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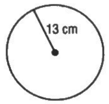
10-1 Study Guide and Intervention *(continued)*

Circles and Circumference

Circumference The **circumference** of a circle is the distance around the circle.

Circumference	For a circumference of C units and a diameter of d units of a radius of r units, $C = \pi d$ or $C = 2\pi r$

Example **Find the circumference of the circle to the nearest hundredth.**



$C = 2\pi r$ — Circumference formula
$= 2\pi(13)$ — $r = 13$
$= 26\pi$ — Simplify.
≈ 81.68 — Use a calculator.

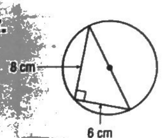
The circumference is 26π or about 81.68 centimeters.

Exercises

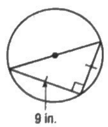
Find the diameter and radius of a circle with the given circumference. Round to the nearest hundredth.

1. $C = 40$ in. **12.73 in.; 6.37 in.**
2. $C = 256$ ft **81.49 ft; 40.74 ft**
3. $C = 15.62$ m **4.97 m; 2.49 m**
4. $C = 9$ cm **2.86 cm; 1.43 cm**
5. $C = 79.5$ yd **25.31 yd; 12.65 yd**
6. $C = 204.16$ m **64.99 m; 32.49 m**

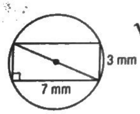
Find the exact circumference of each circle using the given inscribed or circumscribed polygon.

7. 

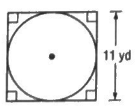
10π cm

8. 

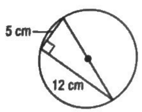
$9\sqrt{2}\pi$ in.

9. 

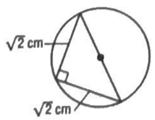
$\sqrt{58}\pi$ mm

10. 

11π yd

11. 

13π cm

12. 

2π cm

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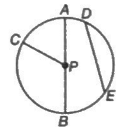
10-1 Skills Practice

Circles and Circumference

For Exercises 1–7, refer to ⊙P.



1. Name the circle.
⊙P

2. Name a radius.
$\overline{PA}$, $\overline{PB}$, or $\overline{PC}$

3. Name a chord.
$\overline{AB}$ or $\overline{DE}$

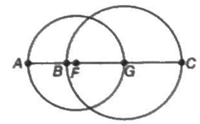
4. Name a diameter.
$\overline{AB}$

5. Name a radius not drawn as part of a diameter.
$\overline{PC}$

6. Suppose the diameter of the circle is 16 centimeters. Find the radius.
8 cm

7. If $PC = 11$ inches, find AB.
22 in.

The diameters of ⊙F and ⊙G are 5 and 6 units, respectively. Find each measure.



8. BF
0.5

9. AB
2

Find the diameter and radius of a circle with the given circumference. Round to the nearest hundredth.

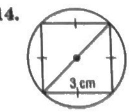
10. $C = 36$ m
11.46 m; 5.73 m

11. $C = 17.2$ ft
5.47 ft; 2.74 ft

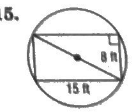
12. $C = 81.3$ cm
25.88 cm; 12.94 cm

13. $C = 5$ yd
1.59 yd; 0.8 yd

Find the exact circumference of each circle.

14. 

$3\pi\sqrt{2}$ cm

15. 

17π ft

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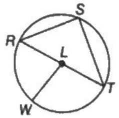
10-1 Practice

Circles and Circumference

For Exercises 1–7, refer to ⊙L.



1. Name the circle.
⊙L

2. Name a radius.
$\overline{LR}$, $\overline{LT}$, or $\overline{LW}$

3. Name a chord.
$\overline{RT}$, $\overline{RS}$, or $\overline{ST}$

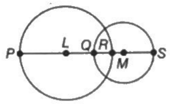
4. Name a diameter.
$\overline{RT}$

5. Name a radius not drawn as part of a diameter. **$\overline{LW}$**

6. Suppose the radius of the circle is 3.5 yards. Find the diameter. **7 yd**

7. If $RT = 19$ meters, find LW. **9.5 m**

The diameters of ⊙L and ⊙M are 20 and 13 units, respectively, and $QR = 4$. Find each measure.



8. LQ
6

9. RM
2.5

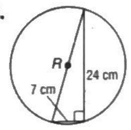
Find the diameter and radius of a circle with the given circumference. Round to the nearest hundredth.

10. $C = 21.2$ ft
6.75 ft; 3.37 ft

11. $C = 5.9$ m
1.88 m; 0.94 m

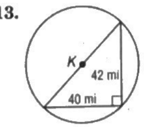
Find the exact circumference of each circle using the given inscribed or circumscribed polygon.

12.



25π cm

13.



58π mi

14. **SUNDIALS** Herman purchased a sundial to use as the centerpiece for a garden. The diameter of the sundial is 9.5 inches.

a. Find the radius of the sundial. **4.75 in.**

b. Find the circumference of the sundial to the nearest hundredth. **29.85 in.**

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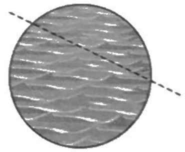
10-1 Word Problem Practice

Circles and Circumference

1. **WHEELS** Zack is designing wheels for a concept car. The diameter of the wheel is 18 inches. Zack wants to make spokes in the wheel that run from the center of the wheel to the rim. In other words, each spoke is a radius of the wheel. How long are these spokes?

9 in.

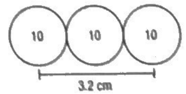
2. **CAKE CUTTING** Kathy slices through a circular cake. The cake has a diameter of 14 inches. The slice that Kathy made is straight and has a length of 11 inches.



Did Kathy cut along a *radius*, a *diameter*, or a *chord* of the circle?

chord

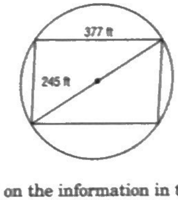
3. **COINS** Three identical circular coins are lined up in a row as shown.



The distance between the centers of the first and third coins is 3.2 centimeters. What is the radius of one of these coins?

0.8 cm

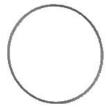
4. **PLAZAS** A rectangular plaza has a surrounding circular fence. The diagonals of the rectangle pass from one point on the fence through the center of the circle to another point on the fence.



Based on the information in the figure, what is the diameter of the fence? Round your answer to the nearest tenth of a foot.

449.6 ft

5. **EXERCISE HOOPS** Taiga wants to make a circular loop that he can twirl around his body for exercise. He will use a tube that is 2.5 meters long.



a. What will be the diameter of Taiga's exercise hoop? Round your answer to the nearest thousandth of a meter.

0.796 m

b. What will be the radius of Taiga's exercise hoop? Round your answer to the nearest thousandth of a meter.

0.398 m

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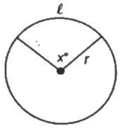
10-2 Study Guide and Intervention *(continued)*

Measuring Angles and Arcs

Arc Length An arc is part of a circle and its length is a part of the circumference of the circle.

