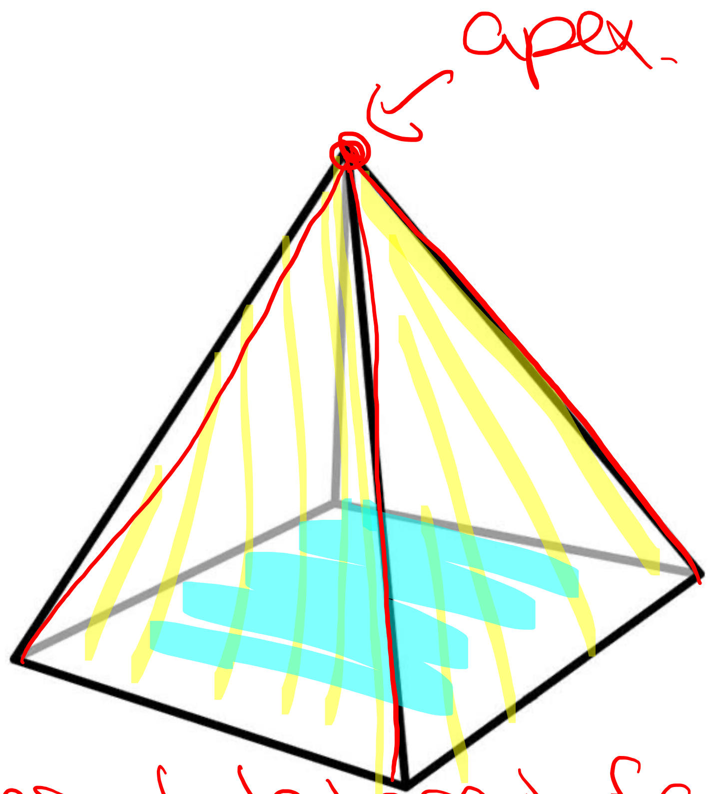


Edges of lateral faces are parallel.

Prism

Lateral faces are rectangular.

Two bases.



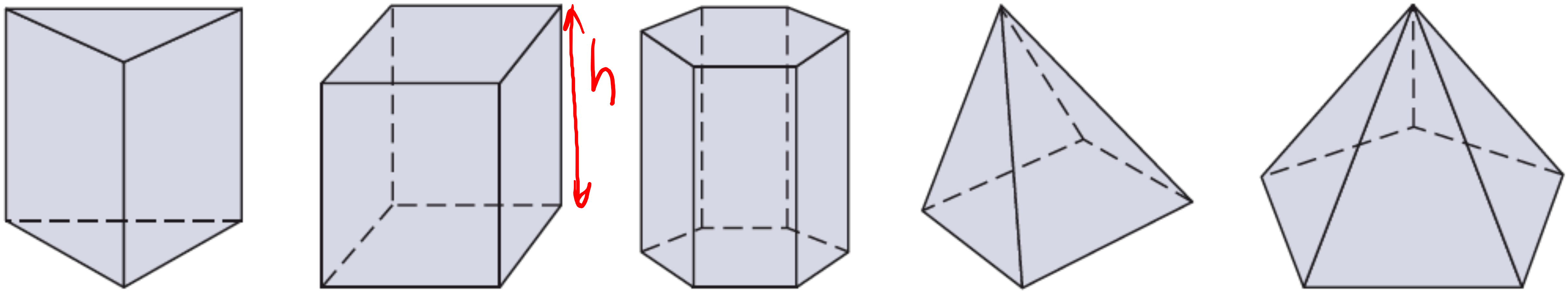
Edges of lateral faces meet at the apex.

Pyramid

- Lateral faces are triangular.
- One base.

CLASSIFICATION

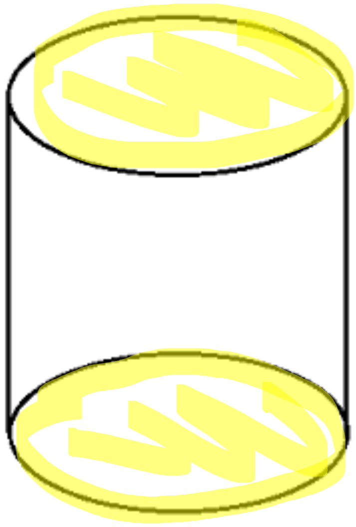
Prisms and pyramids are classified according to the shape of their base.



triangular prism | rectangular prism | hexagonal prism | rectangular pyramid | pentagonal pyramid

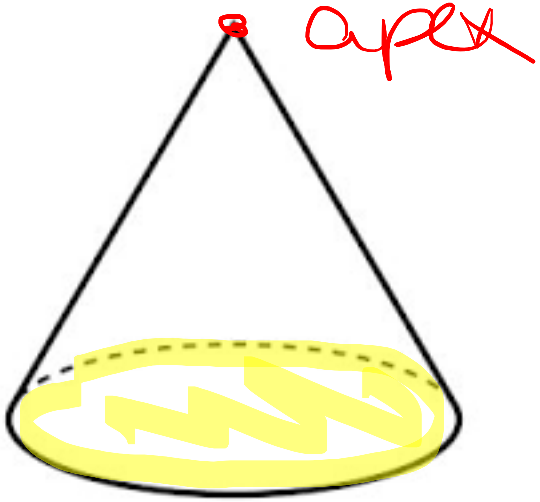
CYLINDERS AND CONES

Cylinders and cones are not polyhedral because they have curved lateral faces.



Cylinder

- Two bases.



Cone

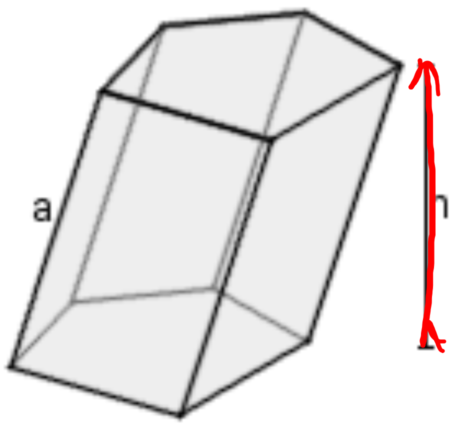
- One base.
- apex

12.2 – SURFACE AREA OF PRISMS AND CYLINDERS

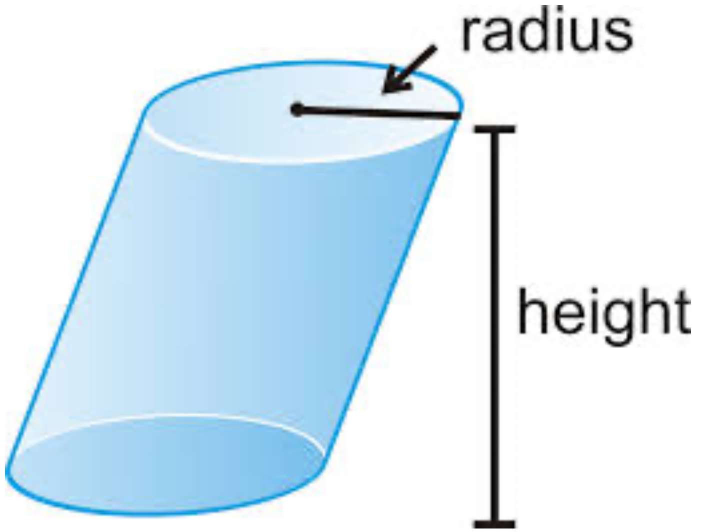
OBLIQUE PRISMS AND CYLINDERS.

