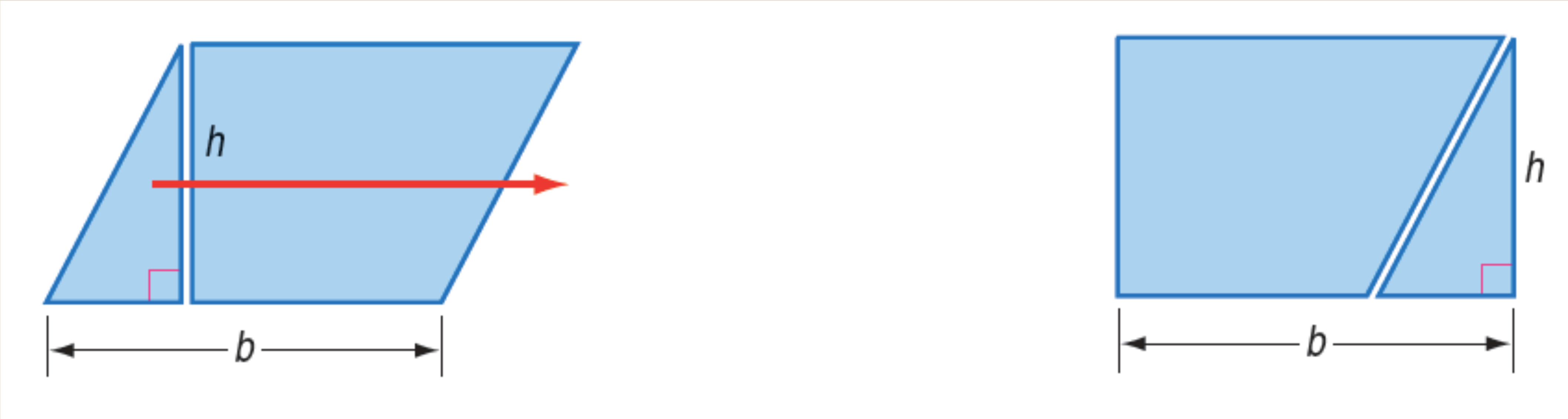
CHAPTER 11: AREAS OF POLYGONS AND CIRCLES

11.1 – AREAS OF PARALLELOGRAMS AND TRIANGLES

Parallelogram



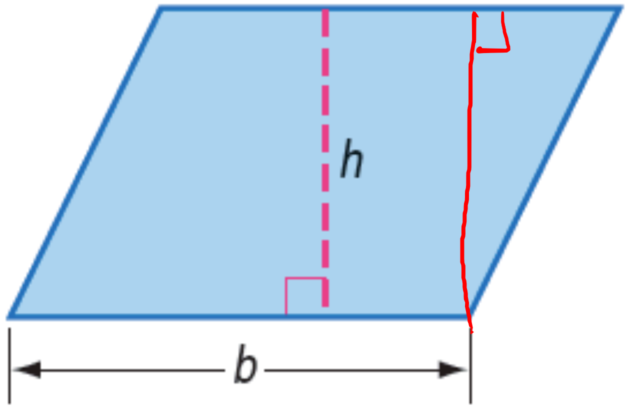
https://www.geogebra.org/m/VCUCx4jh

Parallelogram

KeyConcept Area of a Parallelogram

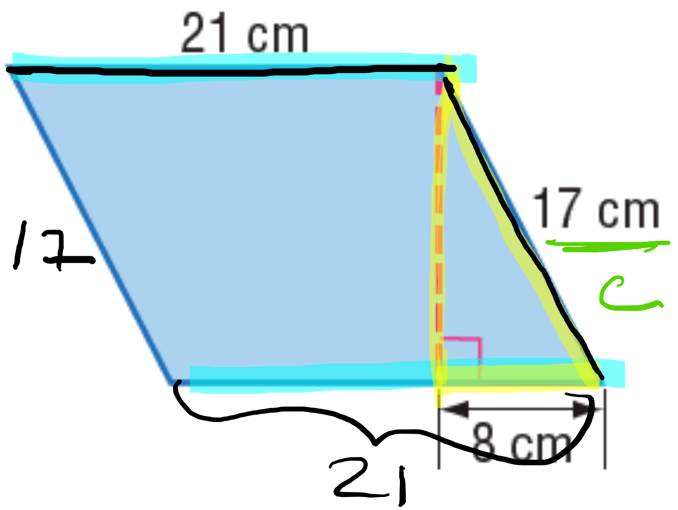
Words The area A of a parallelogram is the product of a base b and its corresponding height h.

Symbols $A = bh$



Find the perimeter and area of each parallelogram.

1A.



$P = 2 \times 21 + 2 \times 17 = 76$ cm

$A = b \times h$

$b = 21$

$h = 25$

$A = 21 \times 25$

$A = 315 \text{ cm}^2$

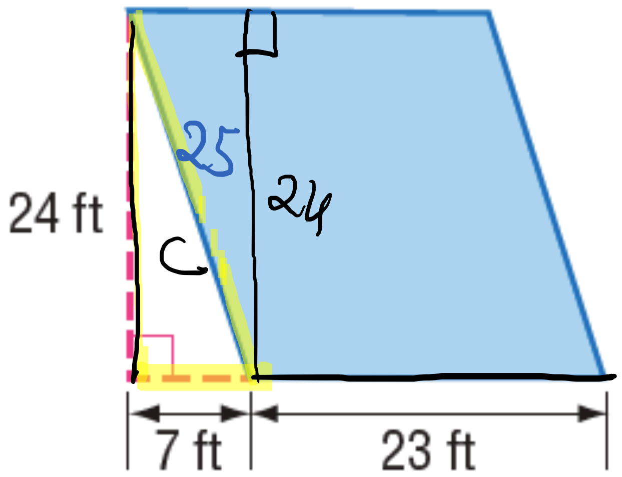
289 64

$17^2 = 8^2 + h^2$

$225 = h^2$

$225 = h$

1B.



$A = 23 \times 24$

$A = 552 \text{ ft}^2$

$b = 23$

$h = 24$

$c^2 = 24^2 + 7^2$

$c^2 = 625$

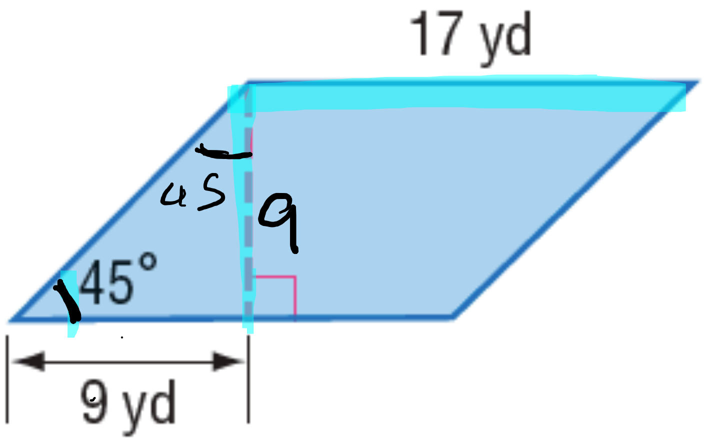
$c = 25$

$P = 2 \cdot 25 + 2 \cdot 23$

$P = 96$ ft

Find the area of each parallelogram. Round to the nearest tenth if necessary.

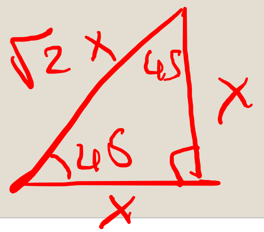
2A.



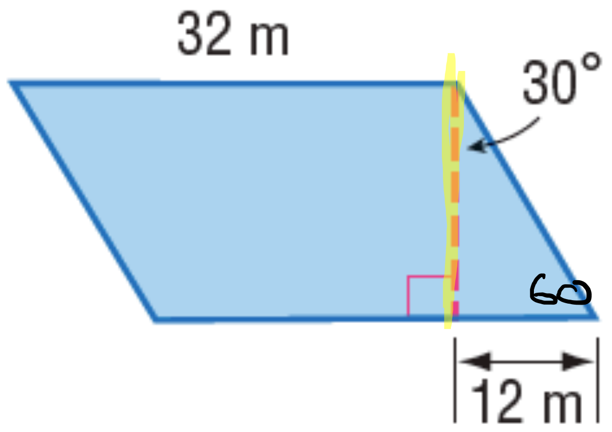
45	45	90
x	x	$\sqrt{2}x$
9	9	

$A = b \times h$

$A = 17 \times 9 = 153\,yd^2$



2B.



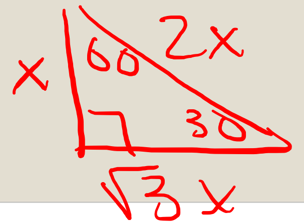
30	60	90
x	$\sqrt{3}x$	$2x$
12	$12\sqrt{3}$	

