CHAPTER 9 – RATIONAL EQUATIONS AND FUNCTIONS

9.1 – INVERSE AND JOINT VARIATION

DIRECT VARIATION

- Direct variation occurs if $y = kx$, where k is a constant.
- To check for direct variation, divide the output by the input. Direct variation will always yield the same ratio.

x	y
31	217
20	140
17	119
12	84

INVERSE VARIATION

- Inverse variation occurs if $y = \frac{k}{x}$, where k is a non-zero constant.
- To check for direct variation, multiply the output by the input. Inverse variation will always yield the same product.

x	y
1.5	20
2.5	12
4	7.5
5	6

CLASSIFYING DIRECT AND INVERSE VARIATION

GIVEN EQUATION	REWRITTEN EQUATION	TYPE OF VARIATION
a. $\frac{y}{5} = x$		
b. $y = x + 2$		
c. $xy = 4$		

CLASSIFYING DIRECT AND INVERSE VARIATION

GIVEN EQUATION	REWRITTEN EQUATION	TYPE OF VARIATION
$xy = \frac{1}{4}$		
$\frac{x}{y} = 5$		
$y = x - 3$		
$\frac{1}{2}xy = 9$		

WRITING INVERSE VARIATION EQUATIONS

The variables x and y vary inversely, and $y = 8$ when $x = 3$.

a. Write an equation that relates x and y.

b. Find y when $x = -4$.

WRITING INVERSE VARIATION EQUATIONS

INVERSE VARIATION MODELS **The variables *x* and *y* vary inversely. Use the given values to write an equation relating *x* and *y*. Then find *y* when *x* = 2.**

29. $x = 5, y = -2$

30. $x = 4, y = 8$

31. $x = 7, y = 1$

JOINT VARIATION

- Joint variation occurs when a quantity varies directly as the product of two or more other quantities. $z = kxy$

JOINT VARIATION

12. $x = 15yz$

13. $\frac{x}{z} = 0.5y$

14. $xy = 4z$

15. $x = \frac{yz}{2}$

Joint Variation Models **The variable z varies jointly with x and y. Use the given values to write an equation relating x, y, and z. Then find z when $x = -4$ and $y = 7$.**

39. $x = 3, y = 8, z = 6$

40. $x = -12, y = 4, z = 2$

41. $x = 1, y = \frac{1}{3}, z = 5$

42. $x = -6, y = 3, z = \frac{2}{5}$

Write an equation for the given relationship.

RELATIONSHIP	EQUATION
a. y varies directly with x.	
b. y varies inversely with x.	
c. z varies jointly with x and y.	
d. y varies inversely with the square of x.	
e. z varies directly with y and inversely with x.	

9.2 – GRAPHING SIMPLE RATIONAL EQUATIONS

BASIC RATIONAL FUNCTION

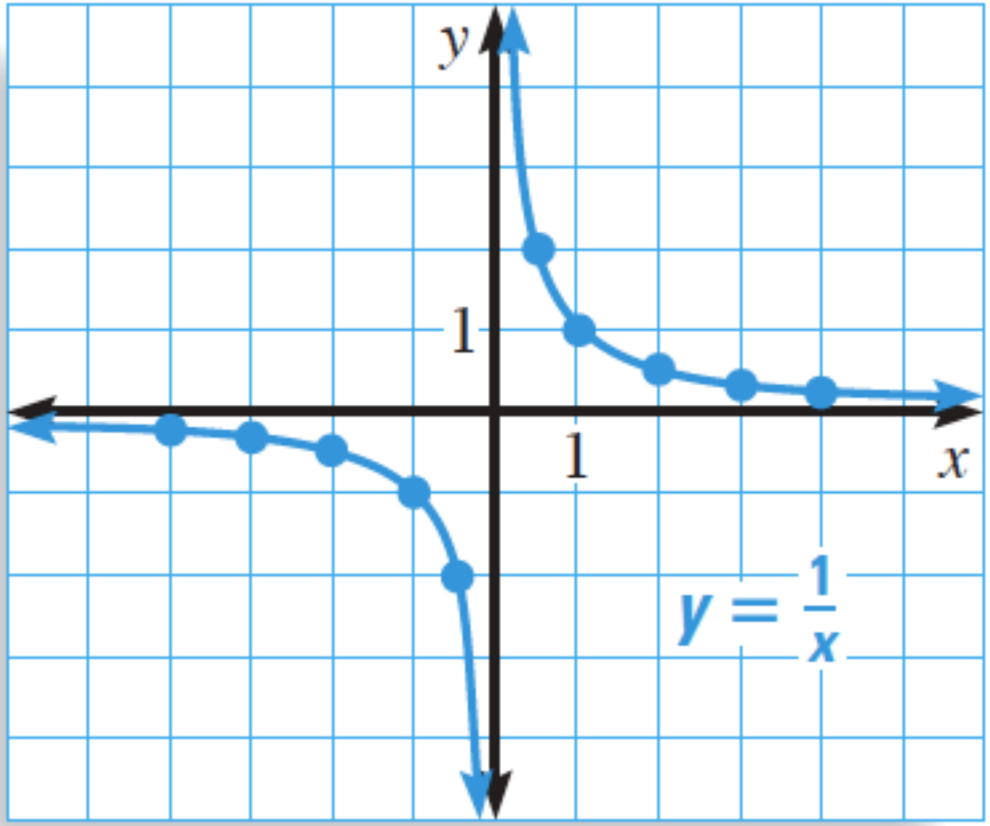
- A **rational function** is of the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$.
- It has two parts called **branches.**