The length of arc ℓ can be found using the following equation:

$\ell = \frac{x}{360} \cdot 2\pi r$



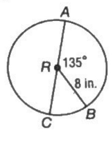
Example **Find the length of $\widehat{AB}$. Round to the nearest hundredth.**

The length of arc $\widehat{AB}$, can be found using the following equation: $\widehat{AB} = \frac{x}{360} \cdot 2\pi r$

$\widehat{AB} = \frac{x}{360} \cdot 2\pi r$ Arc Length Equation

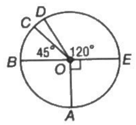
$\widehat{AB} = \frac{135}{360} \cdot 2\pi(8)$ Substitution

$\widehat{AB} \approx 18.85$ in. Use a calculator.



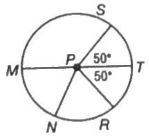
Exercises

Use ⊙O to find the length of each arc. Round to the nearest hundredth.



1. $\widehat{DE}$ if the radius is 2 meters **4.19 m**
2. $\widehat{DEA}$ if the diameter is 7 inches **12.83 in.**
3. $\widehat{BC}$ if $BE = 24$ feet **9.42 ft**
4. $\widehat{CBA}$ if $DO = 3$ millimeters **7.07 mm**

Use ⊙P to find the length of each arc. Round to the nearest hundredth.



5. $\widehat{RT}$, if $MT = 7$ yards **3.05 yd**
6. $\widehat{MR}$, if $PR = 13$ feet **29.50 ft**
7. $\widehat{MST}$, if $MP = 2$ inches **6.28 in.**
8. $\widehat{MRS}$, if $PS = 10$ centimeters **40.14 cm**

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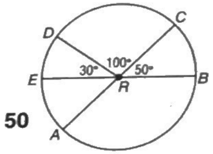
10-2 Skills Practice

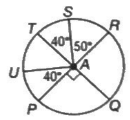
Measuring Angles and Arcs

$\overline{AC}$ and $\overline{EB}$ are diameters of ⊙R. Identify each arc as a *major arc*, *minor arc*, or *semicircle* of the circle. Then find its measure.



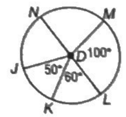
1. $m\widehat{EA}$ **minor arc; 50**
2. $m\widehat{CB}$ **minor arc; 50**
3. $m\widehat{DC}$ **minor arc; 100**
4. $m\widehat{DEB}$ **major arc; 210**
5. $m\widehat{AB}$ **minor arc; 130**
6. $m\widehat{CDA}$ **semicircle; 180**

$\overline{PR}$ and $\overline{QT}$ are diameters of ⊙A. Find each measure.



7. $m\widehat{UPQ}$ **130**
8. $m\widehat{PQR}$ **180**
9. $m\widehat{UTS}$ **90**
10. $m\widehat{RS}$ **50**
11. $m\widehat{RSU}$ **140**
12. $m\widehat{STP}$ **130**
13. $m\widehat{PQS}$ **230**
14. $m\widehat{PRU}$ **320**

Use ⊙D to find the length of each arc. Round to the nearest hundredth.



15. $\widehat{LM}$ if the radius is 5 inches **8.73 in.**
16. $\widehat{MN}$ if the diameter is 3 yards **2.09 yd**
17. $\widehat{KL}$ if $JD = 7$ centimeters **7.33 cm**
18. $\widehat{NJK}$ if $NL = 12$ feet **12.57 ft**
19. $\widehat{KLM}$ if $DM = 9$ millimeters **25.13 mm**
20. $\widehat{JK}$ if $KD = 15$ inches **13.09 in.**

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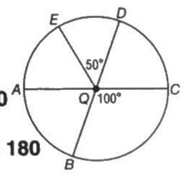
10-2 Practice

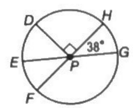
Measuring Angles and Arcs

$\overline{AC}$ and $\overline{DB}$ are diameters of ⊙Q. Identify each arc as a *major arc*, *minor arc*, or *semicircle* of the circle. Then find its measure.



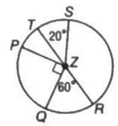
1. $m\widehat{AE}$ **minor arc; 50**
2. $m\widehat{AB}$ **minor arc; 80**
3. $m\widehat{EDC}$ **minor arc; 130**
4. $m\widehat{ADC}$ **semicircle; 180**
5. $m\widehat{ABC}$ **semicircle; 180**
6. $m\widehat{BC}$ **minor arc; 100**

$\overline{FH}$ and $\overline{EG}$ are diameters of ⊙P. Find each measure.



7. $m\widehat{EF}$ **38**
8. $m\widehat{DE}$ **52**
9. $m\widehat{FG}$ **142**
10. $m\widehat{DHG}$ **128**
11. $m\widehat{DFG}$ **232**
12. $m\widehat{DGE}$ **308**

Use ⊙Z to find each arc length. Round to the nearest hundredth.



13. $\widehat{QPT}$, if $QZ = 10$ inches
20.94 in.
14. $\widehat{QR}$, if $PZ = 12$ feet
12.57 ft
15. $\widehat{PQR}$, if $TR = 15$ meters
19.63 m
16. $\widehat{QPS}$, if $ZQ = 7$ centimeters
17.10 cm

17. **HOMEWORK** Refer to the table, which shows the number of hours students at Leland High School say they spend on homework each night.

Homework	
Less than 1 hour	8%
1–2 hours	29%
2–3 hours	58%
3–4 hours	3%
Over 4 hours	2%

a. If you were to construct a circle graph of the data, how many degrees would be allotted to each category?
28.8, 104.4, 208.8, 10.8, 7.2

b. Describe the arcs associated with each category.
The arc associated with 2–3 hours is a major arc; minor arcs are associated with the remaining categories.

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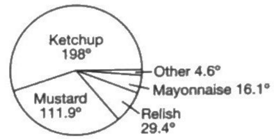
10-2 Word Problem Practice

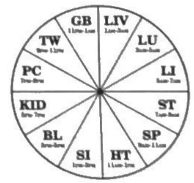
Measuring Angles and Arcs

1. **CONDIMENTS** A number of people in a park were asked to name their favorite condiment for hot dogs. The results are shown in the circle graph.



What was the second most popular hot dog condiment?
mustard

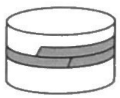
2. **CLOCKS** Shiatsu is a Japanese massage technique. One of the beliefs is that various body functions are most active at various times during the day. To illustrate this, they use a Chinese clock that is based on a circle divided into 12 equal sections by radii.