$b = 32$

$h = 12\sqrt{3}$

$A = 32 \cdot 12\sqrt{3}$

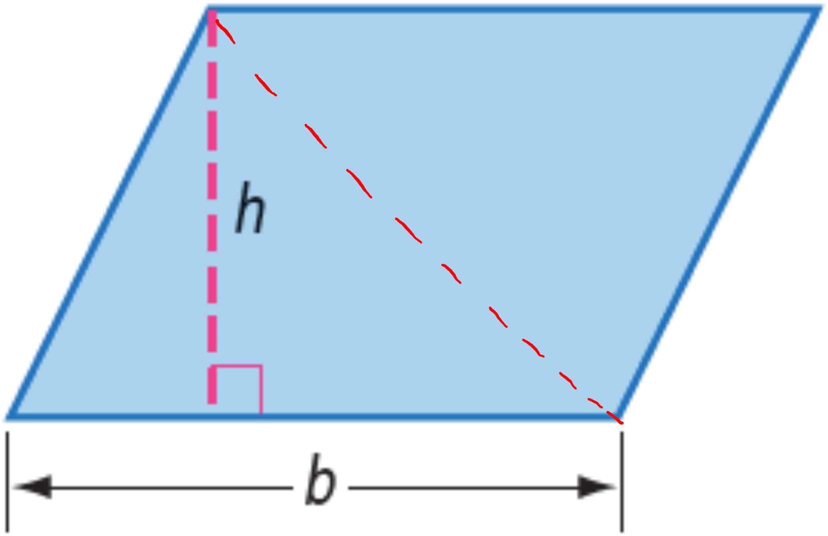
$A = 665.1\,m^2$

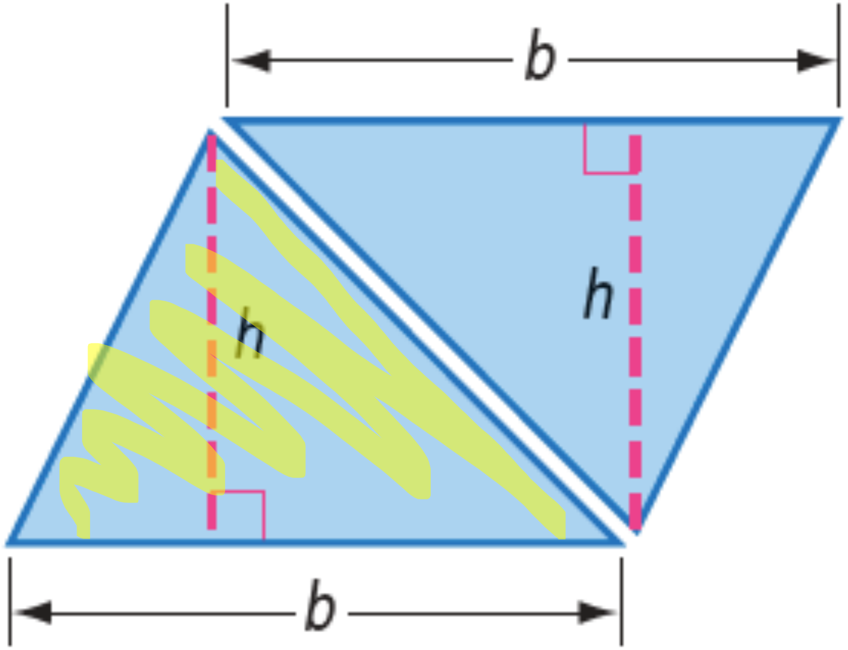


Triangles

$A = \frac{b \times h}{2}$

$A = b \times h$



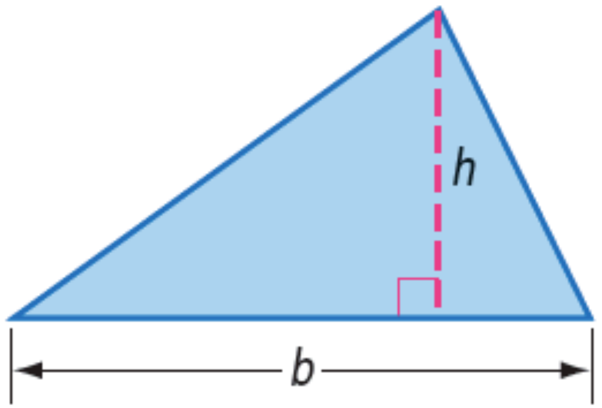


Triangles

KeyConcept Area of a Triangle

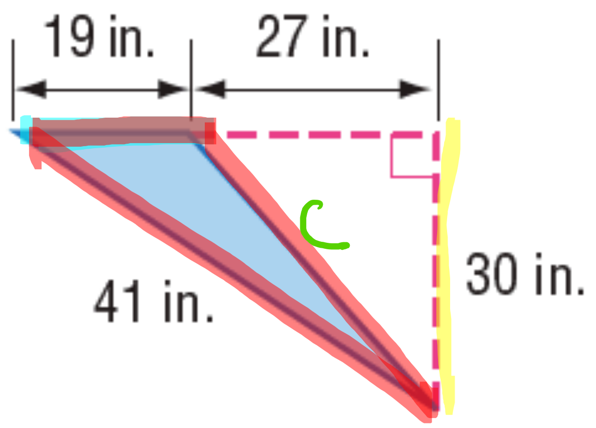
Words The area A of a triangle is one half the product of a base b and its corresponding height h.

Symbols $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$



Find the perimeter and area of each triangle.

3A.



$b = 19$

$h = 30$

$$A = \frac{b \times h}{2} = \frac{19 \times 30}{2} = 285\text{in}^2$$

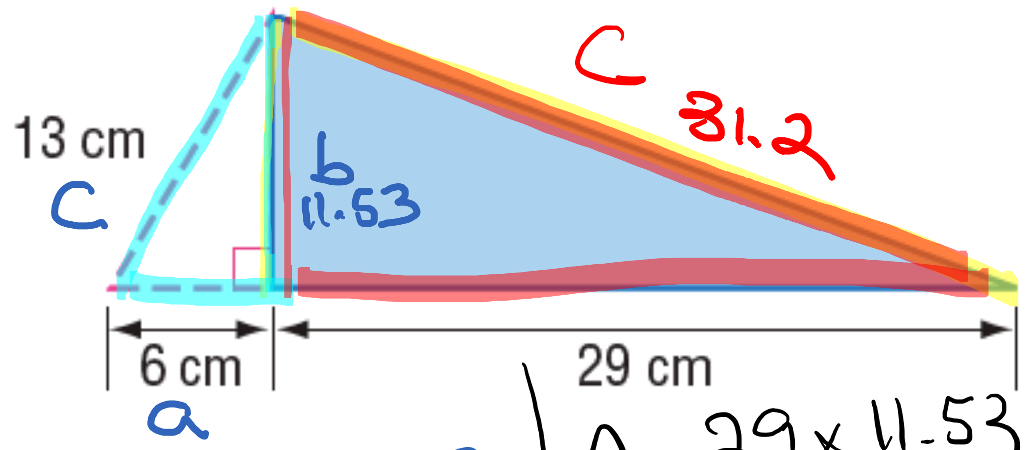
$c^2 = 27^2 + 30^2$

$c^2 = 1629$

$c = 40.36$

$P = 41 + 19 + 40.36 = 100.36\text{in}$

3B.



1. $13^2 = b^2 + 6^2$

   $169 = b^2 + 36$

   $133 = b^2$

   $11.53 = b$

2. $c^2 = 29^2 + 11.53^2$

   $c^2 = 974$

   $c = 31.2$

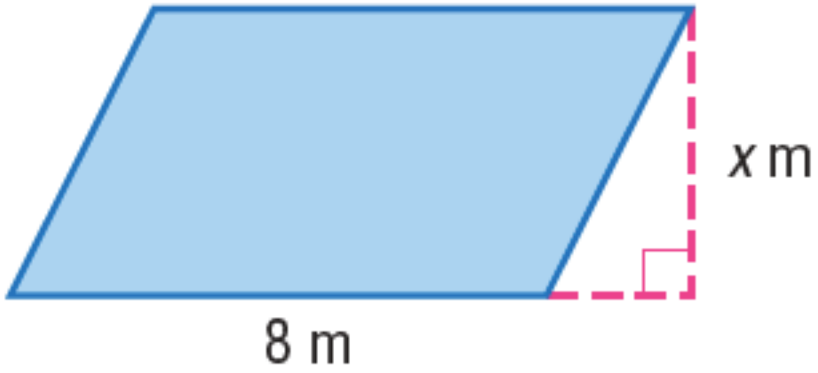
$$A = \frac{29 \times 11.53}{2}$$

$A = 167.19\text{cm}^2$

$P = 31.2 + 29 + 11.53$

$P = 71.73\text{cm}$

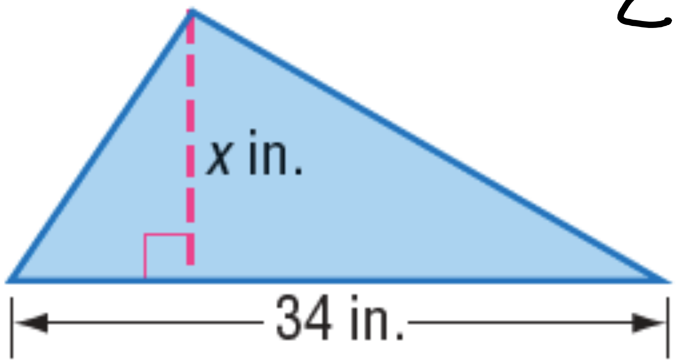
4A. $A = 148\ \text{m}^2$ $A = b \times h$



$$\frac{148}{8} = \frac{8 \cdot x}{8}$$

$18.5\text{m} = x$

4B. $A = 357\ \text{in}^2$ $A = \frac{b \cdot h}{2}$

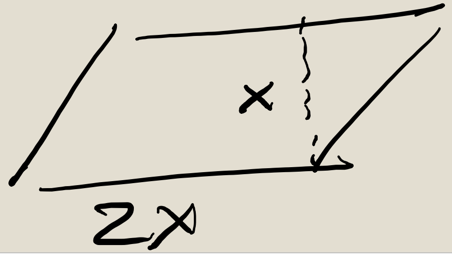


$$357 = \frac{34 \cdot x}{2}$$

$357 = 17x$

$21\text{in} = x$

4C. **ALGEBRA** The base of a parallelogram is twice its height. If the area of the parallelogram is 72 square feet, find its base and height.