BASIC RATIONAL FUNCTION

- A **rational function** where the top and bottom polynomial are both linear (first degree).
- The graph is called a **hyperbola**.
- It has a horizontal and a vertical asymptote.
- It has two parts called **branches.**
- Domain and range are all real number except for the values of the asymptotes.

$f(x) = \frac{1}{x}$ Domain and range are all real number except for x =0 and y = 0.

x	y
-4	$-\frac{1}{4}$
-3	$-\frac{1}{3}$
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2



x	y
4	$\frac{1}{4}$
3	$\frac{1}{3}$
2	$\frac{1}{2}$
1	1
$\frac{1}{2}$	2

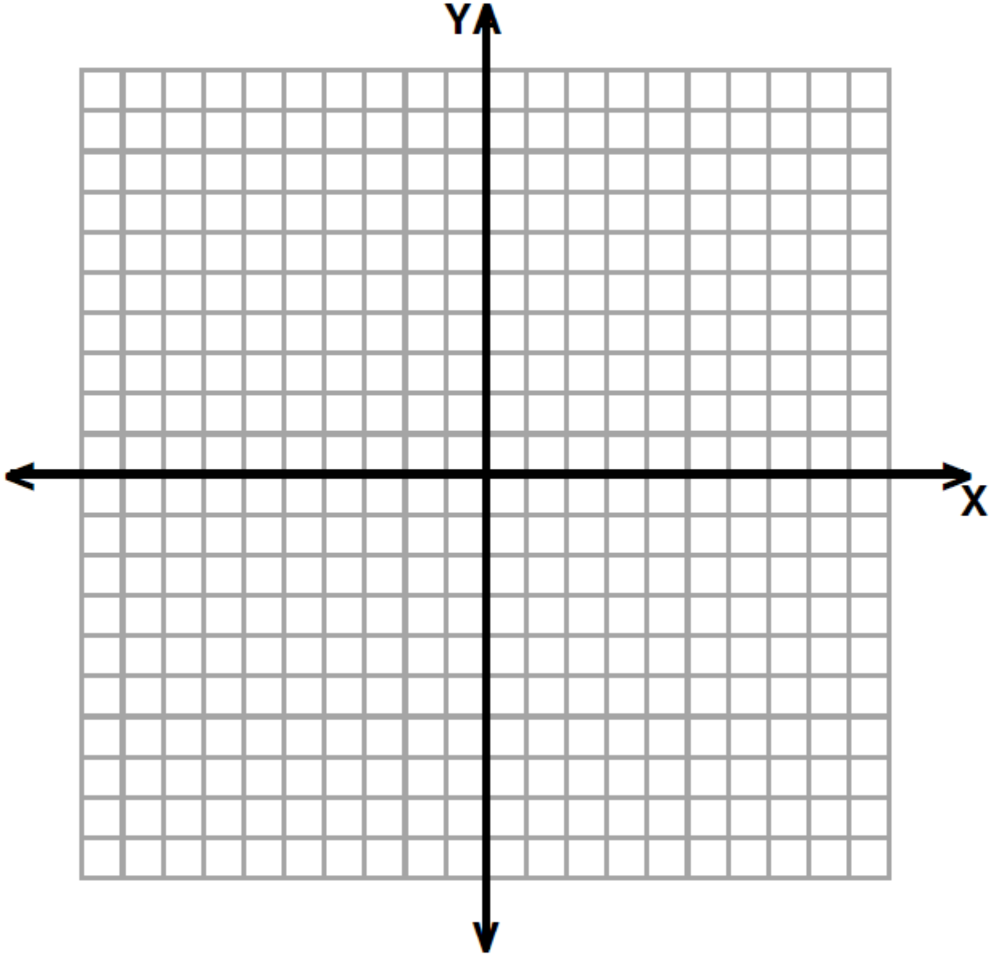
RATIONAL FUNCTION WITH TRANSFORMATIONS

- Is of the form $f(x) = \frac{a}{x-h} + k$ or $f(x) = \frac{ax+b}{cx+d}$.
- In the form $f(x) = \frac{a}{x-h} + k$, the asymptotes are $x = h$ and $y = k$.
- In the form $f(x) = \frac{ax+b}{cx+d}$, the asymptotes are $x = \frac{-d}{c}$ and $y = \frac{a}{c}$.

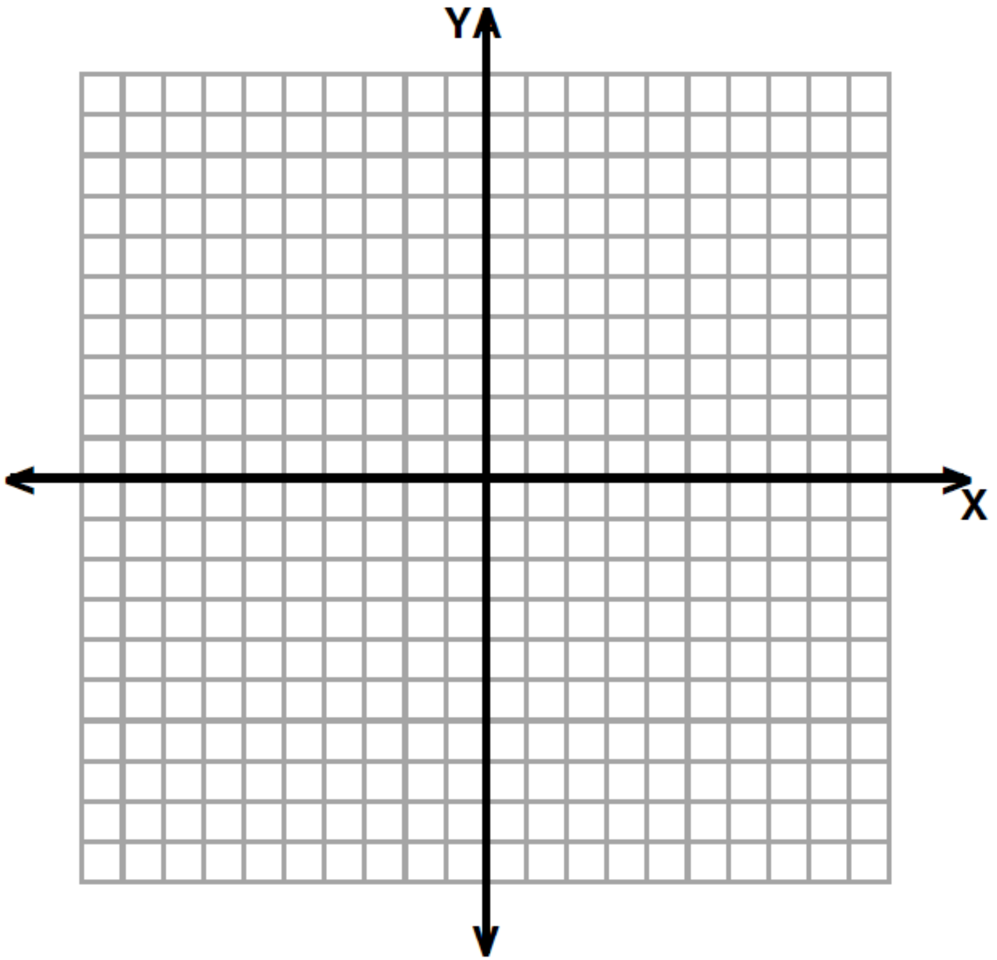
GRAPHING RATIONAL FUNCTIONS WITH TRANSFORMATIONS

- 1) Find and graph the asymptotes.
- 2) Graph two points on each side of the vertical asymptote.