What is the measure of any one of the 12 equal central angles?
30

3. **PIES** Yolanda has divided a circular apple pie into 4 slices by cutting the pie along 4 radii. The central angles of the 4 slices are $3x$, $6x - 10$, $4x + 10$, and $5x$ degrees. What exactly are the numerical measures of the central angles?
60, 110, 90, and 100

4. **RIBBONS** Cora is wrapping a ribbon around a cylinder-shaped gift box. The box has a diameter of 15 inches and the ribbon is 60 inches long. Cora is able to wrap the ribbon all the way around the box once, and then continue so that the second end of the ribbon passes the first end. What is the central angle formed between the ends of the ribbon? Round your answer to the nearest tenth of a degree.
98.4°



5. **BIKE WHEELS** Lucy has to buy a new wheel for her bike. The bike wheel has a diameter of 20 inches.

a. If Lucy rolls the wheel one complete rotation along the ground, how far will the wheel travel? Round your answer to the nearest hundredth of an inch.
62.83 in.

b. If the bike wheel is rolled along the ground so that it rotates 45°, how far will the wheel travel? Round your answer to the nearest hundredth of an inch.
7.85 in.

c. If the bike wheel is rolled along the ground for 10 inches, through what angle does the wheel rotate? Round your answer to the nearest tenth of a degree.
57.3

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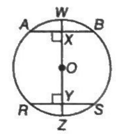
10-3 Study Guide and Intervention *(continued)*

Arcs and Chords

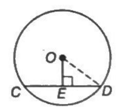
Diameters and Chords

- In a circle, if a diameter (or radius) is perpendicular to a chord, then it bisects the chord and its arc.
- In a circle, the perpendicular bisector of a chord is the diameter (or radius).
- In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.



If $\overline{WZ} \perp \overline{AB}$, then $\overline{AX} \cong \overline{XB}$ and $\widehat{AW} \cong \widehat{WB}$.
If $OX = OY$, then $\overline{AB} \cong \overline{RS}$.
If $\overline{AB} \cong \overline{RS}$, then $\overline{AB}$ and $\overline{RS}$ are equidistant from point O.

Example **In $\odot O$, $\overline{CD} \perp \overline{OE}$, $OD = 15$, and $CD = 24$. Find OE.**



A diameter or radius perpendicular to a chord bisects the chord, so ED is half of CD.

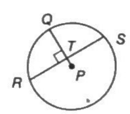
$ED = \frac{1}{2}(24)$
$= 12$

Use the Pythagorean Theorem to find x in $\triangle OED$.

$(OE)^2 + (ED)^2 = (OD)^2$	Pythagorean Theorem
$(OE)^2 + 12^2 = 15^2$	Substitution
$(OE)^2 + 144 = 225$	Simplify.
$(OE)^2 = 81$	Subtract 144 from each side.
$OE = 9$	Take the positive square root of each side.

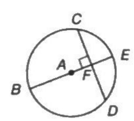
Exercises

In $\odot P$, the radius is 13 and $RS = 24$. Find each measure. Round to the nearest hundredth.



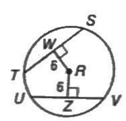
1. RT **12**
2. PT **5**
3. TQ **8**

In $\odot A$, the diameter is 12, $CD = 8$, and $m\widehat{CD} = 90$. Find each measure. Round to the nearest hundredth.

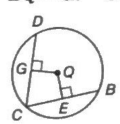


4. $m\widehat{DE}$ **45**
5. FD **4**
6. AF **4.47**

7. In $\odot R$, $TS = 21$ and $UV = 3x$. What is x? **7**



8. In $\odot Q$, $\overline{CD} \cong \overline{CB}$, $GQ = x + 5$ and $EQ = 3x - 6$. What is x? **5.5**



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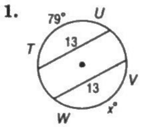
NAME ______________________ DATE ____________ PERIOD ____________

10-3 Skills Practice

Arcs and Chords

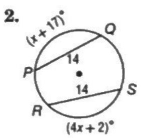
ALGEBRA Find the value of x in each circle.

1.



79

2.



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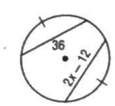
3.



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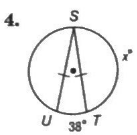
4.



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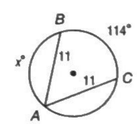
5.



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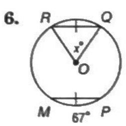
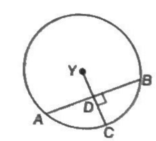
6.



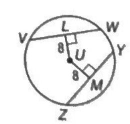
67

In $\odot Y$ the radius is 34, $AB = 60$, and $m\widehat{AC} = 71$. Find each measure.



7. $m\widehat{BC}$ **71**
8. $m\widehat{AB}$ **142**
9. AD **30**
10. BD **30**
11. YD **16**
12. DC **18**

13. In $\odot U$, $VW = 20$ and $YZ = 5x$. What is x?



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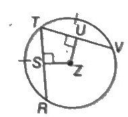
14. In $\odot Z$, $\widehat{TR} \cong \widehat{TV}$, $SZ = x + 4$, and $UZ = 2x - 1$. What is x?



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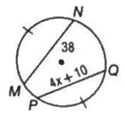
10-3 Practice

Arcs and Chords

ALGEBRA **Find the value of x in each circle.**

1. 

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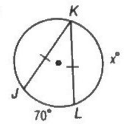
2. 

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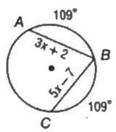
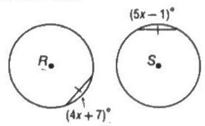
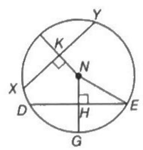
3. 

4.5

4. $\odot R \cong \odot S$ 

8

The radius of $\odot N$ is 18, $NK = 9$, and $m\widehat{DE} = 120$. Find each measure.



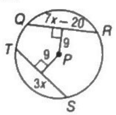
5. $m\widehat{GE}$ **60**

6. $m\angle HNE$ **60**

7. $m\angle HEN$ **30**

8. HN **9**

9. In $\odot P$, $QR = 7x - 20$ and $TS = 3x$. What is x?



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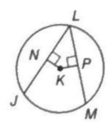
10. In $\odot K$, $\overline{JL} \cong \overline{LM}$, $KN = 3x - 2$, and $KP = 2x + 1$. What is x?



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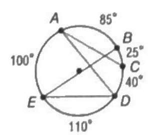
11. **GARDEN PATHS** A circular garden has paths around its edge that are identified by the given arc measures. It also has four straight paths, identified by segments $\overline{AC}$, $\overline{AD}$, $\overline{BE}$, and $\overline{DE}$, that cut through the garden's interior. Which two straight paths have the same length?



$\overline{DE}$ and $\overline{AC}$

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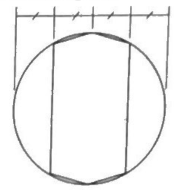
10-3 Word Problem Practice

Arcs and Chords

1. **HEXAGON** A hexagon is constructed as shown in the figure.