$A = b \cdot h$

$72 = 2x \cdot x$

$72 = 2x^2$

$36 = x^2$

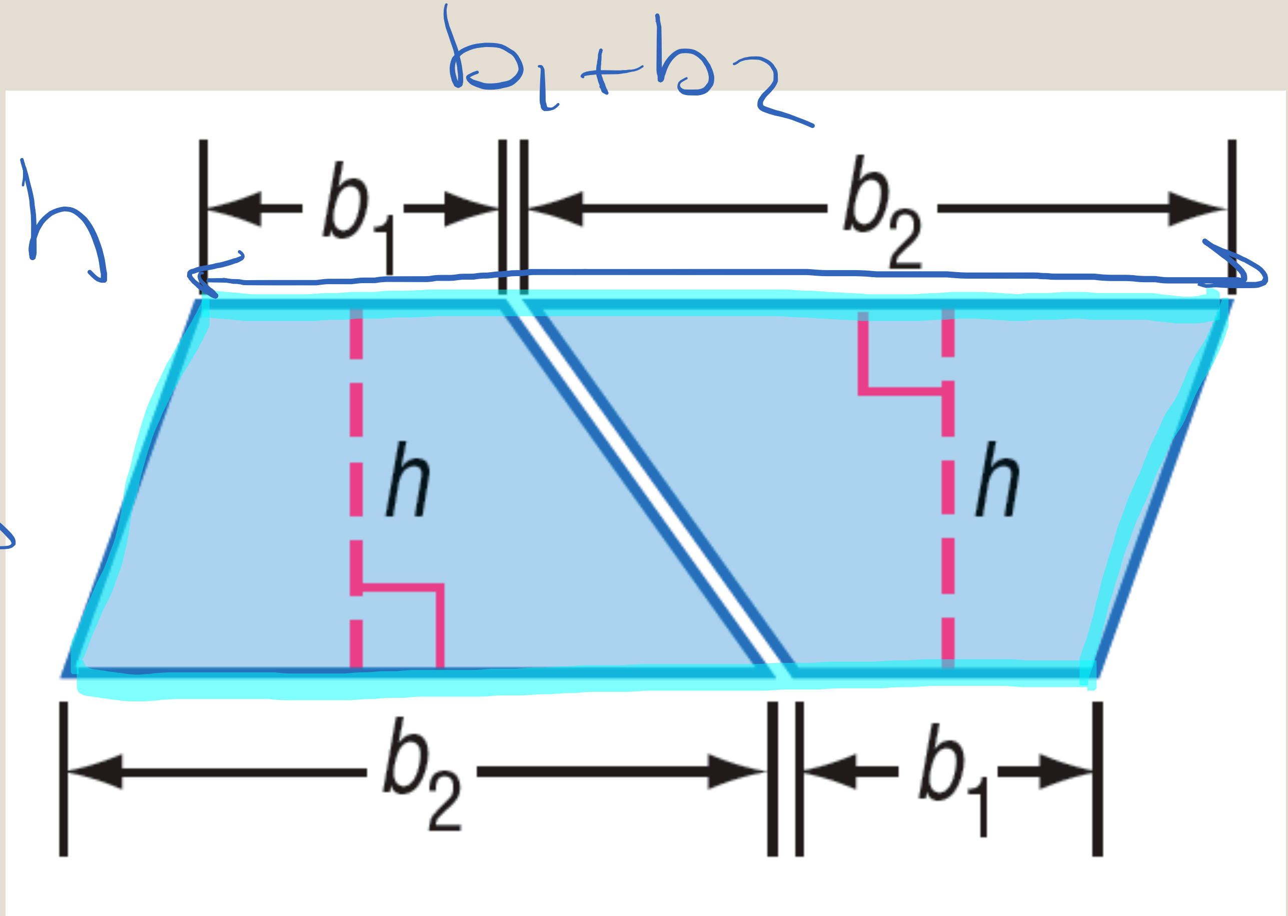
$6\text{ft} = x$

11.2 – AREAS OF TRAPEZOIDS, RHOMBI AND KITES

Trapezoid

$$A = (b_1 + b_2) \cdot h$$

$$A_t = \frac{(b_1 + b_2) \cdot h}{2}$$



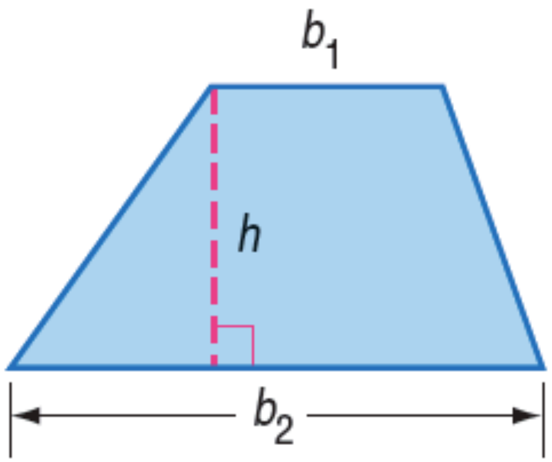
https://www.geogebra.org/m/T8ZfVMFK

Trapezoid

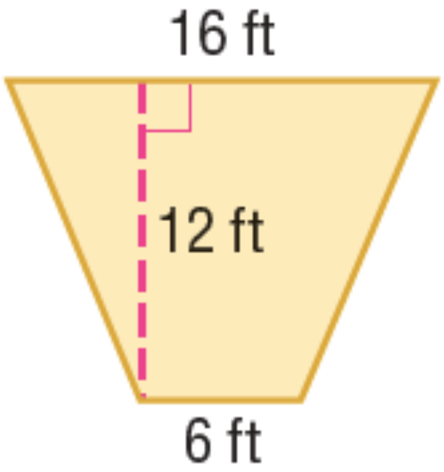
KeyConcept Area of a Trapezoid

Words The area A of a trapezoid is one half the product of the height h and the sum of its bases, b_1 and b_2.

Symbols $A = \frac{1}{2}h(b_1 + b_2) = \frac{b_1 + b_2}{2} \cdot h$



Find the area of each trapezoid



$$A = \frac{b_1 + b_2}{2} \cdot h$$

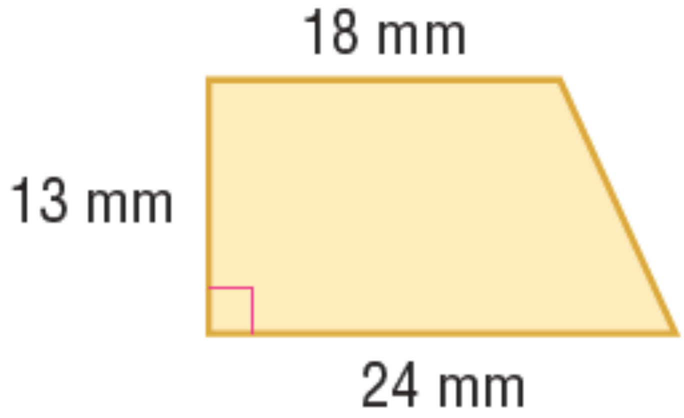
$b_1 = 16$

$b_2 = 6$

$h = 12$

$$A = \frac{16 + 6}{2} \cdot 12$$

$$A = 132\ ft^2$$



$b_1 = 24$

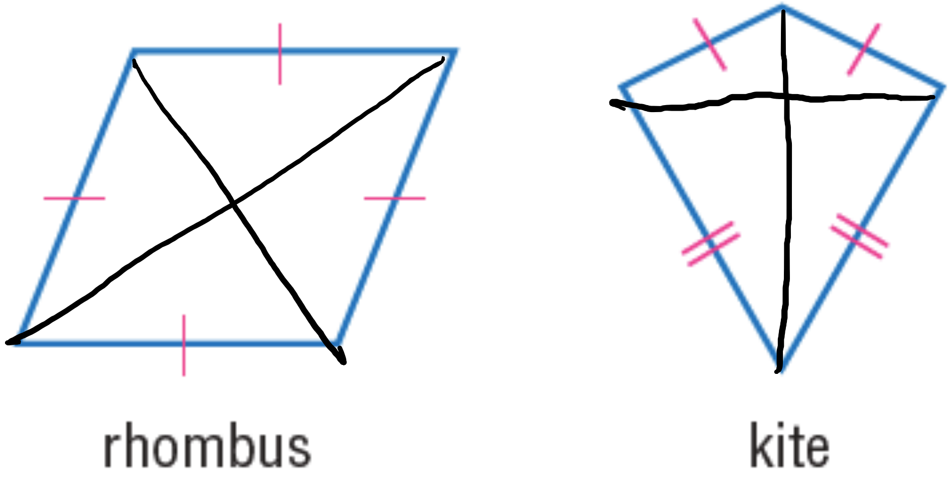
$b_2 = 18$

$h = 13$

$$A = \frac{24 + 18}{2} \cdot 13$$

$$A = 273\ m^2$$

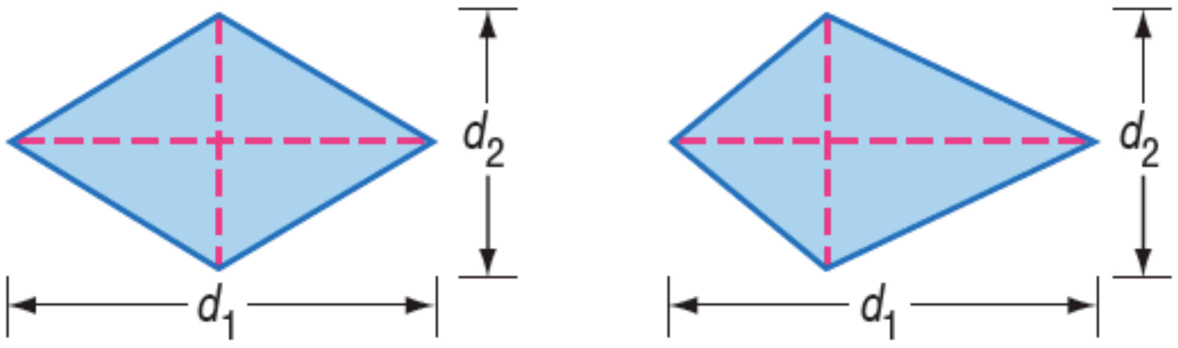
Rhombus and Kite



KeyConcept **Area of a Rhombus or Kite**

Words The area A of a rhombus or kite is one half the product of the lengths of its diagonals, d_1 and d_2.

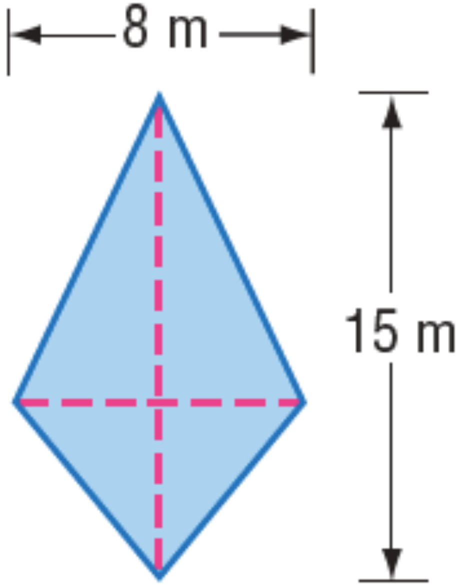
Symbols $A = \frac{1}{2}d_1 d_2 = \frac{d_1 \cdot d_2}{2}$



Find the area of each rhombus or kite.

$$A = \frac{d_1 \cdot d_2}{2}$$

a.

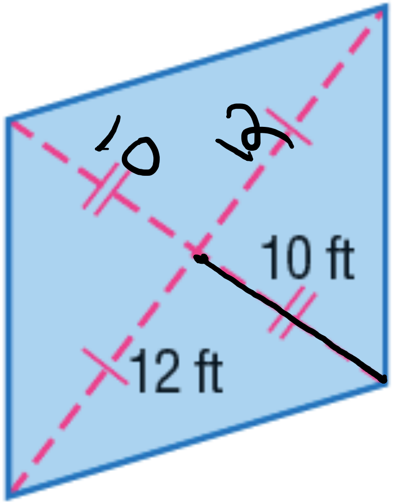


$d_1 = 8$

$d_2 = 15$

$$A = \frac{8 \cdot 15}{2} = 60\ m^2$$

b.