Graph $y = \dfrac{-2}{x+3} - 1$. State the domain and range.



Graph $y = \dfrac{x + 1}{2x - 4}$. State the domain and range.



9.3 – GRAPHING GENERAL RATIONAL FUNCTIONS

CHARACTERISTICS OF GENERAL RATIONAL FUNCTION

For a **rational function** of the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$:

- The x-intercepts of the graph are the real zeros of $p(x)$.
- The graph has vertical asymptotes at all the real zeros of $q(x)$.

HORIZONTAL ASYMPTOTES OF GENERAL RATIONAL FUNCTION

For a **rational function** of the form

$$f(x) = \frac{p(x)}{q(x)} = \frac{a_m x^m + a_{m-1} x^{m-1} + \cdots + a_1 x + a_0}{b_n x^n + b_{n-1} x^{n-1} + \cdots + b_1 x + b_0}$$

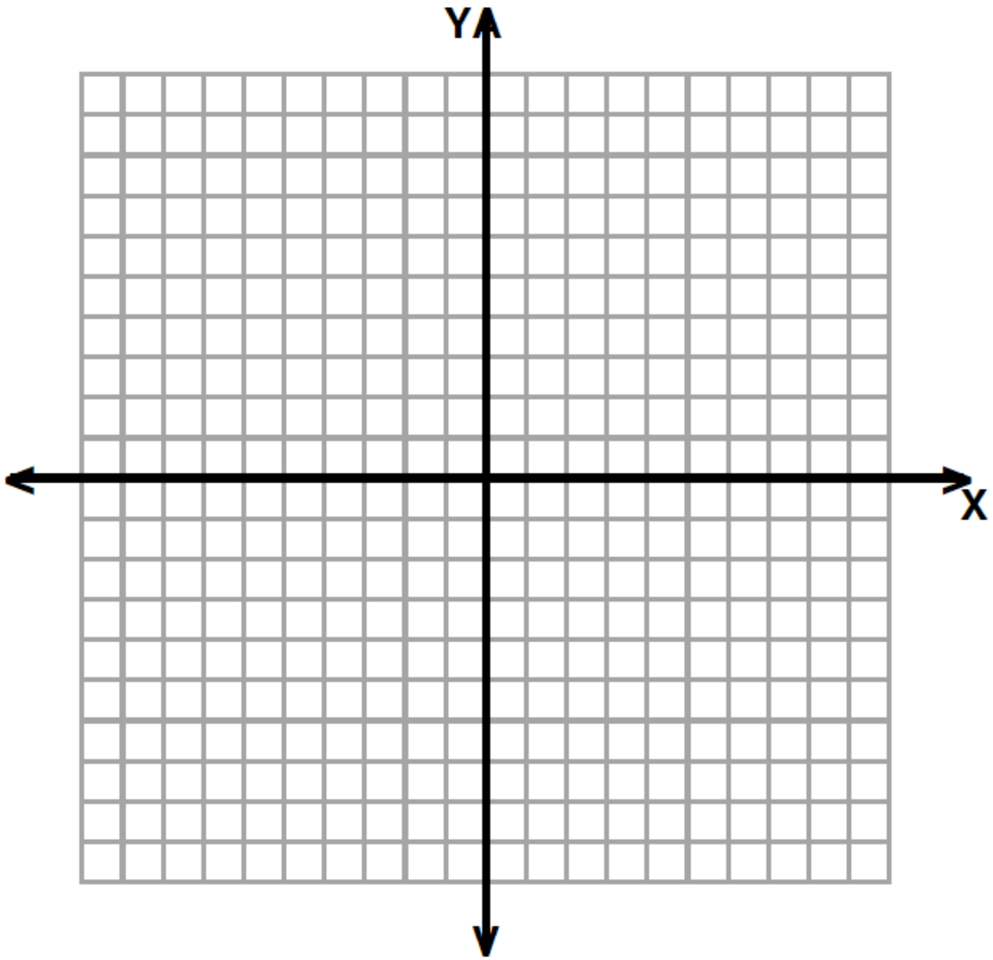