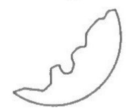
How many different chord lengths occur as side lengths of the hexagon?

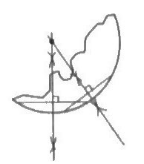
2

2. **WATERMARKS** For security purposes a jewelry company prints a hidden watermark on the logo of all its official documents. The watermark is a chord located 0.7 cm from the center of a circular ring that has a 2.5 cm radius. To the nearest tenth, what is the length of the chord?

4.8 cm

3. **ARCHAEOLOGY** Only one piece of a broken plate is found during an archaeological dig. Use the sketch of the pottery piece below to demonstrate how constructions with chords and perpendicular bisectors can be used to draw the plate's original size.





Draw two chords on the arc and find the perpendicular bisector of each chord. The chords will intersect at the center of the original circle. Then use a compass and the radius to draw the circle for the original size plate.

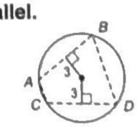
4. **CENTERS** Neil wants to find the center of a large circle. He draws what he thinks is a diameter of the circle and then marks its midpoint and declares that he has found the center. His teacher asks Neil how he knows that the line he drew is the diameter of the circle and not a smaller chord. Neil realizes that he does not know for sure. What can Neil do to determine if it is an actual diameter.

Sample answer: Neil can draw a line perpendicular to the line he just drew through the mark he made. If the midpoint of the first line is also the midpoint of the second line, it is a diameter.

5. **QUILTING** Miranda is following directions for a quilt pattern "In a 10-inch diameter circle, measure 3 inches from the center of the circle and mark a chord $\overline{AB}$ perpendicular to the radius of the circle. Then cut along the chord." Miranda is to repeat this for another chord, $\overline{CD}$. Finally, she is to cut along chord $\overline{DB}$ and $\overline{AC}$. The result should be four curved pieces and one quadrilateral.

   a. If Miranda follows the directions, is she guaranteed that the resulting quadrilateral is a rectangle? Explain.

   No; the method will produce congruent chords $\overline{AB}$ and $\overline{CD}$, but they are not necessarily parallel.



   b. Assume the resulting quadrilateral is a rectangle. One of the curved pieces has an arc measure of 74. What are the measures of the arcs on the other three curved pieces?

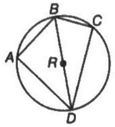
   74, 106, 106

NAME ______________________ DATE __________ PERIOD __________

10-4 Study Guide and Intervention *(continued)*

Inscribed Angles

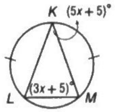
Angles of Inscribed Polygons An **inscribed polygon** is one whose sides are chords of a circle and whose vertices are points on the circle. Inscribed polygons have several properties.



- An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle. — If $\widehat{BCD}$ is a semicircle, then $m\angle BCD = 90$.
- If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary. — For inscribed quadrilateral $ABCD$, $m\angle A + m\angle C = 180$ and $m\angle ABC + m\angle ADC = 180$.

Example Find $m\angle K$.

$\widehat{KL} \cong \widehat{KM}$, so $KL = KM$. The triangle is an isosceles triangle, therefore $m\angle L = m\angle M = 3x + 5$.



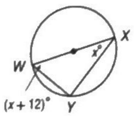
$m\angle L + m\angle M + m\angle K = 180$	Angle Sum Theorem
$(3x + 5) + (3x + 5) + (5x + 5) = 180$	Substitution
$11x + 15 = 180$	Simplify.
$11x = 165$	Subtract 15 from each side.
$x = 15$	Divide each side by 11.

So, $m\angle K = 5(15) + 5 = 80$.

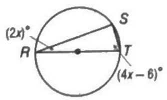
Exercises

ALGEBRA Find each measure.

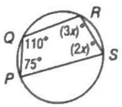
1. x **39**
2. $m\angle W$ **51**



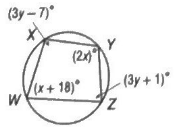
3. x **16**
4. $m\angle T$ **58**



5. $m\angle R$ **105**
6. $m\angle S$ **70**



7. $m\angle W$ **72**
8. $m\angle X$ **86**



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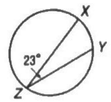
10-4 Skills Practice

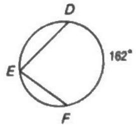
Inscribed Angles

Find each measure.

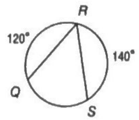
1. $m\widehat{XY}$ **46**



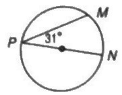
2. $m\angle E$ **81**



3. $m\angle R$ **50**

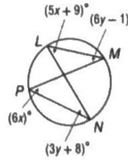


4. $m\widehat{MP}$ **118**

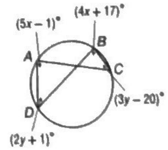


ALGEBRA Find each measure.

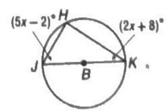
5. $m\angle N$ **17**
6. $m\angle L$ **54**



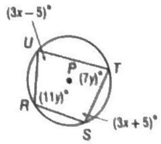
7. $m\angle C$ **43**
8. $m\angle A$ **89**



9. $m\angle J$ **58**
10. $m\angle K$ **32**



11. $m\angle S$ **95**
12. $m\angle R$ **110**



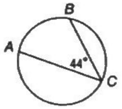
NAME ______________________ DATE __________ PERIOD __________

10-4 Practice

Inscribed Angles

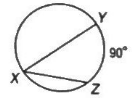
Find each measure.

1. $m\widehat{AB}$



88

2. $m\angle X$



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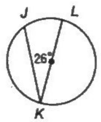
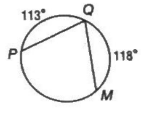
3. $m\widehat{JK}$



128

4. $m\angle Q$

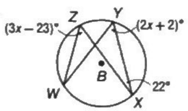


64.5

ALGEBRA Find each measure.

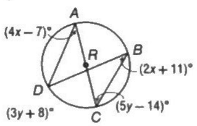
5. $m\angle W$
22

6. $m\angle Y$
52



7. $m\angle A$
29

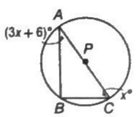
8. $m\angle D$
41



ALGEBRA Find each measure.

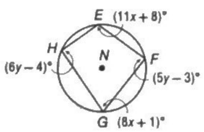
9. $m\angle A$
69

10. $m\angle C$
21



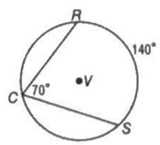
11. $m\angle G$
73

12. $m\angle H$
98



13. **PROBABILITY** In ⊙V, point C is randomly located so that it does not coincide with points R or S. If $m\widehat{RS} = 140$, what is the probability that $m\angle RCS = 70$?

$\frac{11}{18}$



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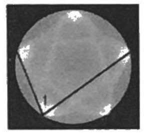
10-4 Word Problem Practice

Inscribed Angles

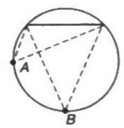
1. **ARENA** A circus arena is lit by five lights equally spaced around the perimeter.