$d_1 = 20\ ft$

$d_2 = 24\ ft$

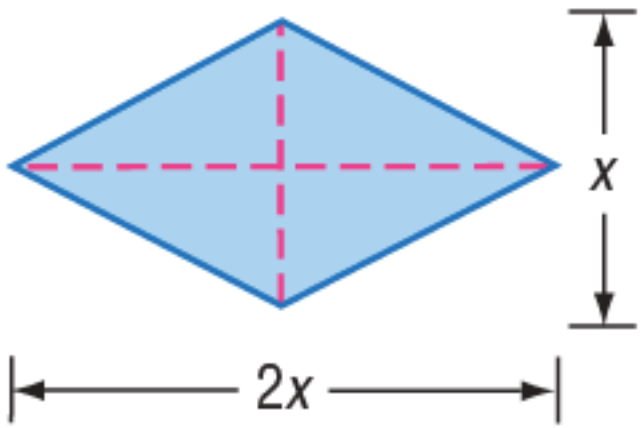
$$A = \frac{20 \cdot 24}{2} = 240\ ft^2$$

Solving for unknowns

ALGEBRA One diagonal of a rhombus is twice as long as the other diagonal. If the area of the rhombus is 169 square millimeters, what are the lengths of the diagonals?

$A = \frac{d_1 d_2}{2}$

$A = 169\ mm^2$



$169 = \frac{2x \cdot x}{2}$

$169 = x^2$

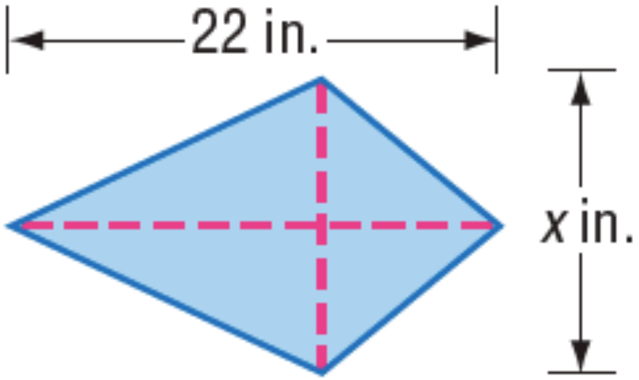
$13 = x$ mm

$2x = 26$ mm

ALGEBRA Find x.

$A = \frac{d_1 d_2}{2}$

4A. $A = 92 \text{ in}^2$



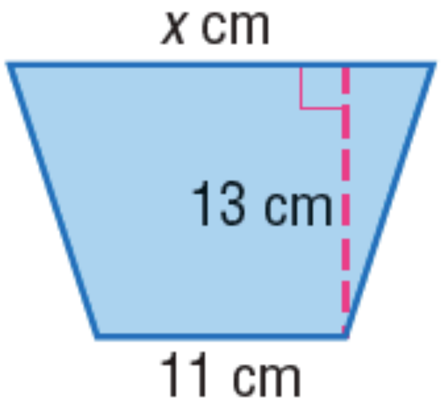
$92 = \frac{22x}{2}$

$92 = 11x$

$8.36 \text{ in} = x$

$A = \frac{b_1 + b_2}{2} \cdot h$

4B. $A = 177 \text{ cm}^2$



$177 = \frac{11 + x}{2} \cdot 13$

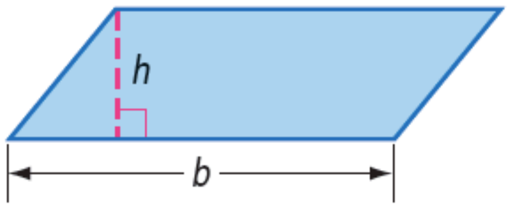
$354 = (11 + x) \cdot 13$

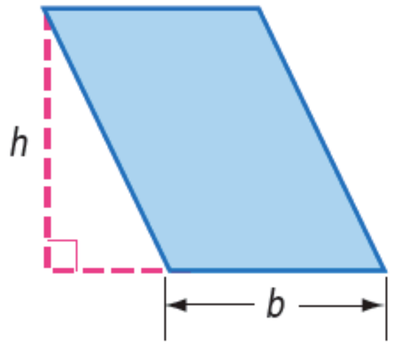
$27.23 = 11 + x$

$16.23 \text{ cm} = x$

ConceptSummary Areas of Polygons

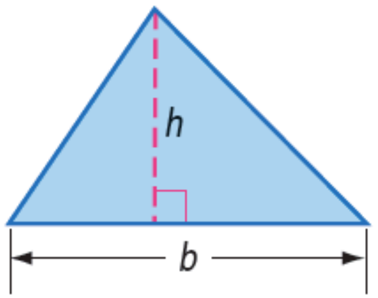
Parallelogram

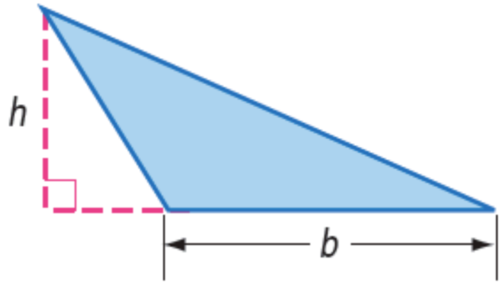




$A = bh$

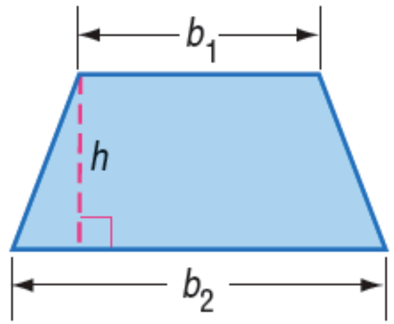
Triangles

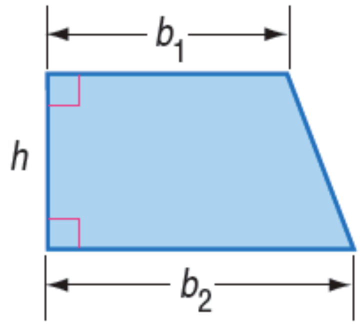




$A = \frac{1}{2}bh$

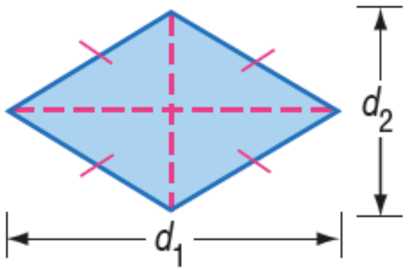
Trapezoids

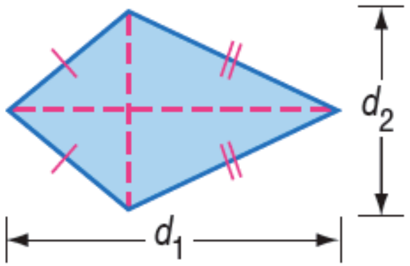




$A = \frac{1}{2}h(b_1 + b_2)$

Rhombi and Kites





$A = \frac{1}{2}d_1 d_2$

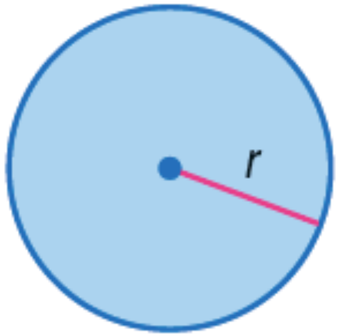
11.3 – AREAS OF CIRCLES AND SECTORS

$C = 2\pi r = \pi d$

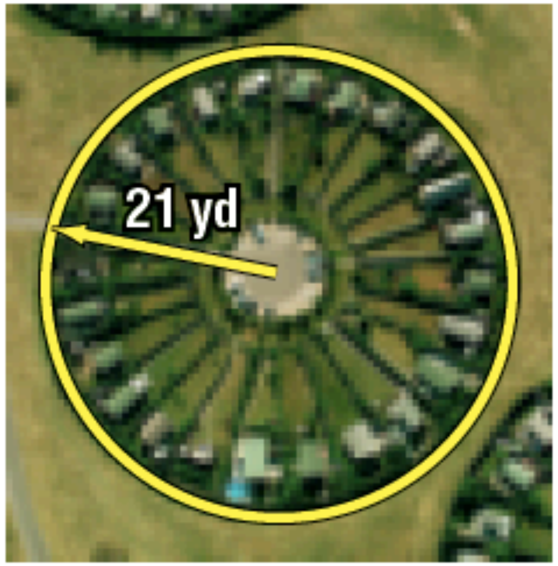
KeyConcept Area of a Circle

Words The area A of a circle is equal to π times the square of the radius r.

Symbols $\boxed{A = \pi r^2}$



CONSTRUCTION Find the area of each circle. Round to the nearest tenth.

1. 

2. 

$r = 21$

$A = \pi (21)^2 = 1385.44 \text{ yd}^2$

$d = 0.4 \text{ km}$

$r = 0.2 \text{ km}$

$A = \pi (0.2)^2 = 0.13 \text{ km}^2$

Finding missing measures

ALGEBRA Find the radius of a circle with an area of 95 square centimeters.

$A = \pi r^2$

$95 = \pi r^2$

$\frac{95}{\pi} = \frac{\pi r^2}{\pi}$

$30.24 = r^2$

$r = 5.5$ cm

- divide by π
- take the $\sqrt{\ }$

ALGEBRA The area of a circle is 196π square yards. Find the diameter.

$A = \pi r^2$

$196\cancel{\pi} = \cancel{\pi} r^2$

$196 = r^2$

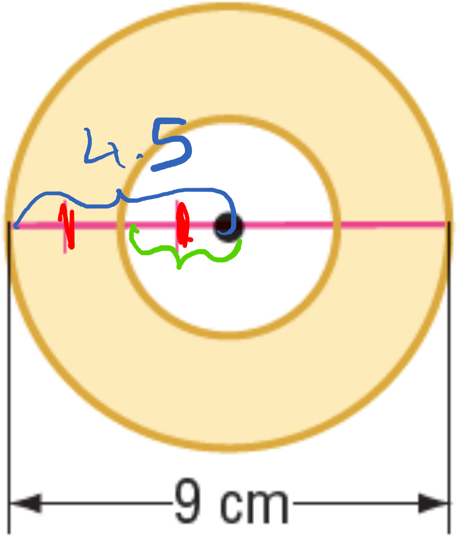
$14 = r$

$d = 2 \times r = 2 \times 14 = 28$ yd

Find the area of each shaded region.