Solids are **oblique** when they are slanted. In this case, the height of the prisms and cylinder does not correspond to the edges.

Oblique prism:

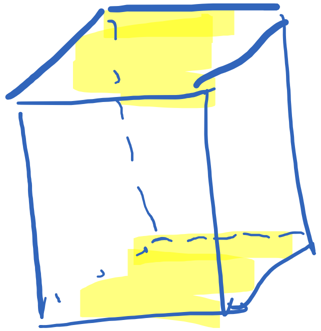


Oblique cylinder:



AREA DEFINITIONS

→ everything except the yellow.



Lateral area includes the area of all the lateral faces.

Surface area includes the area of lateral faces and bases.

↳ total area, includes the yellow faces.

<u>For prisms and cylinders:</u>

lateral area

$$SA = LA + 2 \cdot A_{base}$$

surface area

<u>For pyramids and cones:</u>

$$SA = LA + A_{base}$$

PRISMS AND CYLINDERS AS LAYERING OF SHAPES

Using this method, we can use the formula:

$$LA = P_{base} \cdot h$$

$$SA = \underbrace{P_{base} \cdot h}_{LA} + 2A_{base}$$

P_{base} = perimeter of base.

h = height

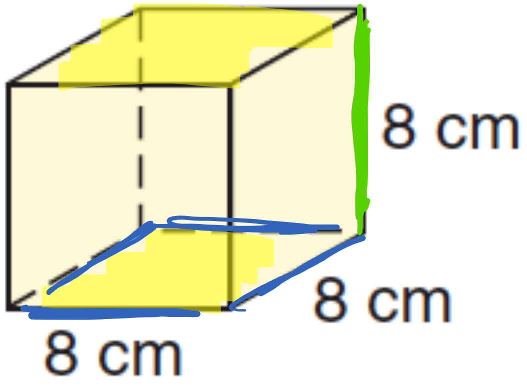
height = distance between the two bases.

Find the lateral area and the surface area of each prism.

$LA = P_b \cdot h$

$SA = LA + 2 \cdot A_b$

a.



$P_b = 8 \times 4 = 32\ cm$

$h = 8\ cm$

$A_b = 8 \times 8 = 64\ cm^2$

$LA = 32 \times 8 = 256\ cm^2$

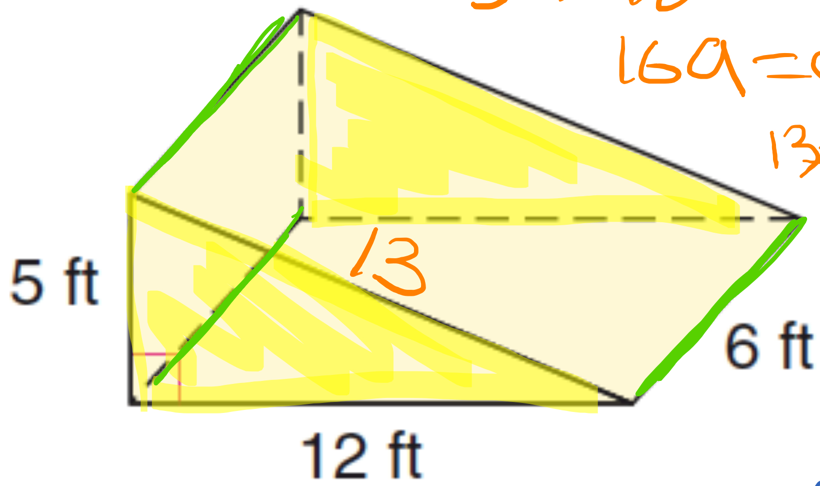
$SA = 256 + 2 \times 64 = 384\ cm^2$

b.

$5^2 + 12^2 = c^2$

$169 = c^2$

$13 = c$



$P_b = 5 + 12 + 13 = 30\ ft$

$h = 6\ ft$

$A_b = \frac{5 \times 12}{2} = 30\ ft^2$

$LA = 30 \times 6 = 180\ ft^2$

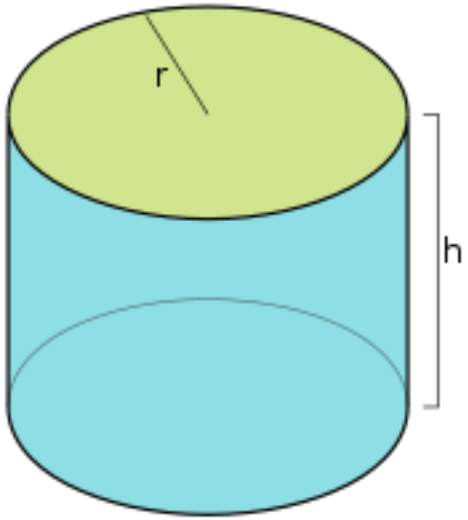
$SA = 180 + 2 \times 30 = 240\ ft^2$

PRISMS AND CYLINDERS AS LAYERING OF SHAPES

$C = 2\pi r$

$$LA = P_{base} \cdot h$$
$$SA = P_{base} \cdot h + 2A_{base}$$

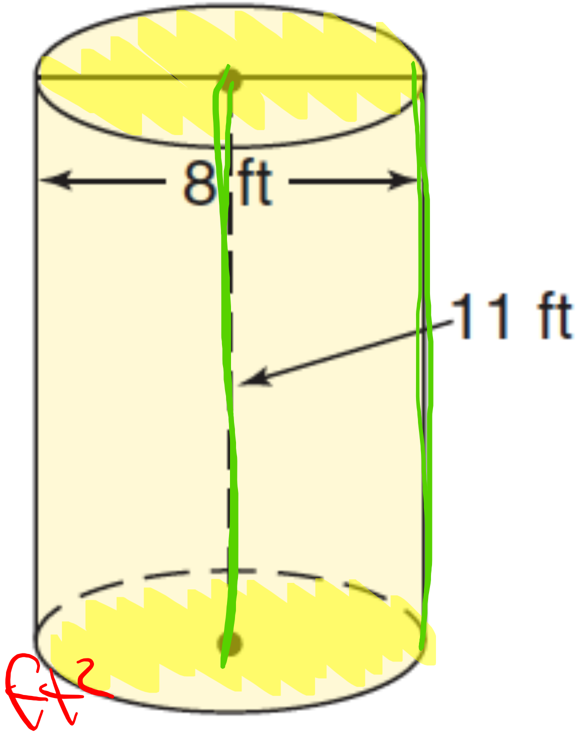
πr^2



For cylinders, we can replace the perimeter and area by their formulas, which give us:

$$LA = 2\pi r \cdot h$$
$$SA = 2\pi rh + 2\pi r^2$$

Find the lateral area and surface area of the cylinder to the nearest hundredth. $r = 8 \div 2 = 4$



$P_b = \pi(8) = 25.13$ ft

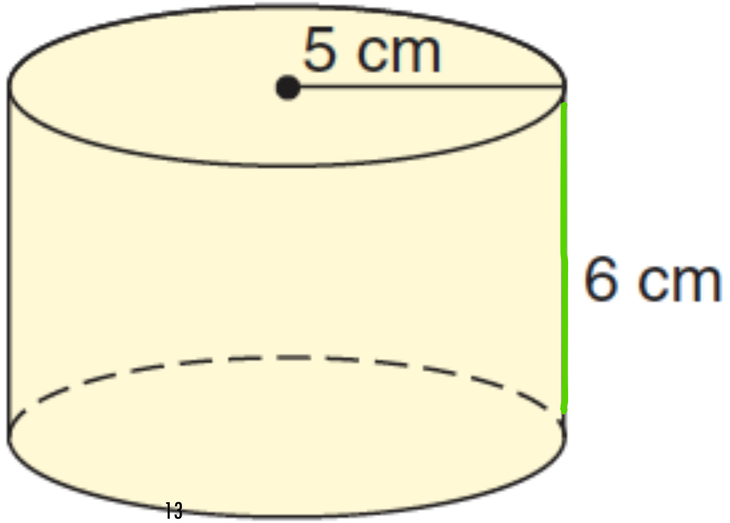
$h = 11$ ft

$A_b = \pi(4)^2 = 50.27$ ft^2

$LA = 25.13 \times 11 = 276.43$ ft^2

$SA = 276.43 + 2 \times 50.27 = 376.97$ ft^2

Find the lateral area and the surface area of the cylinder to the nearest hundredth.



$P_b = 2\pi(5) = 31.42$ cm

$h = 6$ cm

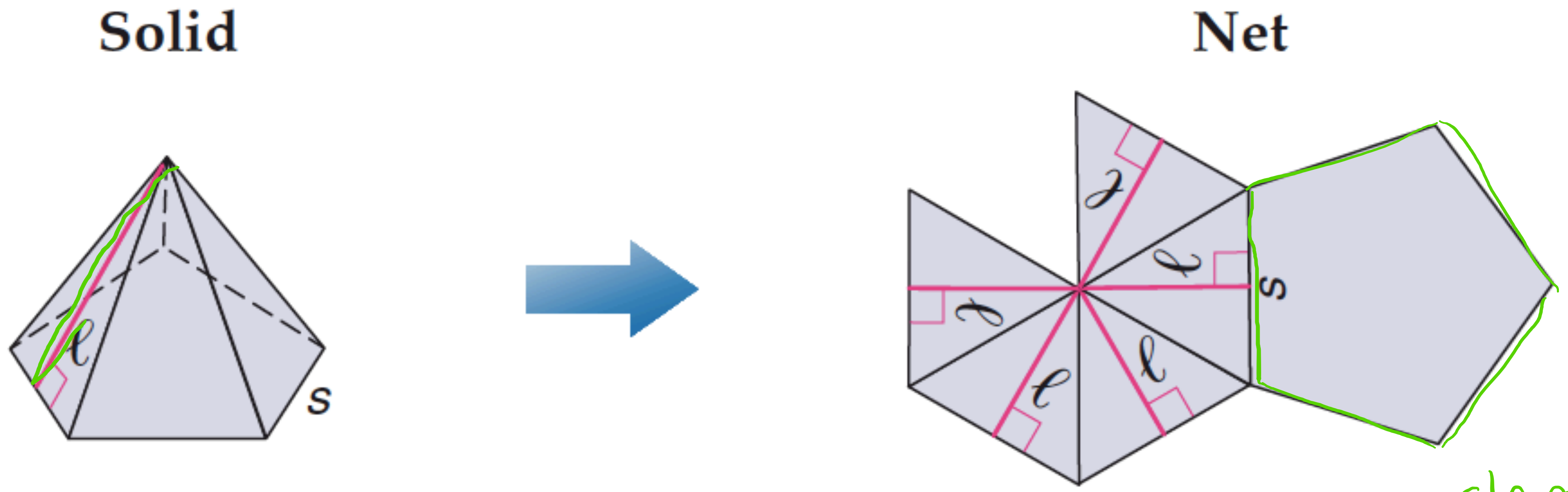
$A_b = \pi(5)^2 = 78.54$ cm^2

$LA = 31.42 \times 6 = 188.52$ cm^2

$SA = 188.52 + 2 \times 78.54 = 345.6$ cm^2

12.3 – SURFACE AREA OF PYRAMIDS AND CONES

AREA USING NETS



slant height

Area of triangle: $\frac{s \times \ell}{2}$

$LA = \frac{s \times \ell}{2} \cdot 5$ — $P_b \cdot LA = \frac{P_b \cdot \ell}{2}$

AREA USING PERIMETER

l = slant height
→ the height of a lateral face.

$$LA = \frac{1}{2}P_{base} \cdot l$$

$$SA = \frac{1}{2}P_{base}l + A_{base}$$

For a cone, you can replace perimeter and area of circle by their formula.