What is $m\angle 1$?



72

2. **FIELD OF VIEW** The figure shows a top view of two people in front of a very tall rectangular wall. The wall makes a chord of a circle that passes through both people.



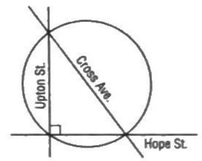
Which person has more of their horizontal field of vision blocked by the wall?

Neither; the same amount of the field of vision is blocked for both viewers.

3. **RHOMBI** Paul is interested in circumscribing a circle around a rhombus that is not a square. He is having great difficulty doing so. Can you help him? Explain.

No, because it's impossible to do. Opposite angles of a rhombus are congruent, and if the rhombus is inscribed in a circle, the measures of the angles must add up to 180. This would imply that all of the angles are 90°. A rhombus with right angles can only be a square.

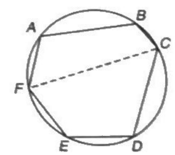
4. **STREETS** Three kilometers separate the intersections of Cross and Upton and Cross and Hope.



What is the distance between the intersection of Upton and Hope and the point midway between the intersections of Upton and Cross and Cross and Hope?

1.5 km

5. **INSCRIBED HEXAGONS** You will prove that the sum of the measures of alternate interior angles in an inscribed hexagon is 360.



a. How are $\angle A$ and $\angle BCF$ related? Similarly, how are $\angle E$ and $\angle DCF$ related?

They are supplementary.

b. Show that $m\angle A + m\angle BCD + m\angle E = 360$.

$m\angle A + m\angle BCD + m\angle E =$
$m\angle A + m\angle BCF + m\angle DCF +$
$m\angle E = 180 + 180 = 360$

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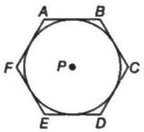
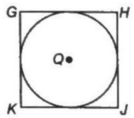
10-5 Study Guide and Intervention *(continued)*

Tangents

Circumscribed Polygons When a polygon is circumscribed about a circle, all of the sides of the polygon are tangent to the circle.

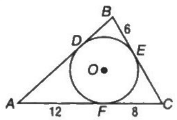


Hexagon *ABCDEF* is circumscribed about ⊙*P*. $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, and $\overline{FA}$ are tangent to ⊙*P*.



Square *GHJK* is circumscribed about ⊙*Q*. $\overline{GH}$, $\overline{JH}$, $\overline{JK}$, and $\overline{KG}$ are tangent to ⊙*Q*.

Example **△*ABC* is circumscribed about ⊙*O*. Find the perimeter of △*ABC*.**



△*ABC* is circumscribed about ⊙*O*, so points *D*, *E*, and *F* are points of tangency. Therefore $AD = AF$, $BE = BD$, and $CF = CE$.

$P = AD + AF + BE + BD + CF + CE$

$= 12 + 12 + 6 + 6 + 8 + 8$

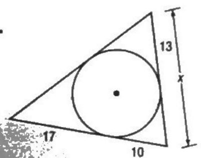
$= 52$

The perimeter is 52 units.

Exercises

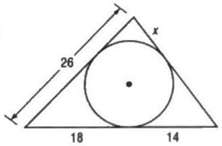
For each figure, find *x*. Then find the perimeter.

1.



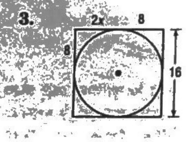
23; 80

2.



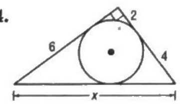
8; 80

3.



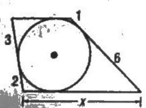
4; 64

4.



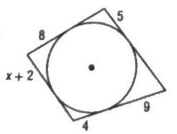
10; 24

5.



8; 24

6.



6; 52

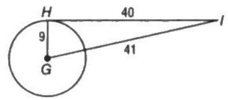
NAME ______________________ DATE __________ PERIOD __________

10-5 Skills Practice

Tangents

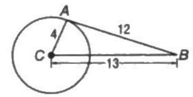
Determine whether each segment is tangent to the given circle. Justify your answer.

1. $\overline{HI}$



yes; $9^2 + 40^2 = 41^2$

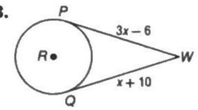
2. $\overline{AB}$



no; $4^2 + 12^2 \neq 13^2$

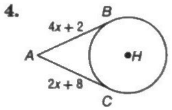
Find *x*. Assume that segments that appear to be tangent are tangent. Round to the nearest tenth if necessary.

3.



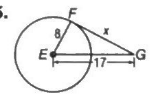
8

4.



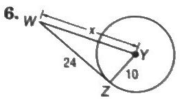
3

5.



15

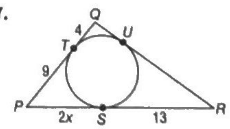
6.



26

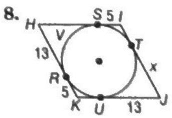
For each figure, find *x*. Then find the perimeter.

7.



x = 4.5; 52

8.



x = 13; 72

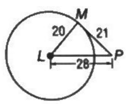
NAME ______________________ DATE __________ PERIOD __________

10-5 Practice

Tangents

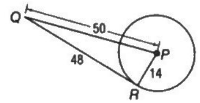
Determine whether each segment is tangent to the given circle. Justify your answer.

1. $\overline{MP}$



no; $20^2 + 21^2 \neq 28^2$

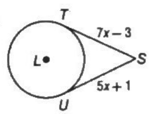
2. $\overline{QR}$



yes; $14^2 + 48^2 = 50^2$

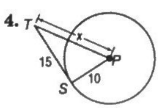
Find x. Assume that segments that appear to be tangent are tangent. Round to the nearest tenth if necessary.

3.



2

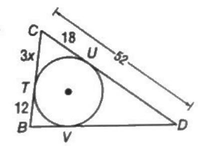
4.



18.0

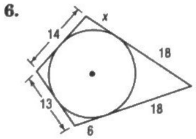
For each figure, find x. Then find the perimeter.

5.



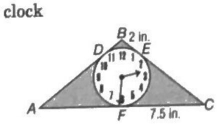
6; 128

6.



7; 76

7. **CLOCKS** The design shown in the figure is that of a circular clock face inscribed in a triangular base. AF and FC are equal.



a. Find AB. **9.5 in.**

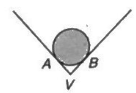
b. Find the perimeter of the clock. **34 in.**

NAME ______________________ DATE __________ PERIOD __________

10-5 Word Problem Practice

Tangents

1. **CANALS** The concrete canal in Landtown is shaped like a "V" at the bottom. One day, Maureen accidentally dropped a cylindrical tube as she was walking and it rolled to the bottom of the dried out concrete canal. The figure shows a cross section of the tube at the bottom of the canal.