$A = \pi r^2$

Plan: Area of big circle − area of small circle.



diameter

Area of big circle:

$r = 9 \div 2 = 4.5$

$A = \pi(4.5)^2 = 63.62 \text{ cm}^2$

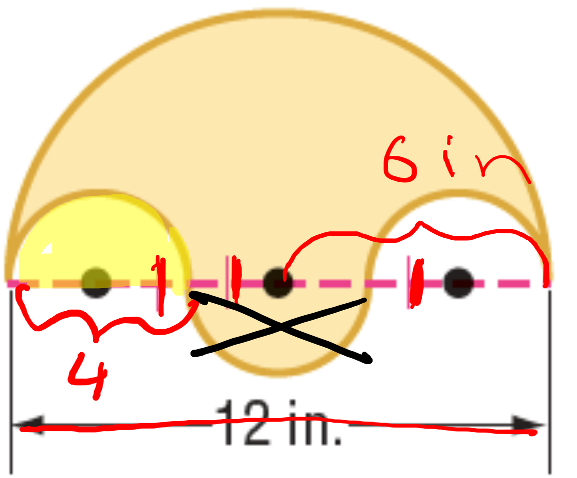
Area of small circle:

$r = 4.5 \div 2 = 2.25 \text{ cm}$

$A = \pi(2.25)^2 = 15.9 \text{ cm}^2$

Area of shaded $= 63.62 - 15.9 = 47.72 \text{ cm}^2$

Find the area of each shaded region.



radius of big circle = $12 \div 2 = 6$ in

diameter of 6 small circle = $12 \div 3 = 4$ in

radius of 6 small circle = $4 \div 2 = 2$ in

Area of big semicircle − area of one small semicircle.

Big ~~circle~~ semi:

$$A = \frac{\pi (6)^2}{2} = 113.1 \div 2 = 56.55 \text{ in}^2$$

Small semi circle:

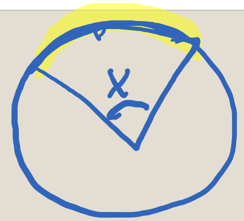
$$A = \frac{\pi (2)^2}{2} = 12.57 \div 2 = 6.29 \text{ in}^2$$

Actual shape:

$56.55 - 6.29$

$= 50.26 \text{ in}^2$

Area of sectors



$$\frac{\text{length of arc}}{\text{circumference}} = \frac{\text{degrees}}{360}$$

KeyConcept Area of a Sector

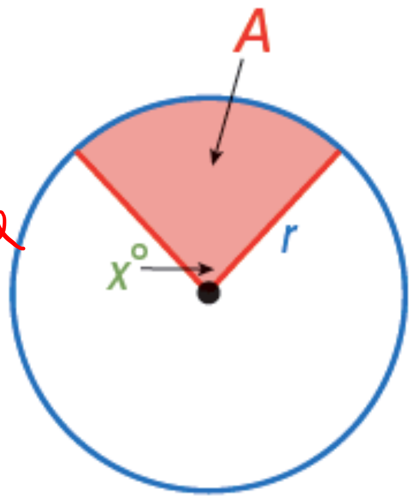
The ratio of the **area A of a sector** to the **area of the whole circle, πr^2**, is equal to the ratio of the **degree measure of the intercepted arc x** to 360.

Proportion: $\dfrac{A}{\pi r^2} = \dfrac{x}{360}$

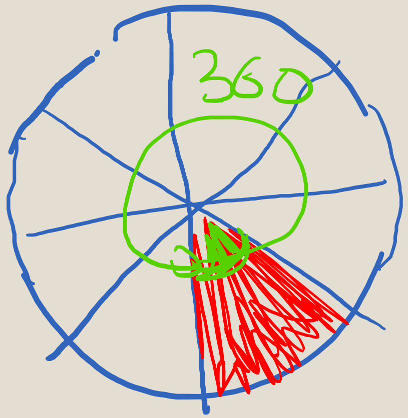
degrees

area of the circle

Equation: $A = \dfrac{x}{360} \cdot \pi r^2$



PIZZA A circular pizza has a diameter of 12 inches and is cut into 8 congruent slices. What is the area of one slice to the nearest hundredth?



d = 12 in

r = 6 in

central angle = 360 ÷ 8 = 45°

$$\frac{A}{\pi(6)^2} = \frac{45}{360}$$

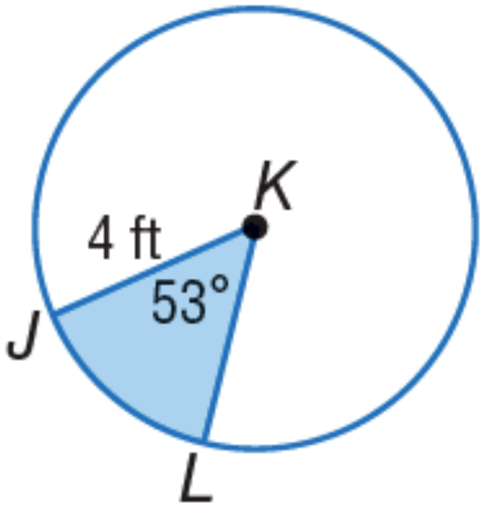
$$360A = 45\pi(6)^2$$

$$A = \frac{45\pi(6)^2}{360}$$

$$A = 14.14 \text{ in}^2$$

Find the area of the shaded sector. Round to the nearest tenth.

3A.



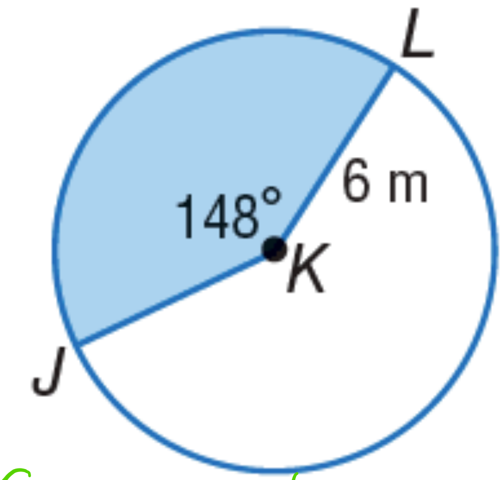
r = 4

angle = 53

$$\frac{A}{\pi(4)^2} = \frac{53}{360}$$

$$A = \frac{53 \cdot \pi(4)^2}{360} = 7.4\ ft^2$$

3B.



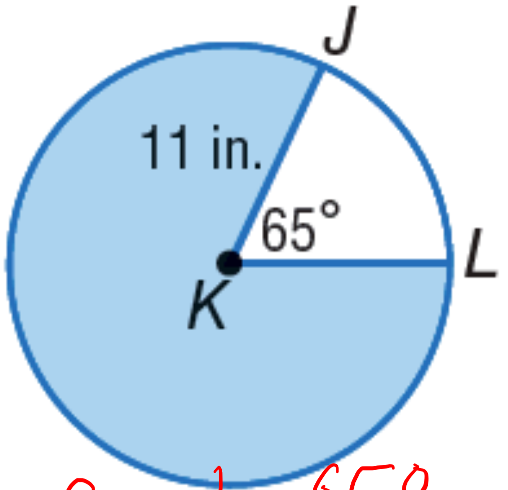
r = 6 angle = 148

$$\frac{A}{\pi(6)^2} = \frac{148}{360}$$

$$A = \frac{148 \times \pi(6)^2}{360}$$

$$A = 46.5\ m^2$$

3C.



r = 11 angle = 65°

$$\frac{A}{\pi(11)^2} = \frac{65}{360}$$

$$A = \frac{65 \cdot \pi(11)^2}{360}$$

A = 1.06 in2

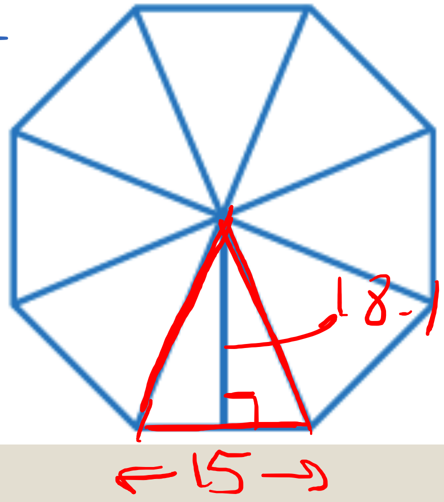
11.4 – AREAS OF REGULAR POLYGONS AND COMPOSITE FIGURES

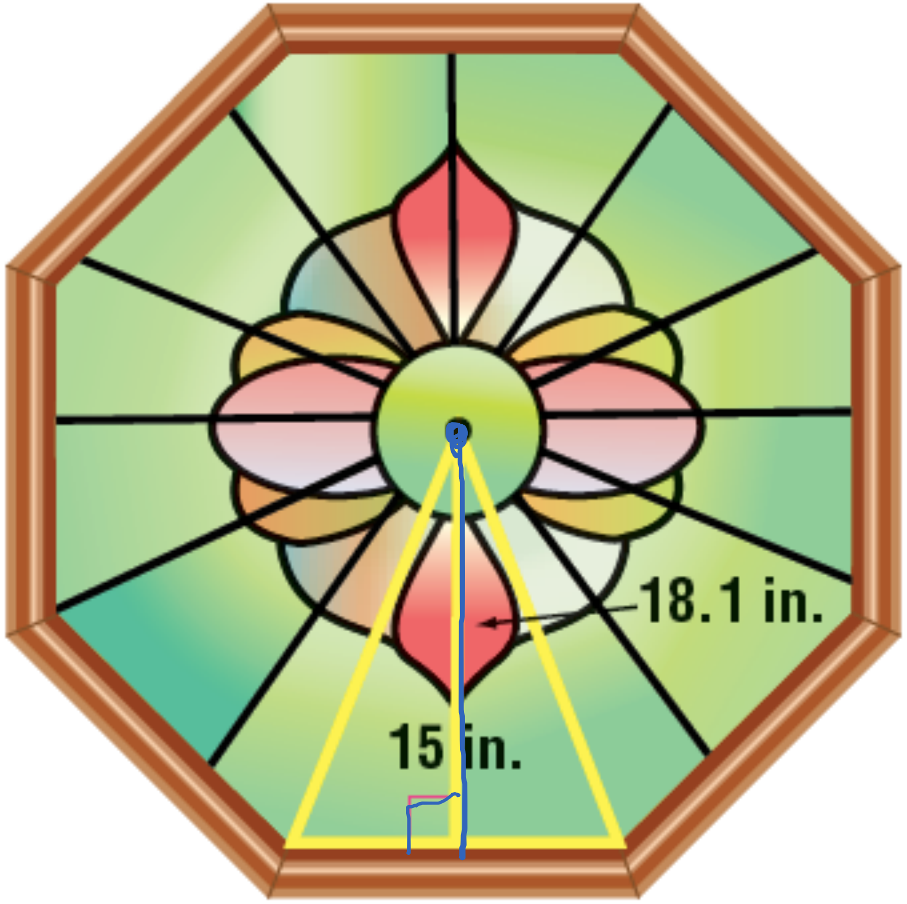
ART Kang created the stained glass window shown. The window is a regular octagon with a side length of 15 inches and an apothem of 18.1 inches. What is the area covered by the window?

same length 8 sides

1) Calculate the area of 1 triangle.