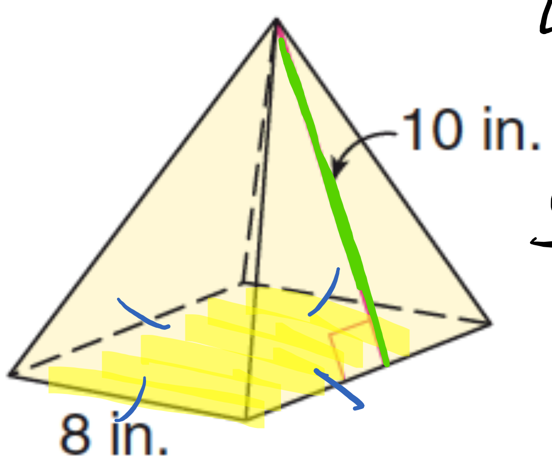
$$LA = \frac{P_b \cdot l}{2} = \pi r l \quad (P_b = 2\pi r)$$

$$LA = \pi r l$$

$$SA = \pi r l + \pi r^2$$

Find the lateral area and the surface area of each regular pyramid.

a.



$LA = \frac{P_b \cdot l}{2}$

$SA = LA + A_b$

$P_b = 4 \times 8 = 32 in$

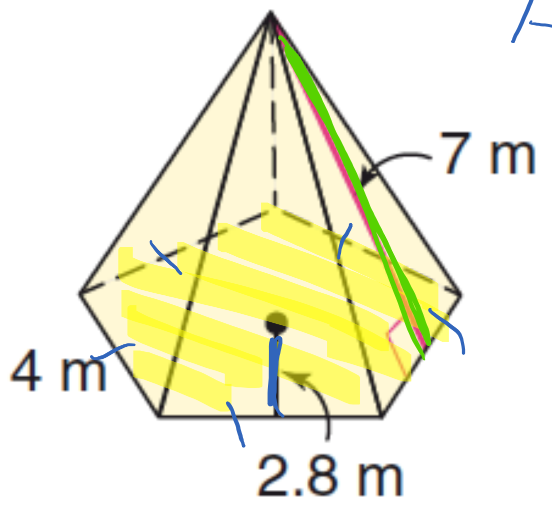
$l = 10 in$

$A_b = 8 \times 8 = 64$

$LA = \frac{32 \times 10}{2} = 160 in^2$

$SA = 160 + 64 = 224 in^2$

b.



$A = \frac{n \cdot s \cdot a}{2}$

$P_b = 5 \times 4 = 20 m$

$l = 7 m$

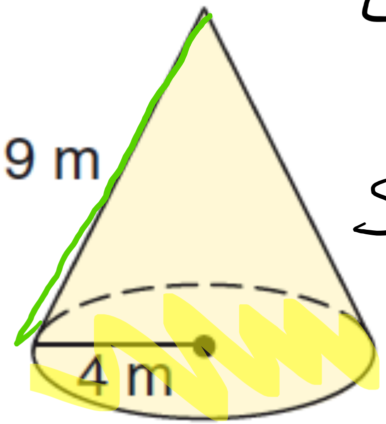
$A_b = \frac{5 \times 4 \times 2.8}{2} = 28 m^2$

$LA = \frac{20 \times 7}{2} = 70 m^2$

$SA = 70 + 28 = 98 m^2$

Find the lateral area and the surface area of each cone. Round to the nearest hundredth.

c.



$LA = \frac{P_b \cdot \ell}{2}$

$SA = LA + A_b$

$P_b = 2\pi(4) = 25.13 \text{ m}$

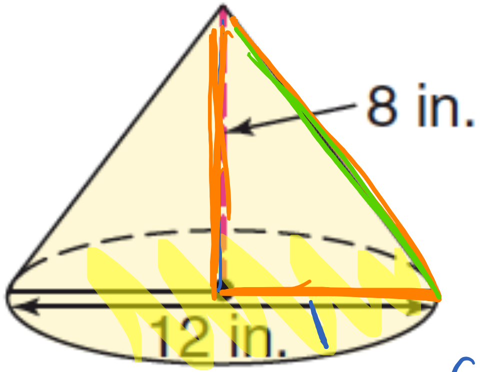
$\ell = 9 \text{ m}$

$A_b = \pi(4)^2 = 50.27 \text{ m}^2$

$LA = \frac{25.13 \times 9}{2} = 113.09 \text{ m}^2$

$SA = 113.09 + 50.27 = 163.35 \text{ m}^2$

d.



$r = 6$

$8^2 + 6^2 = \ell^2$

$100 = \ell^2$

$10 = \ell$

$P_b = 2\pi(6) = 37.70 \text{ in}$

$\ell = 10 \text{ in}$

$A_b = \pi(6)^2 = 113.09 \text{ in}^2$

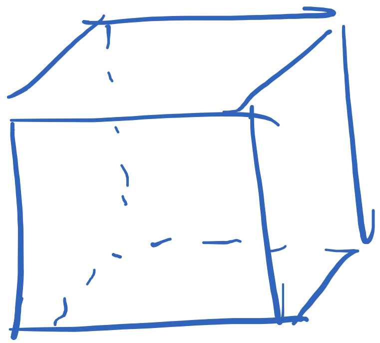
$LA = \frac{37.70 \times 10}{2} = 188.5 \text{ in}^2$

$SA = 188.5 + 113.09 = 301.59 \text{ in}^2$

12.4 – VOLUME OF PRISMS AND CYLINDERS

DEFINITION: VOLUME

Volume is the amount of space contained in a solid. It is measured in cubic units.



VOLUME OF PRISMS AND CYLINDERS

Formula:

$$V = A_{base} \cdot h$$

For cylinders, you can replace the area by the formula for the area of a circle.

$$V = \pi r^2 \cdot h$$

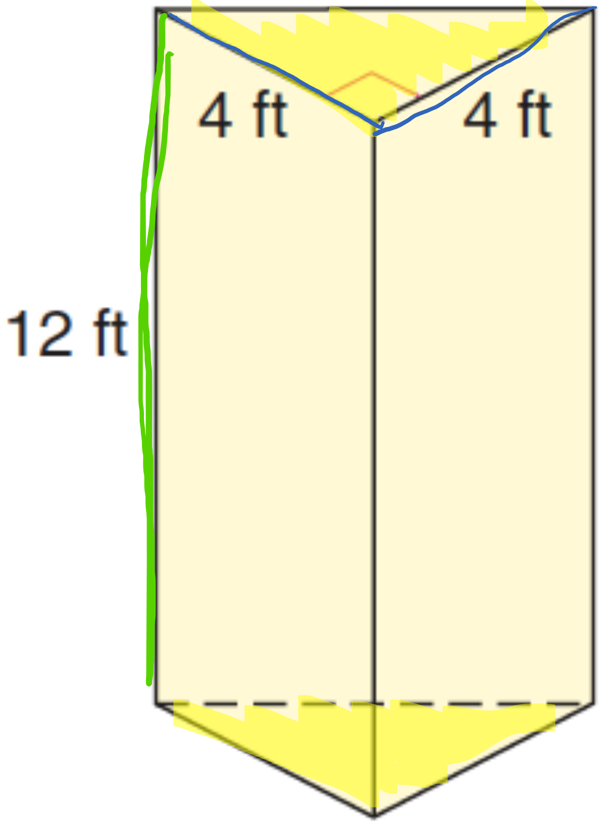
Find the volume of the triangular prism.