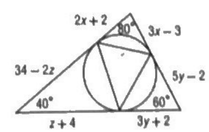
Compare the lengths AV and BV.

They are equal.

2. **PACKAGING** Taylor packed a sphere inside a cubic box. He had painted the sides of the box black before putting the sphere inside. When the sphere was later removed, he discovered that the black paint had not completely dried and there were black marks on the sides of the sphere at the points of tangency with the sides of the box. If the black marks are used as the vertices of a polygon, what kind of polygon results?

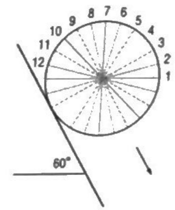
a square

3. **JEWELRY** Juanita is designing a pendant with a circular gem inscribed in a triangle. Find the values of x, y, and z. Then find the perimeter of the triangle.



5, 2, 10, 68

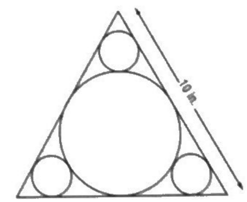
4. **ROLLING** A wheel is rolling down an incline. Twelve evenly spaced diameters form spokes of the wheel.



When spoke 2 is vertical, which spoke will be perpendicular to the incline?

spoke 10

5. **DESIGN** Amanda wants to make this design of circles inside an equilateral triangle.



a. What is the radius of the large circle to the nearest hundredth of an inch?

2.89 in.

b. What are the radii of the smaller circles to the nearest hundredth of an inch?

0.96 in.

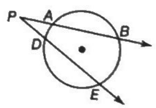
NAME ______________________ DATE __________ PERIOD __________

10-6 Study Guide and Intervention *(continued)*

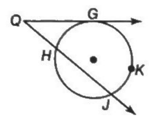
Secants, Tangents, and Angle Measures

Intersections Outside a Circle If secants and tangents intersect outside a circle, they form an angle whose measure is related to the intercepted arcs.

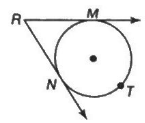
If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one half the difference of the measures of the intercepted arcs.



$\overrightarrow{PB}$ and $\overrightarrow{PE}$ are secants.
$m\angle P = \frac{1}{2}(m\widehat{BE} - m\widehat{AD})$

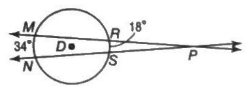


$\overrightarrow{QG}$ is a tangent. $\overrightarrow{QJ}$ is a secant.
$m\angle Q = \frac{1}{2}(m\widehat{GKJ} - m\widehat{GH})$



$\overrightarrow{RM}$ and $\overrightarrow{RN}$ are tangents.
$m\angle R = \frac{1}{2}(m\widehat{MTN} - m\widehat{MN})$

Example Find $m\angle MPN$.



$\angle MPN$ is formed by two secants that intersect in the exterior of a circle.

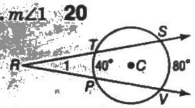
$$m\angle MPN = \frac{1}{2}(m\widehat{MN} - m\widehat{RS})$$
$$= \frac{1}{2}(34 - 18)$$
$$= \frac{1}{2}(16) \text{ or } 8$$

The measure of the angle is 8.

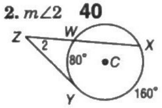
Exercises

Find each measure. Assume that segments that appear to be tangent are tangent.

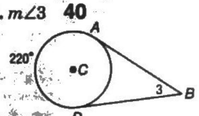
1. $m\angle 1$ **20**



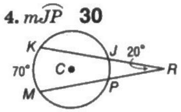
2. $m\angle 2$ **40**



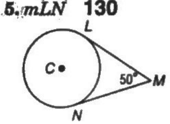
3. $m\angle 3$ **40**



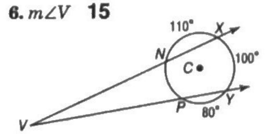
4. $m\widehat{JP}$ **30**



5. $m\widehat{LN}$ **130**



6. $m\angle V$ **15**



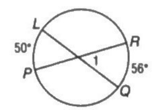
NAME ______________________ DATE __________ PERIOD __________

10-6 Skills Practice

Secants, Tangents, and Angle Measures

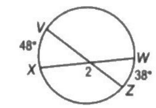
Find each measure. Assume that segment that appear to be tangent are tangent.

1. $m\angle 1$



53

2. $m\angle 2$



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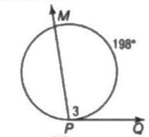
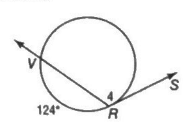
3. $m\angle 3$



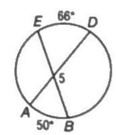
99

4. $m\angle 4$



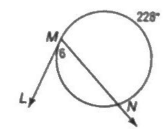
118

5. $m\angle 5$



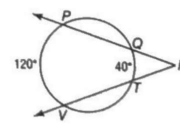
122

6. $m\angle 6$



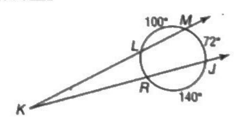
66

7. $m\angle R$



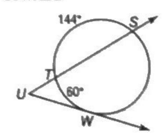
40

8. $m\angle K$



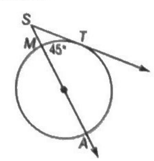
12

9. $m\angle U$



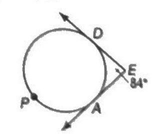
48

10. $m\angle S$



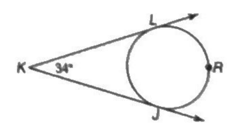
45

11. $m\widehat{DPA}$



264

12. $m\widehat{LJ}$



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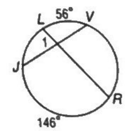
NAME ____________________ DATE __________ PERIOD __________

10-6 Practice

Secants, Tangents, and Angle Measures

Find each measure. Assume that any segments that appear to be tangent are tangent.

1. $m\angle 1$



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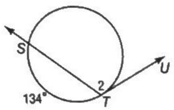
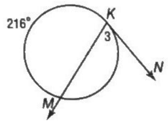
2. $m\angle 2$



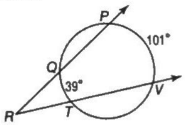
113

3. $m\angle 3$



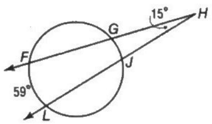
72

4. $m\angle R$



31

5. $m\widehat{GJ}$



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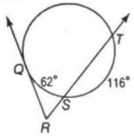
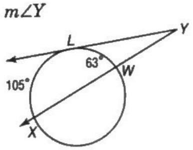
6. $m\angle R$



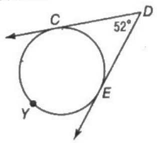
60

7. $m\angle Y$



21

8. $m\widehat{CE}$



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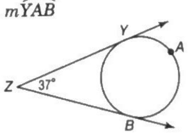
9. $m\widehat{YAB}$



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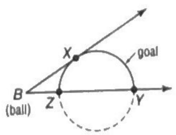
10. **RECREATION** In a game of kickball, Rickie has to kick the ball through a semicircular goal to score. If $m\widehat{XZ} = 58$ and the $m\widehat{XY} = 122$, at what angle must Rickie kick the ball to score? Explain.