2) Multiply by 8.





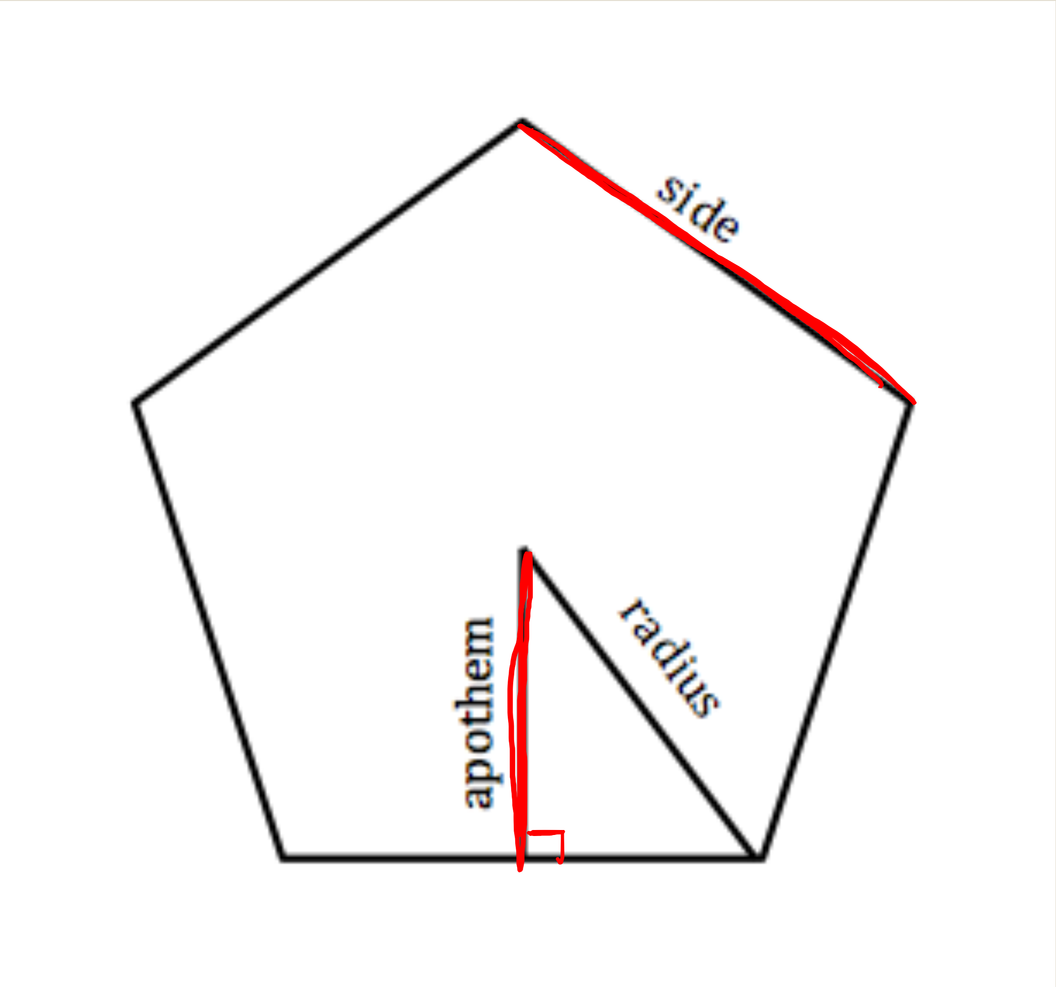
Area of triangle

$$A = \frac{15 \times 18.1}{2} = 135.75 \text{ in}^2$$

Area of octagon

$$135.75 \times 8 = 1086 \text{ in}^2$$

Parts of a polygon



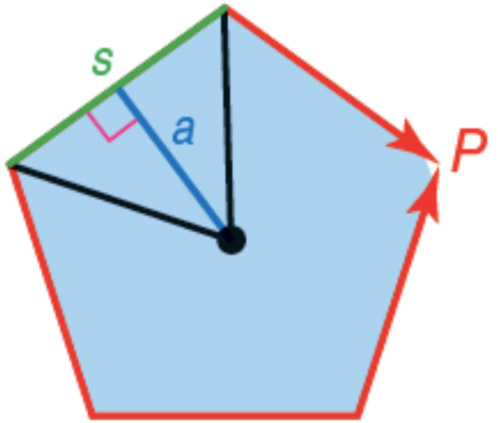
Area of a regular polygon

$\Delta: \frac{\text{base} \times \text{height}}{2} \cdot .5$

KeyConcept Area of a Regular Polygon

Words The area A of a regular n-gon with side length s is one half the product of the apothem a and perimeter P.

Symbols $A = \frac{1}{2}a(ns)$ or $A = \frac{1}{2}aP$. $\rightarrow A = \frac{a \cdot P}{2}$



$A = \frac{n \cdot s \cdot a}{2}$

# of triangle

area of each triangle.

S = side length → base of triangle

a = apothem → height of triangle

n = # of sides → # of triangles.

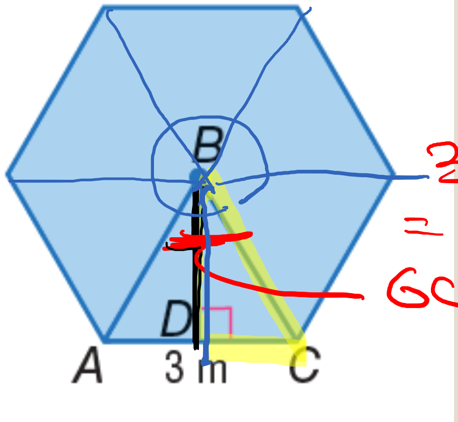
Find the area of each regular polygon. Round to the nearest tenth.

$$A = \frac{n \cdot s \cdot a}{2}$$

a. regular hexagon → $n = 6$

$s = 3$

$a = 2.6$



$360° \div 6 = 60°$

$60°$

30	60	90
x	$\sqrt{3}x$	$2x$
1.5	2.6	3

$\sqrt{3} \cdot 1.5$

$$A = \frac{6 \times 3 \times 2.6}{2}$$

$A = 23.4\,m^2$

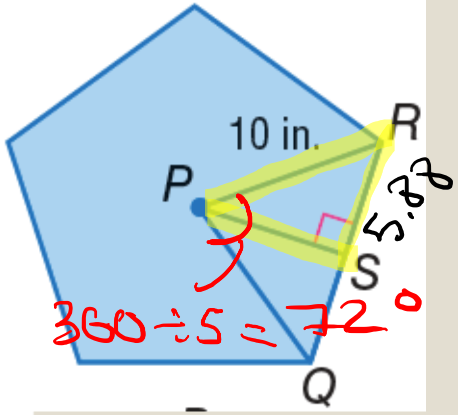
b. regular pentagon

$n = 5$

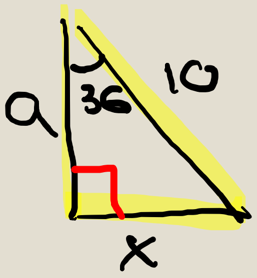
$s = 2 \times 5.88 = 11.76$ in

$a = 8.1$ in

$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$



$360 \div 5 = 72°$



$\sin 36 = \frac{x}{10}$

$x = 10 \sin 36 \approx 5.88$

$\cos 36 = \frac{a}{10}$

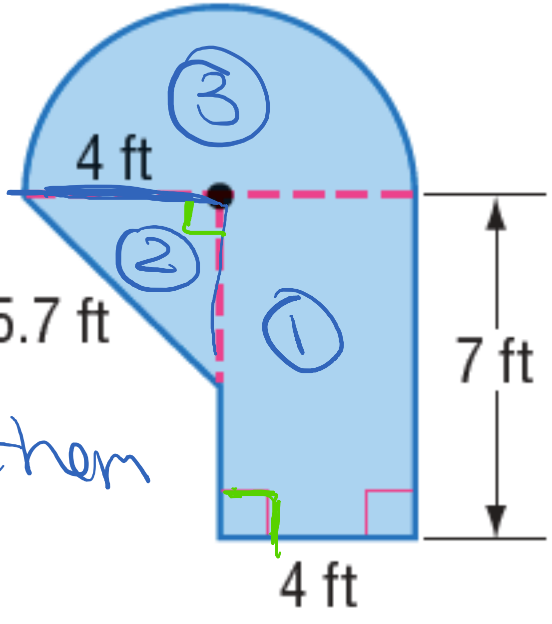
$a = 10 \cdot \cos 36$

$a = 8.1$

$$A = \frac{5 \times 11.76 \times 8.1}{2} = 238.6\ in^2$$

Area of composite figures

MINIATURE GOLF **The dimensions of a putting green at a miniature golf course are shown. How many square feet of carpet are needed to cover this green?**



Plan: calculate all 3 areas and add them

Area 1

$A = b \times h$

$A = 4 \times 7 = 28\,ft^2$

Area 3

$A = \frac{\pi r^2}{2} = \frac{\pi (4)^2}{2} = 25.13\,ft^2$

Area 2

$A = \frac{b \times h}{2}$

$5.7^2 = 4^2 + h^2$

$h^2 = 16.49$

$h = 3.5\,ft$

$A = \frac{4 \times 3.5}{2}$

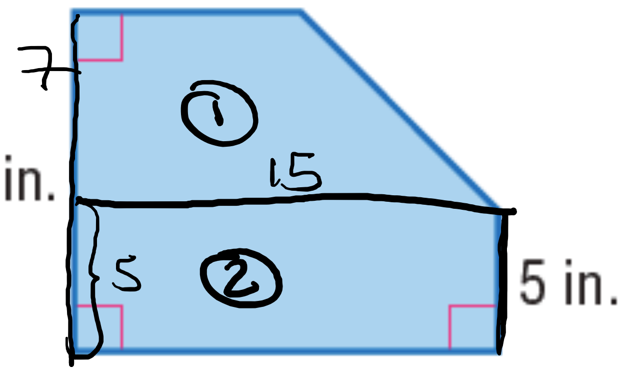
$A = 7\,ft^2$

Total area:

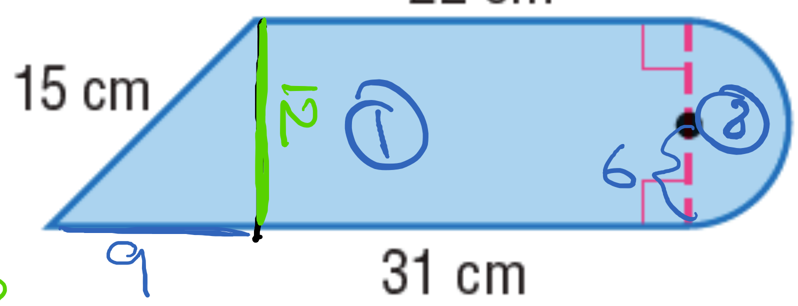
$28 + 7 + 25.13 = 60.13\,ft^2$

Find the area of each figure. Round to the nearest tenth if necessary.