$V = A_b \cdot h$

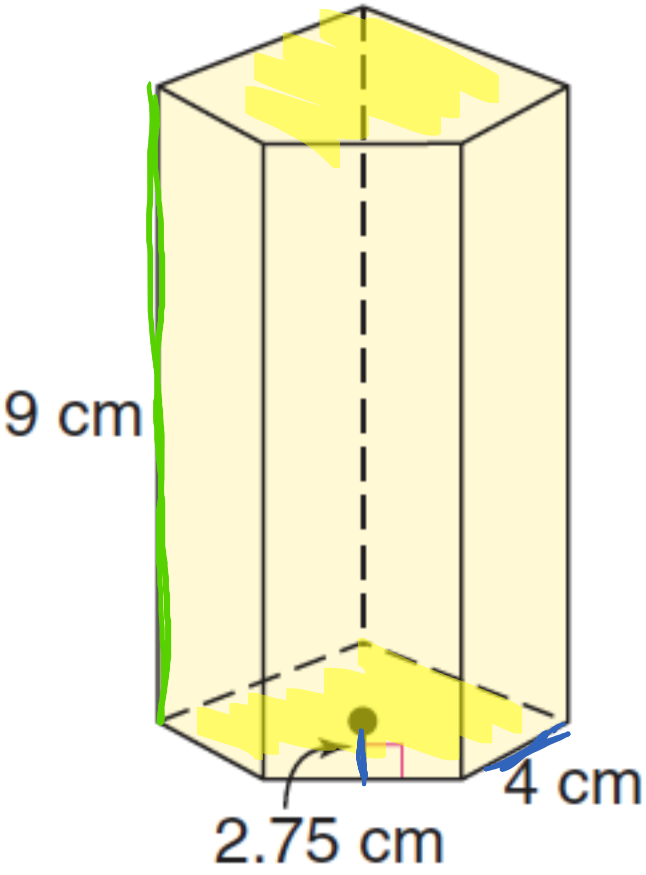
$A_b = \frac{4 \times 4}{2} = 8\ ft^2$

$h = 12ft$

$V = 8 \times 12 = 96\ ft^3$



The base of the prism is a regular pentagon with sides of 4 centimeters and an apothem of 2.75 centimeters. Find the volume of the prism.



$$A = \frac{n \times s \times a}{2}$$

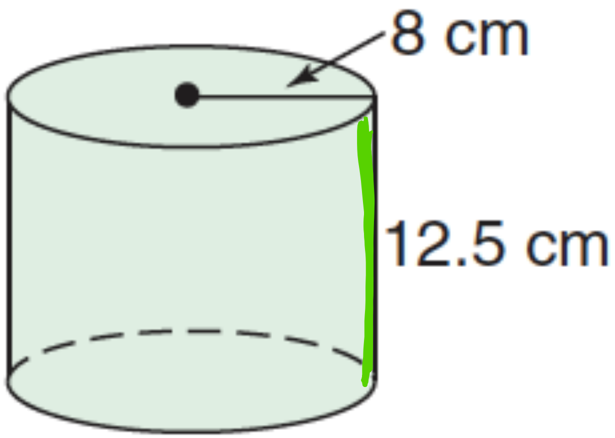
$$A_b = \frac{5 \times 4 \times 2.75}{2} = 27.5\ cm^2$$

$$h = 9\ cm$$

$$V = 27.5 \times 9 = 247.5\ cm^3$$

Find the volume of the cylinder to the nearest hundredth.

$A = \pi r^2$

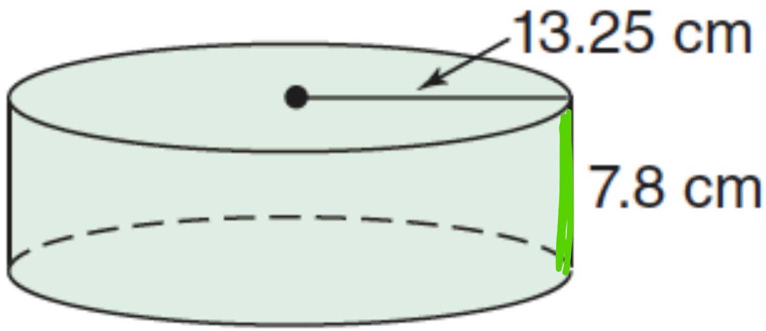


$A_b = \pi \cdot 8^2 = 201.06 \text{ cm}^2$

$h = 12.5 \text{ cm}$

$V = 201.06 \times 12.5 = 2513.25 \text{ cm}^3$

Find the volume of the cylinder to the nearest hundredth.