Rickie must kick the ball at an angle less than 32 since the measure of the angle from the ground that a tangent would make with the goal post is 32.

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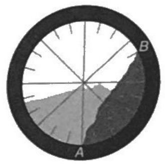
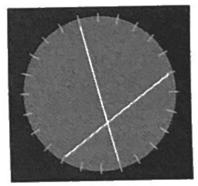
10-6 Word Problem Practice

Secants, Tangents, and Angle Measures

1. **TELESCOPES** Vanessa looked through her telescope at a mountainous landscape. The figure shows what she saw. Based on the view, approximately what angle does the side of the mountain that runs from A to B make with the horizontal? **60**



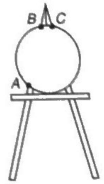
2. **RADAR** Two airplanes were tracked on radar. They followed the paths shown in the figure.



What is the acute angle between their flight paths?

67.5

3. **EASELS** Francisco is a painter. He places a circular canvas on his A-frame easel and carefully centers it. The apex of the easel is 30° and the measure of arc BC is 22°. What is the measure of arc AB?



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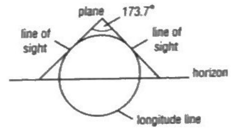
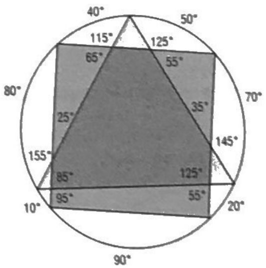
4. **FLYING** When flying at an altitude of 5 miles, the lines of sight to the horizon looking north and south make about a 173.7° angle. How much of the longitude line directly under the plane is visible from 5 miles high?



6.3

5. **STAINED GLASS** Pablo made the stained glass window shown. He used an inscribed square and equilateral triangle for the design.



a. Label the angle measures on the outer edge of the triangle.

See diagram.

b. Label all of the arcs with their degree measure.

See diagram.

Lesson 10-6

Answers

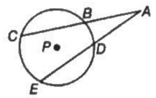
NAME ______________________ DATE __________ PERIOD __________

10-7 Study Guide and Intervention *(continued)*

Special Segments in a Circle

Segments Intersecting Outside a Circle If secants and tangents intersect outside a circle, then two products are equal. A **secant segment** is a segment of a secant line that has exactly one endpoint on the circle. A secant segment that lies in the exterior of the circle is called an **external secant segment**. A **tangent segment** is a segment of a tangent with one endpoint on the circle.

- If two secants are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.

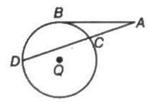


$\overline{AC}$ and $\overline{AE}$ are secant segments.

$\overline{AB}$ and $\overline{AD}$ are external secant segments.

$AC \cdot AB = AE \cdot AD$

- If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.



$\overline{AB}$ is a tangent segment.

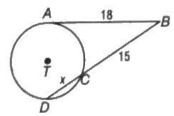
$\overline{AD}$ is a secant segment.

$\overline{AC}$ is an external secant segment.

$(AB)^2 = AD \cdot AC$

Example $\overline{AB}$ **is tangent to the circle. Find** x**. Round to the nearest tenth.**

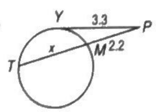
The tangent segment is $\overline{AB}$, the secant segment is $\overline{BD}$, and the external secant segment is $\overline{BC}$.

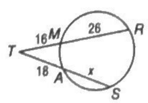


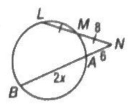
$(AB)^2 = BC \cdot BD$	
$(18)^2 = 15(15 + x)$	Substitution.
$324 = 225 + 15x$	Multiply.
$99 = 15x$	Subtract 225 from both sides.
$6.6 = x$	Divide both sides by 15.

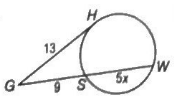
Exercises

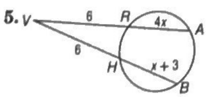
Find x**. Round to the nearest tenth. Assume segments that appear to be tangent are tangent.**

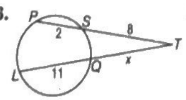
1.  2.8

2.  19.3

3.  7.7

4.  2.0

5.  1.0

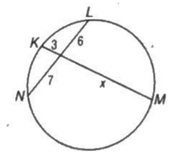
6.  5.0

NAME ______________________ DATE __________ PERIOD __________

10-7 Skills Practice

Special Segments in a Circle

Find x **to the nearest tenth if necessary. Assume that segments that appear to be tangent are tangent.**

1. 

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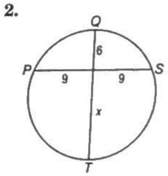
2. 

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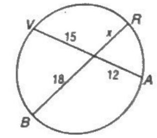
3. 

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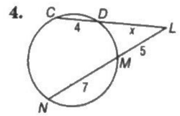
4. 

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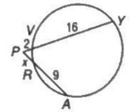
5. 

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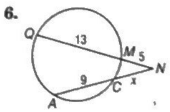
6. 

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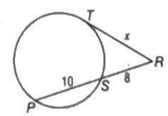
7. 

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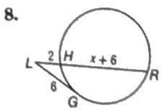
8. 

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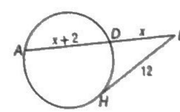
9. 

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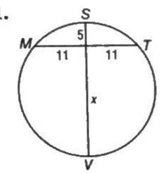
NAME ______________________ DATE __________ PERIOD __________

10-7 Practice

Special Segments in a Circle

Find x. Assume that segments that appear to be tangent are tangent. Round to the nearest tenth if necessary

1. 

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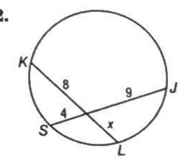
2. 

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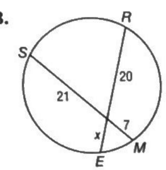
3. 

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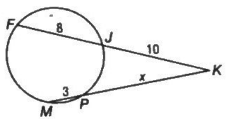
4. 

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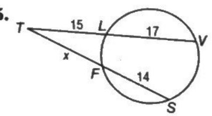
5. 

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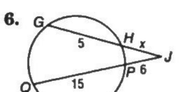
6. 

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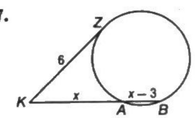
7. 

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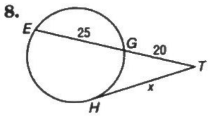
8. 

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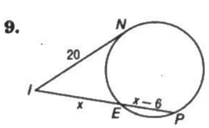
9. 