4A.



4B.



Area① $A = \frac{B+b}{2} \cdot h = \frac{15+8}{2} \cdot 7$

$A = 80.5 \text{ in}^2$

Area② $A = l \times w = 15 \times 5 = 75 \text{in}^2$

Total $80.5 + 75 = 155.5 \text{in}^2$

$15^2 - 9^2 = h^2$

$h^2 = 144$

$h = 12$

Area① (Trapezoid)

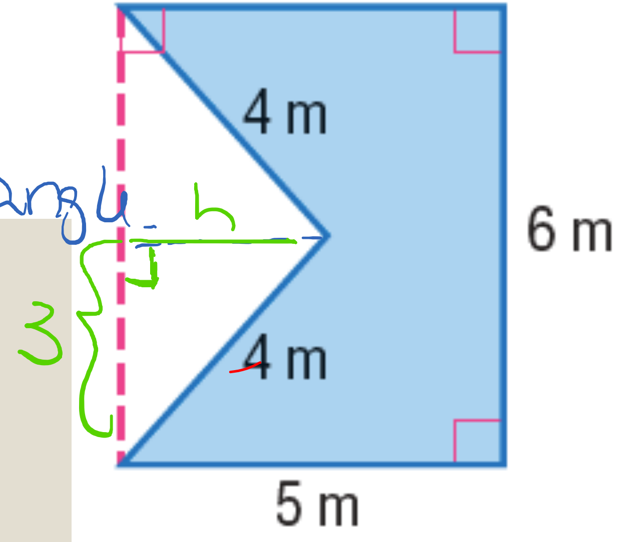
$A = \frac{31+22}{2} \cdot 12 = 318 \text{ cm}^2$

Area②

$A = \frac{\pi(6)^2}{2} = 56.55 \text{ cm}^2$

Total $318 + 56.55 = 374.55 \text{ cm}^2$

Find the area of the figure. Round to the nearest tenth if necessary.



Plan: Area of rectangle – area of triangle

Rectangle:

$A = 5 \times 6 = 30\ m^2$

Triangle:

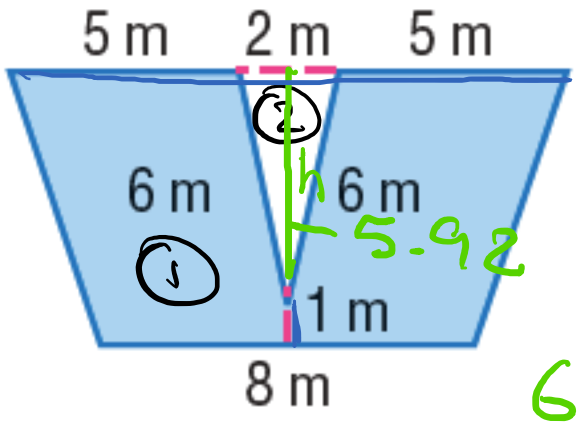
$h^2 = 4^2 - 3^2$

$h = 2.65$

$A = \frac{6 \times 2.65}{2} = 7.95$

Total $30 - 7.95 = 22.05\ m^2$

5A.



h | 6

1

$6^2 - 1^2 = h^2$

$h^2 = 35$

$h = 5.92$

Trapezoid

B = 12m

b = 8m

h = 6.92

$A = \frac{12+8}{2} \cdot 6.92$

$A = 69.2m^2$

Triangle

$A = \frac{2 \times 5.92}{2}$

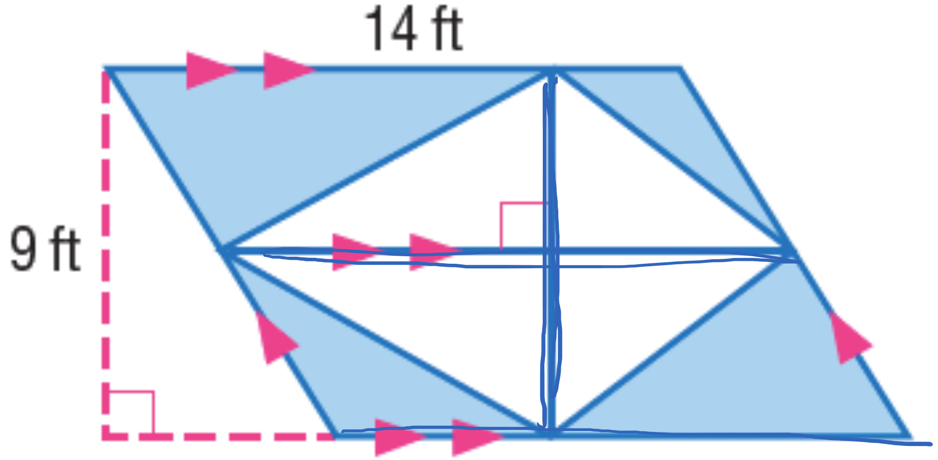
$A = 5.92m^2$

Total

69.2 − 5.92

$= 63.28 m^2$

5B.



Parallelogram

$A = b \times h$

$A = 14 \times 9 = 126 ft^2$

Kite

$A = \frac{d_1 \cdot d_2}{2}$

$A = \frac{9 \times 14}{2} = 63$

Total

126 − 63

$= 63 ft^2$

11.5 – AREAS OF SIMILAR FIGURES

Similar figures

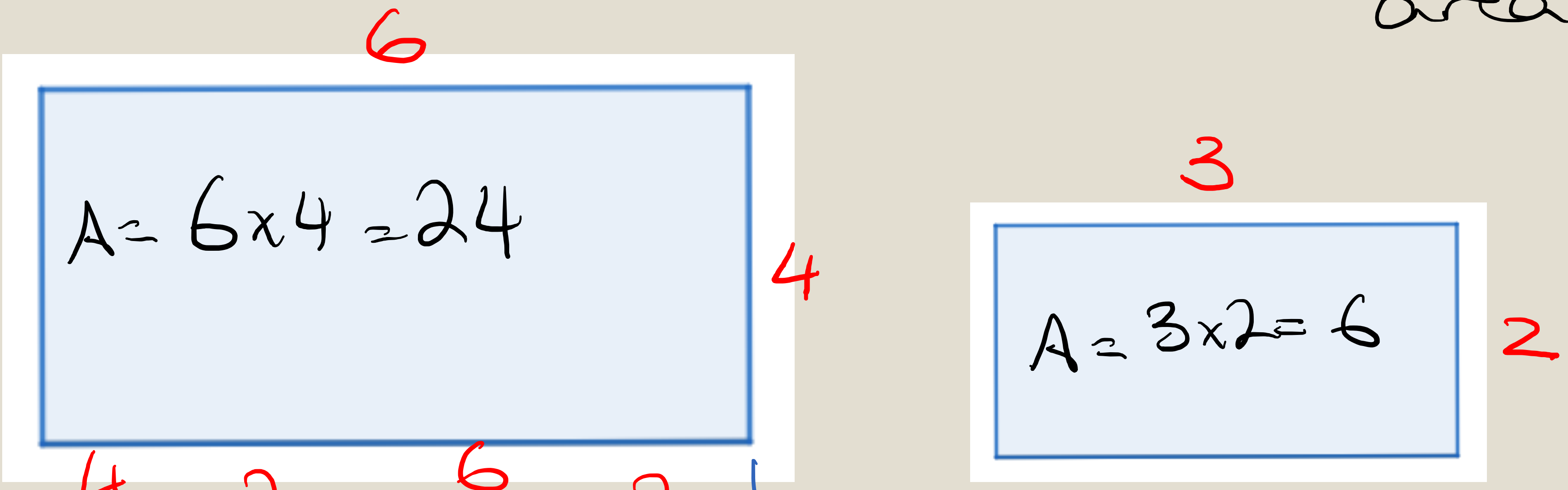
- Two figures are similar when their corresponding sides are proportional → the ratio is called the **scale factor.** → the number you get when you divide the length of corresponding sides in similar figures.

- The perimeter of these figures is also proportional.

Area of Similar figures

The areas of similar figures are **proportional to the square of the scale factor.** $k = 2$

scale factor of areas: $\frac{24}{6} = 4 = 2^2$



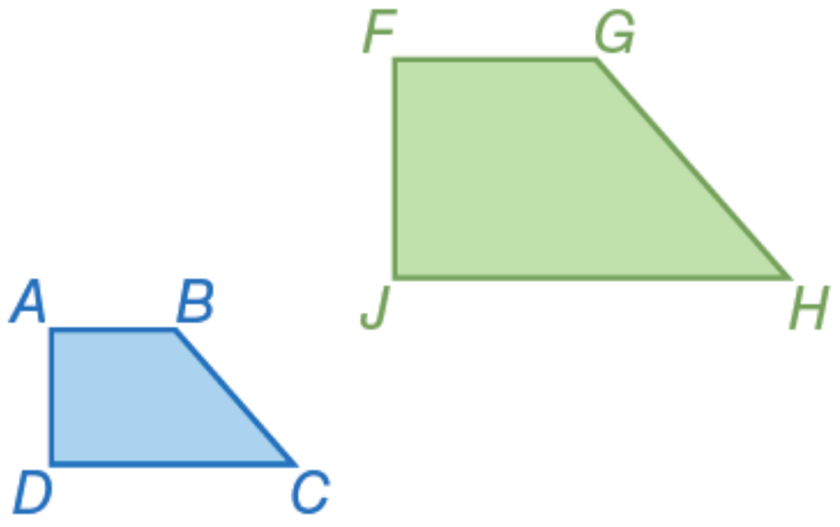
$\frac{\text{big}}{\text{small}}$ $\frac{4}{2} = 2$ $\frac{6}{3} = 2$

Theorem 11.1 Areas of Similar Polygons

Words If two polygons are similar, then their areas are proportional to the square of the scale factor between them.