$A_b = \pi(13.25)^2 = 551.55\ cm^2$

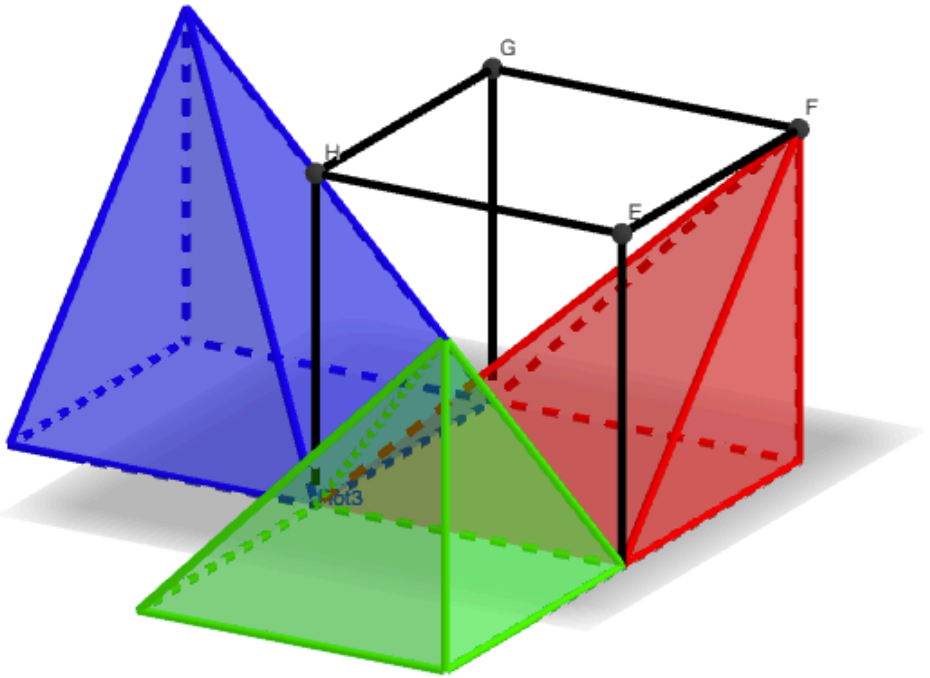
$h = 7.8cm$

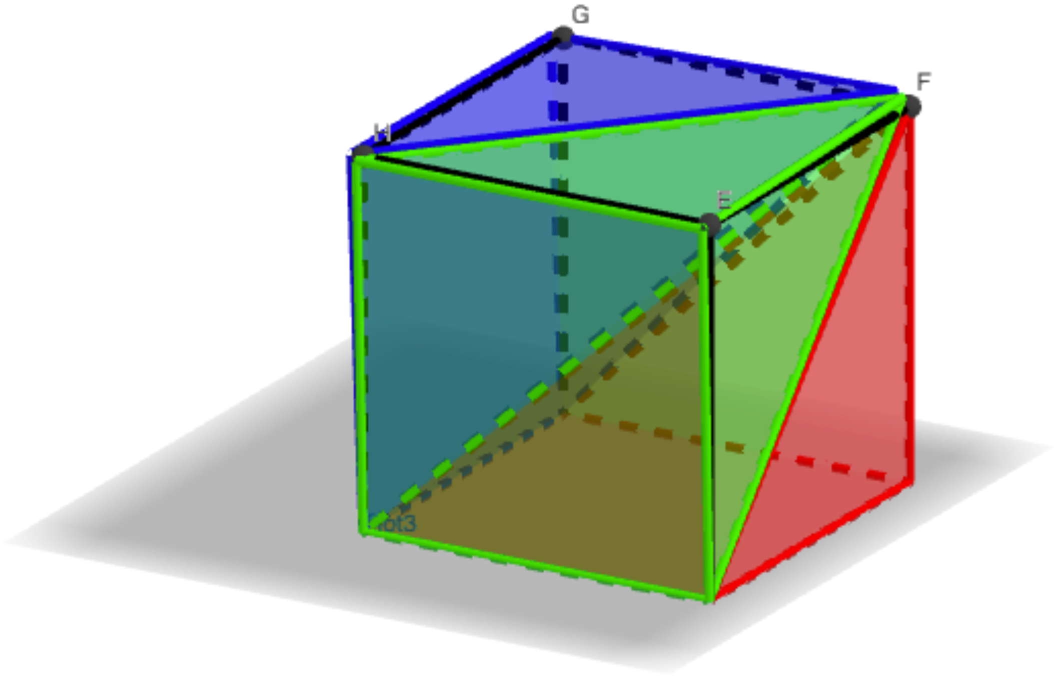
$V = 551.55 \times 7.8 = 4302.09\ cm^3$

12.5 – VOLUME OF PYRAMIDS AND CONES

VOLUME OF A PYRAMID DEMO

https://www.geogebra.org/m/jwf5y73q





$V_{pyramid} = \frac{V_{prism}}{3}$

VOLUME OF PYRAMIDS AND CONES

$$V = \frac{1}{3} A_{base} \cdot h$$

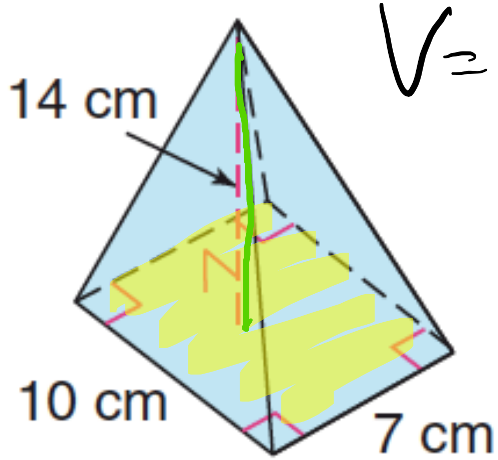
For a cone, replace with formula for area of a circle:

$$V = \frac{1}{3} \pi r^2 h$$

Find the volume of each pyramid. Round to the nearest hundredth.

$V = \frac{A_b \cdot h}{3}$

a.

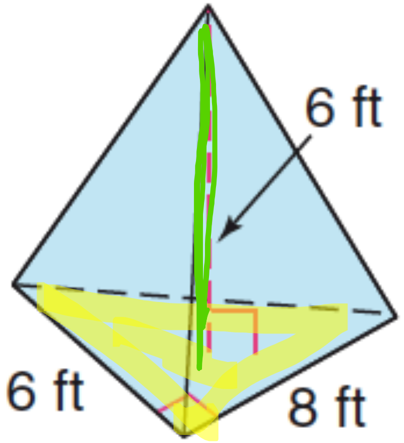


$A_b = 10 \times 7 = 70 cm^2$

$h = 14 cm$

$V = \frac{70 \times 14}{3} = 326.67 cm^3$

b.



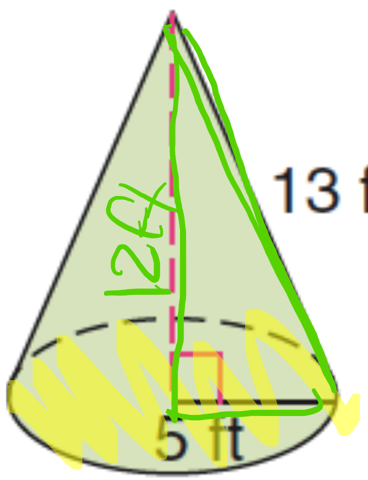
$A_b = \frac{6 \times 8}{2} = 24 ft^2$

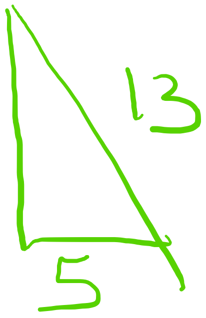
$h = 6 ft$

$V = \frac{24 \times 6}{3} = 48 ft^3$

Find the volume of each cone to the nearest hundredth.

c.