15.7

10. **CONSTRUCTION** An arch over an apartment entrance is 3 feet high and 9 feet wide. Find the radius of the circle containing the arc of the arch. **4.875 ft**

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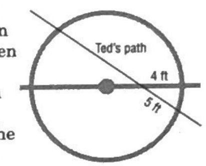
10-7 Word Problem Practice

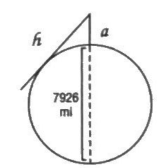
Special Segments in a Circle

1. **ICE SKATING** Ted skated through one of the face-off circles at a skating rink. His path through the circle is shown in the figure. Given that the face-off circle is 15 feet in diameter, what distance within the face-off circle did Ted travel?



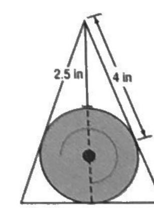
13.8 ft

2. **HORIZONS** Assume that Earth is a perfect sphere with a diameter of 7926 miles. From an altitude of a miles, how long is the horizon line h?



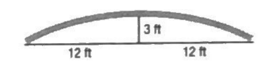
$h = \sqrt{a(a + 7926)}$

3. **AXLES** The figure shows the cross-section of an axle held in place by a triangular sleeve. A brake extends from the apex of the triangle. When the brake is extended 2.5 inches into the sleeve, it comes into contact with the axle. What is the diameter of the axle?



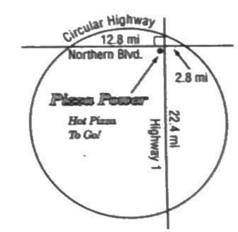
3.9 in.

4. **ARCHEOLOGY** Scientists unearthed part of a circular wall. They made the measurements shown in the figure.



Based on the information in the figure, what was the radius of the circle?
25.5 ft

5. **PIZZA DELIVERY** Pizza Power is located at the intersection of Northern Boulevard and Highway 1 in a city with a circular highway running all the way around its outskirts. The radius of the circular highway is 13 miles. Pizza Power puts the map shown below on its take-out menus.



a. How many miles away is the Circular Highway from Pizza Power if you travel north on Highway 1?
1.6 mi

b. The city builds a new road along the diameter of Circular Highway that passes through the intersection of Northern Boulevard and Highway 1. Along this new road, about how many miles is it (the shorter way) to the Circular Highway from Pizza Power?

about 1.46 mi

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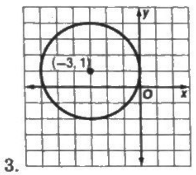
10-8 Study Guide and Intervention *(continued)*

Equations of Circles

Graph Circles If you are given an equation of a circle, you can find information to help you graph the circle.

Example Graph $(x + 3)^2 + (y + 1)^2 = 9$.



Use the parts of the equation to find (h, k) and r.

Rewrite $(x + 3)^2 + (y - 1)^2 = 9$ to find the center and the radius.

$$[x - (-3)]^2 + (y - 1)^2 = 3^2$$

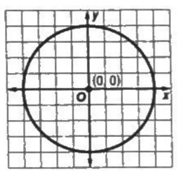
$$(x - h)^2 + (y - k)^2 = r^2$$

So $h = -3$, $k = 1$, and $r = 3$. The center is at $(-3, 1)$ and the radius is 3.

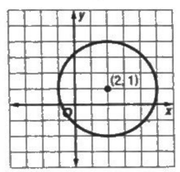
Exercises

For each circle with the given equation, state the coordinates of the center and the measure of the radius. Then graph the equation.

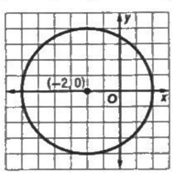
1. $x^2 + y^2 = 16$ **(0, 0); 4**



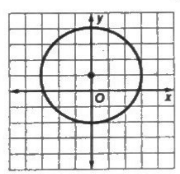
2. $(x - 2)^2 + (y - 1)^2 = 9$ **(2, 1); 3**



3. $(x + 2)^2 + y^2 = 16$ **(−2, 0); 4**

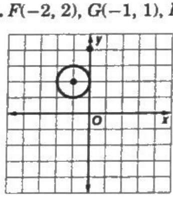


4. $x^2 + (y - 1)^2 = 9$ **(0, 1); 3**



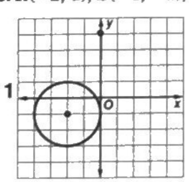
Write an equation of a circle that contains each set of points. Then graph the circle.

5. $F(-2, 2)$, $G(-1, 1)$, $H(-1, 3)$



$(x + 1)^2 + (y - 2)^2 = 1$

6. $R(-2, 1)$, $S(-4, -1)$, $T(0, -1)$



$(x + 2)^2 + (y + 1)^2 = 4$

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10-8 Skills Practice

Equations of Circles

Write the equation of each circle.

1. center at origin, radius 6

$x^2 + y^2 = 36$

2. center at (0, 0), radius 2

$x^2 + y^2 = 4$

3. center at (4, 3), radius 9

$(x - 4)^2 + (y - 3)^2 = 81$

4. center at (7, 1), diameter 24

$(x - 7)^2 + (y - 1)^2 = 144$

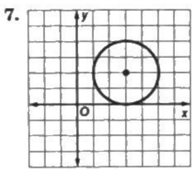
5. center at (−4, −1), passes through (−2, 3)

$(x + 4)^2 + (y + 1)^2 = 20$

6. center at (5, −2), passes through (4, 0)

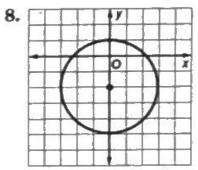
$(x - 5)^2 + (y + 2)^2 = 5$

7.



$(x - 3)^2 + (y - 2)^2 = 4$

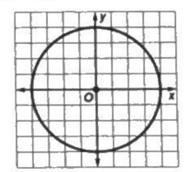
8.



$x^2 + (y + 2)^2 = 9$

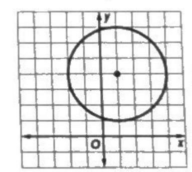
For each circle with the given equation, state the coordinates of the center and the measure of the radius. Then graph the equation.

9. $x^2 + y^2 = 16$



(0, 0); $r = 4$

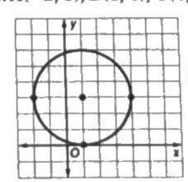
10. $(x - 1)^2 + (y - 4)^2 = 9$



(1, 4); $r = 3$

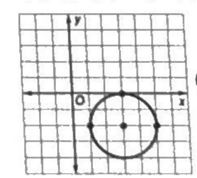
Write an equation of a circle that contains each set of points. Then graph the circle.

11. $A(-2, 3)$, $B(1, 0)$, $C(4, 3)$



$(x - 1)^2 + (y - 3)^2 = 9$

12. $F(3, 0)$, $G(5, -2)$, $H(1, -2)$



$(x - 3)^2 + (y + 2)^2 = 4$