Example If *ABCD* ~ *FGHJ*, then

$$\frac{\text{area of } FGHJ}{\text{area of } ABCD} = \left(\frac{FG}{AB}\right)^2.$$



If two figures are similar, the scale factor of their areas is the square of the scale factor of the lengths.

$k \to$ scale factor for length

$k^2 \to$ scale factor for area.

Finding the area from length measures.

If $\triangle JKL \sim \triangle PQR$ and the area of $\triangle JKL$ is 30 square inches, find the area of $\triangle PQR$.

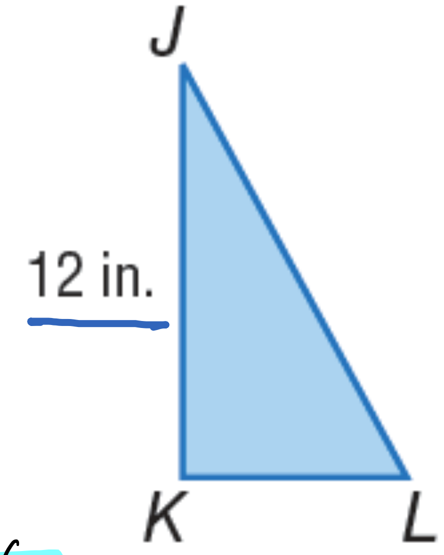
- length scale factor: $k = \frac{12}{15} = \frac{4}{5}$
- area scale factor: $k^2 = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$

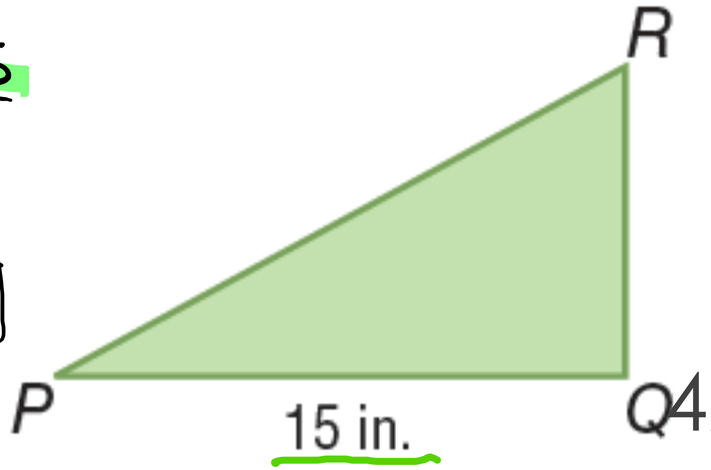
$$\frac{16}{25} = \frac{30}{x}$$

$$16x = 30 \times 25$$

$$16x = 750$$

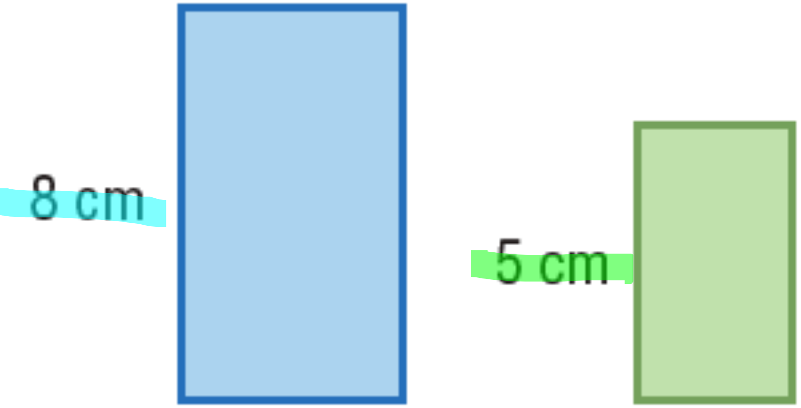
$$\boxed{x = 46.875 \text{ in}^2}$$





For each pair of similar figures, find the area of the green figure.

1A.



$A = 32\text{ cm}^2$

$k = \frac{8}{5}$ $\quad k^2 = \left(\frac{8}{5}\right)^2 = \frac{64}{25}$

$\frac{64}{25} = \frac{32}{x}$

$64x = 32 \times 25$

$x = 12.5\text{ cm}^2$

1B.

$A = 13.5\text{ ft}^2$

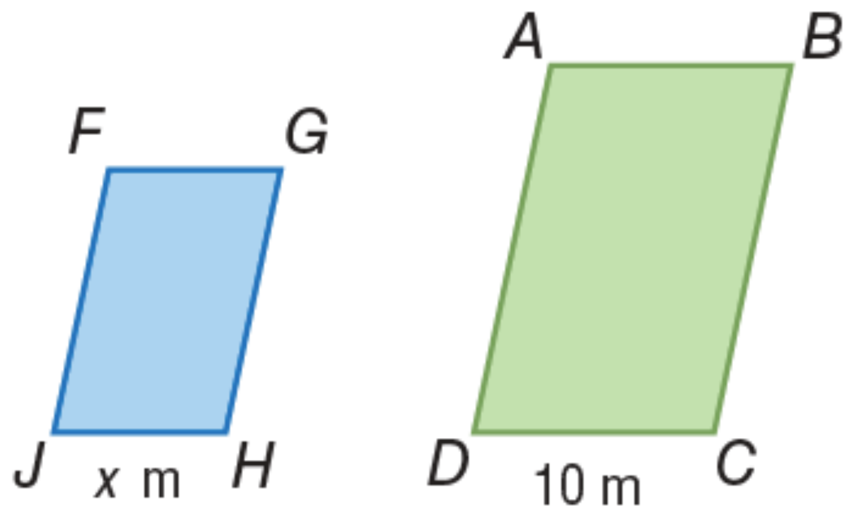
$k = \frac{6}{8}$ $\quad k^2 = \frac{36}{64} = \frac{9}{16}$

$\frac{9}{16} = \frac{13.5}{x}$

$x = 24\text{ ft}^2$

Finding lengths from areas.

The area of ▱*ABCD* is 150 square meters.
The area of ▱*FGHJ* is 54 square meters.
If ▱*ABCD* ~ ▱*FGHJ*, find the scale factor of ▱*FGHJ* to ▱*ABCD* and the value of *x*.



$k^2 = \frac{54}{150} = \frac{9}{25}$

$k = \sqrt{\frac{9}{25}} = \frac{3}{5}$

$\frac{3}{5} = \frac{x}{10}$

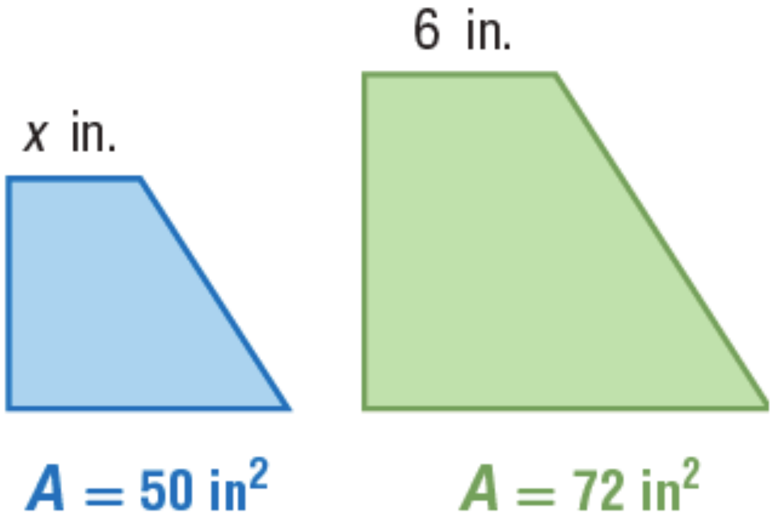
$5x = 3 \times 10$

$5x = 30$

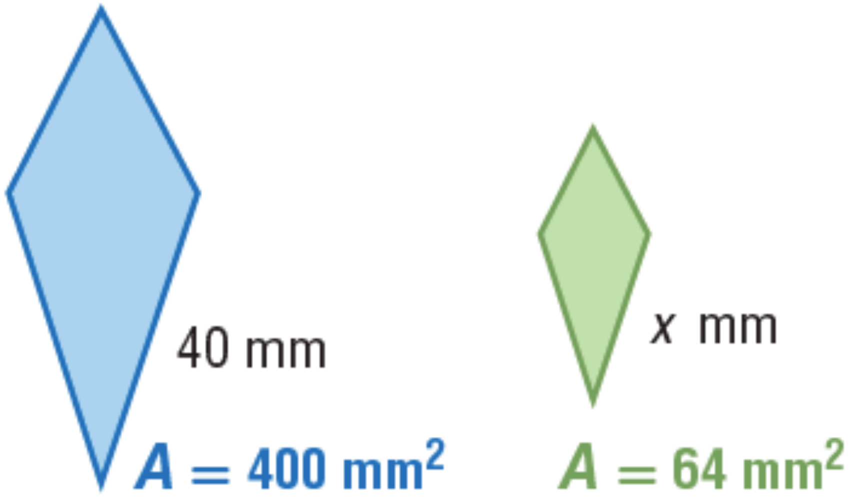
$x = 6$

For each pair of similar figures, use the given areas to find the scale factor of the blue to the green figure. Then find x.

2A.



2B.



$k^2 = \frac{50}{72} = \frac{25}{36}$ $k = \sqrt{\frac{25}{36}} = \frac{5}{6}$

$\frac{5}{6} = \frac{x}{6}$

$\boxed{x = 5}$

$k^2 = \frac{400}{64} = \frac{25}{4}$ $k = \sqrt{\frac{25}{4}} = \frac{5}{2}$

$\frac{5}{2} = \frac{40}{x}$

$5x = 2 \times 40$

$5x = 80$ $x = 16$