$13^2 - 5^2 = h^2$

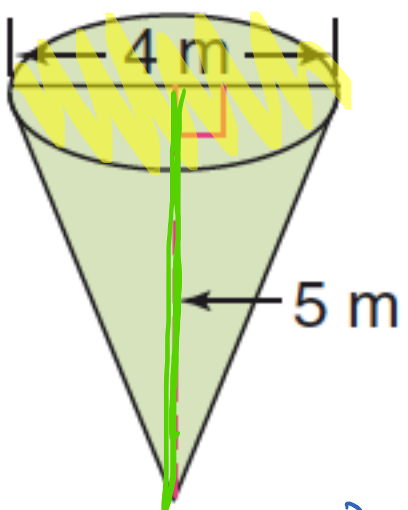
$144 = h^2$

$A_b = \pi(5)^2 = 78.54\ ft^2$

$h = 12\ ft$

$V = \frac{78.54 \times 12}{3} = 314.16\ ft^3$

d.



$r = 4 \div 2 = 2m$

$A_b = \pi(2)^2 = 12.57 m^2$

$h = 5m$

$V = \frac{12.57 \times 5}{3} = 20.95\ m^3$

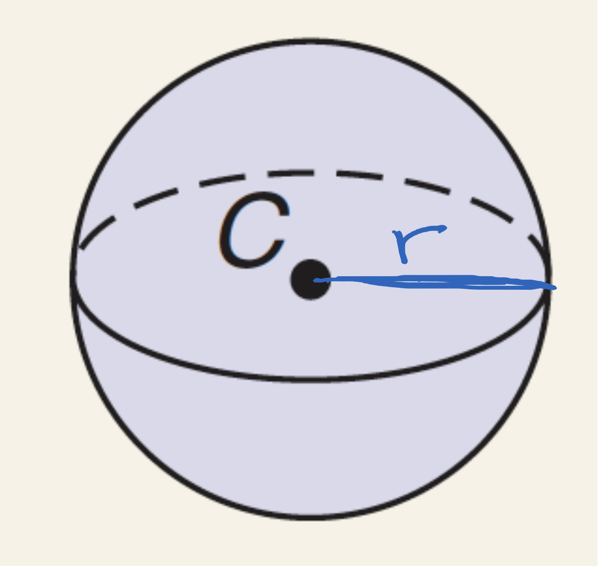
12.6 – SURFACE AREA AND VOLUME OF SPHERES

AREA AND VOLUME FORMULAS

Spheres have no base, so there is only one area (no distinction between lateral and surface areas.)

Surface Area: $S = 4\pi r^2$

Volume: V$= \frac{4}{3}\pi r^3$

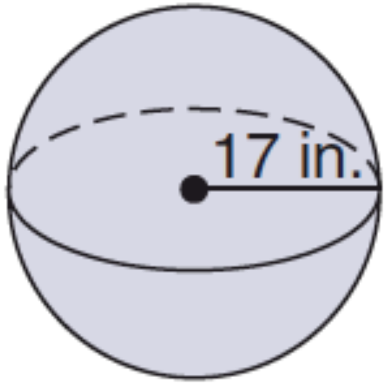


Find the surface area and volume of each sphere. Round to the nearest hundredth.

$S = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

a.

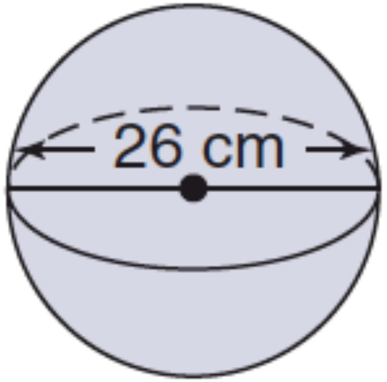


$r = 17\text{ in}$

$S = 4\pi(17)^2 \approx 3631.68\text{ in}^2$

$V = \frac{4}{3}\pi(17)^3 \approx 20579.53\text{ in}^3$

b.



$d = 26\text{ cm}$

$r = 26 \div 2$

$r = 13\text{ cm}$

$S = 4\pi(13)^2 \approx 2123.72\text{ cm}^2$

$V = \frac{4}{3}\pi(13)^3 = 9202.78\text{ cm}^3$

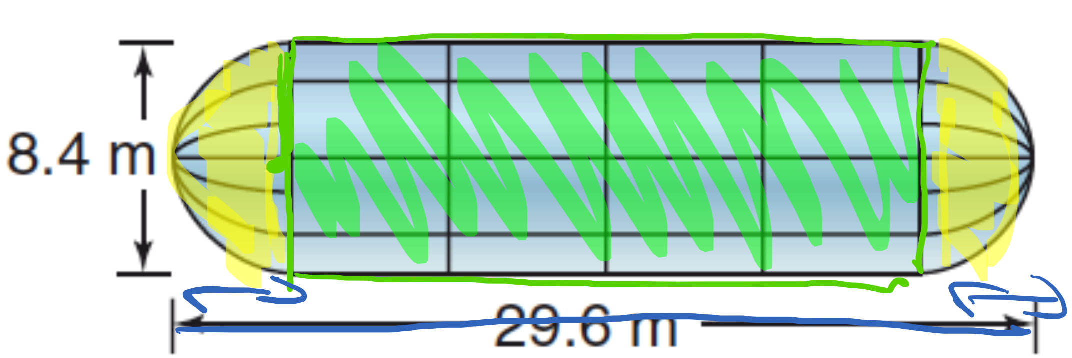
AREA AND VOLUME OF COMPOSITE SOLIDS

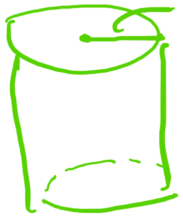
To find the area or volume of composite solids, calculate the area or volume of the individual solids they are made up of and add them together.

Total volume: $1174.9 + 310.34 = 1485.24 \text{ m}^3$

The large external tank attached to the space shuttle at the time of launch contains the propellants for takeoff. It holds three tanks, including the liquid hydrogen tank. If the ends of the liquid hydrogen tank are hemispheres, find the volume of this tank to the nearest hundredth.

Liquid Hydrogen Tank





Cylinder:

$A_b = \pi(4.2)^2 = 55.42 \text{ m}^2$

$h = 29.6 - 8.4 = 21.2 \text{ m}$

$V = 55.42 \times 21.2 = 1174.90 \text{ m}^3$

Sphere

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(4.2)^3 = 310.34 \text{ m}^3$

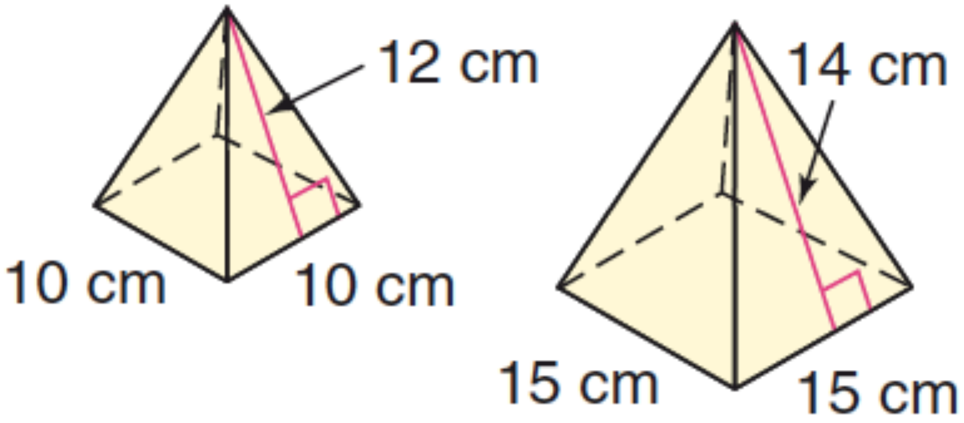
12.8 – CONGRUENT AND SIMILAR SOLIDS

SIMILAR SOLIDS

Just like similar figures, **similar solids** have the same shape, but not the same size. All their measures are proportional.

Determine whether each pair of solids is similar.

1

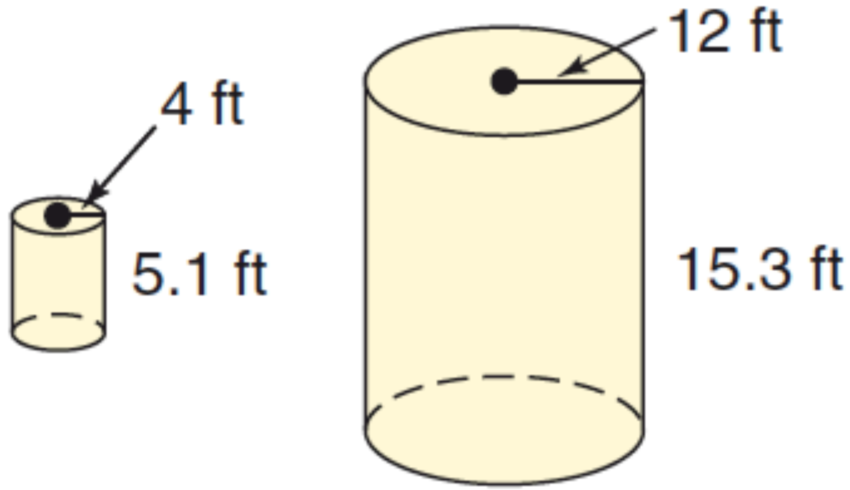


$\frac{10}{15}$ $\frac{12}{14}$

180 ≠ 140

Not similar

2



$\frac{4}{12}$ $\frac{5.1}{15.3}$

61.2 = 61.2

similar.

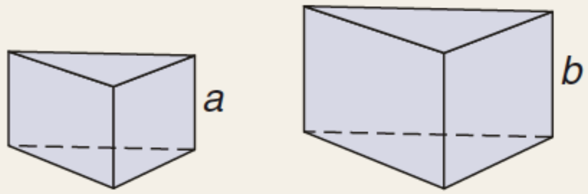
SCALE FACTOR RELATIONSHIPS

In similar solids, the areas and volumes are also proportional, but their scale factors are squares for area and cubed for volume.

Theorem 12–15

Words: If two solids are similar with a scale factor of $a:b$, then the surface areas have a ratio of $a^2:b^2$ and the volumes have a ratio of $a^3:b^3$.

Model:



Solid A Solid B

Symbols: scale factor of solid A to solid B $= \frac{a}{b} = k$

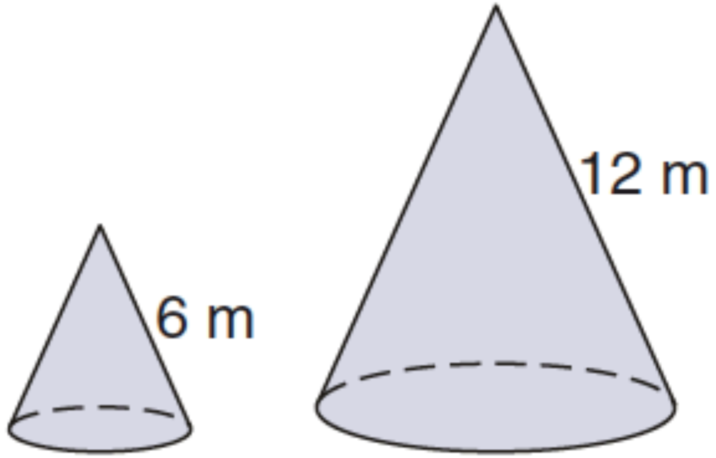
$$\frac{\text{surface area of solid A}}{\text{surface area of solid B}} = \frac{a^2}{b^2} = k^2$$

$$\frac{\text{volume of solid A}}{\text{volume of solid B}} = \frac{a^3}{b^3} = k^3$$

→ go back to original ratio

For each pair of similar solids, find the scale factor of the solid on the left to the solid on the right. Then find the ratios of the surface areas and the volumes.

c.

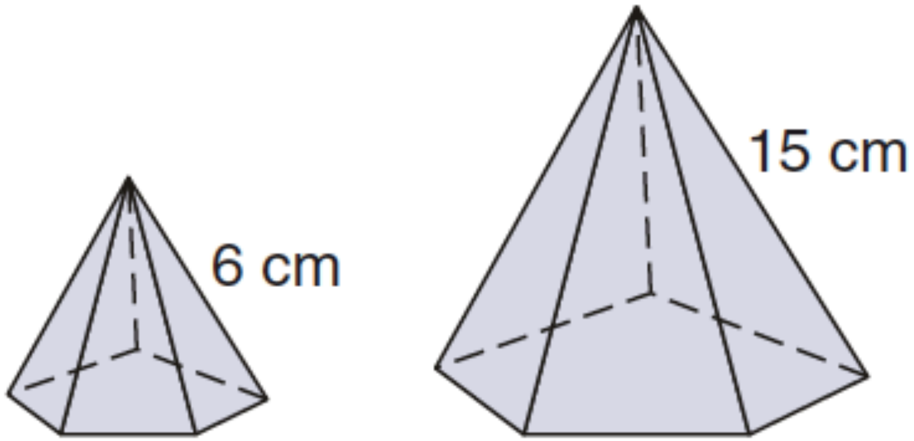


$k = \frac{6}{12} = \frac{1}{2}$

$k^2 = \frac{1}{4}$

$k^3 = \frac{1}{8}$

d.



$k = \frac{6}{15} = \frac{2}{5}$

$k^2 = \frac{4}{25}$

$k^3 = \frac{8}{125}$