HORIZONTAL ASYMPTOTES OF GENERAL RATIONAL FUNCTION

Polynomial characteristic	Asymptote / End behavior
Bottom polynomial has higher degree.	$y = 0$
Polynomials have same degree. $f(x) = \frac{a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0}{b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0}$	$y = \frac{a_m}{b_n}$
Top polynomial has higher degree. $f(x) = \frac{a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0}{b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0}$	$y = \frac{a_m}{b_n} x^{m-n}$

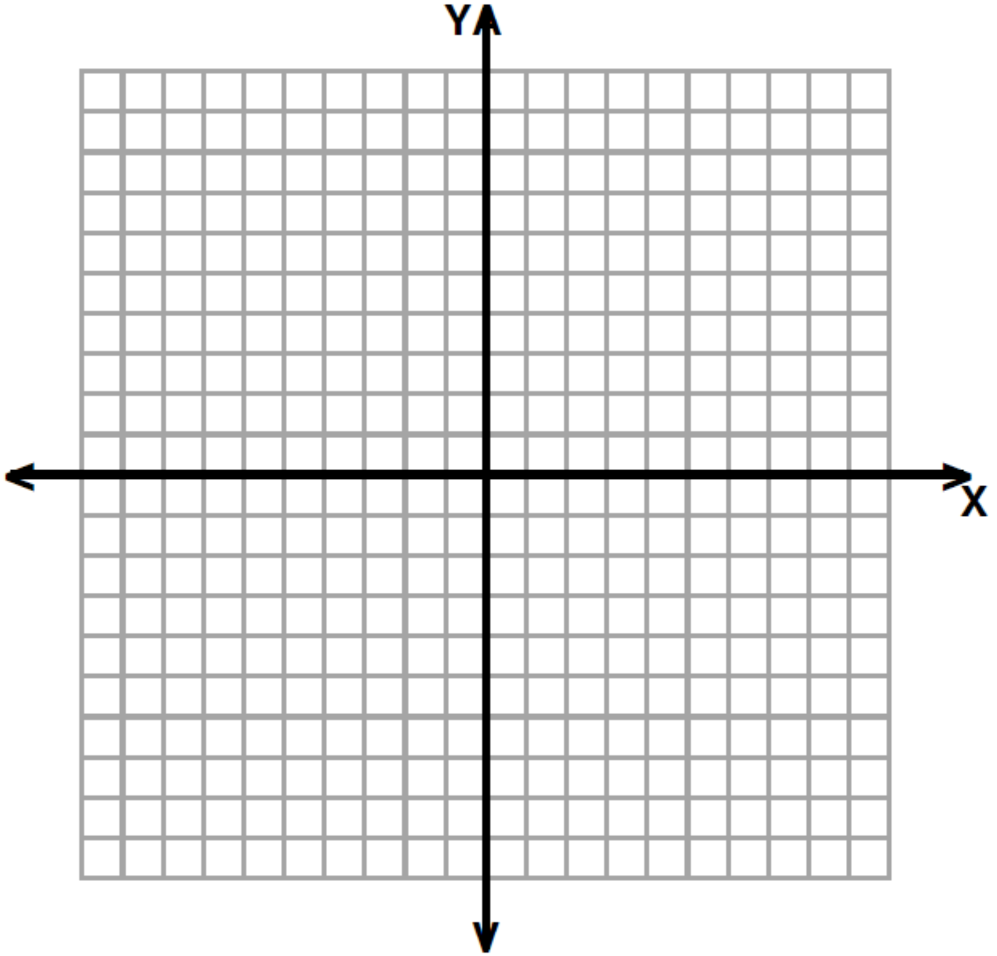
GRAPHING RATIONAL FUNCTIONS WITH TRANSFORMATIONS

- 1) Find and graph the asymptotes.
- 2) Find and graph the zeros.
- 3) Find and graph some points around the asymptotes.

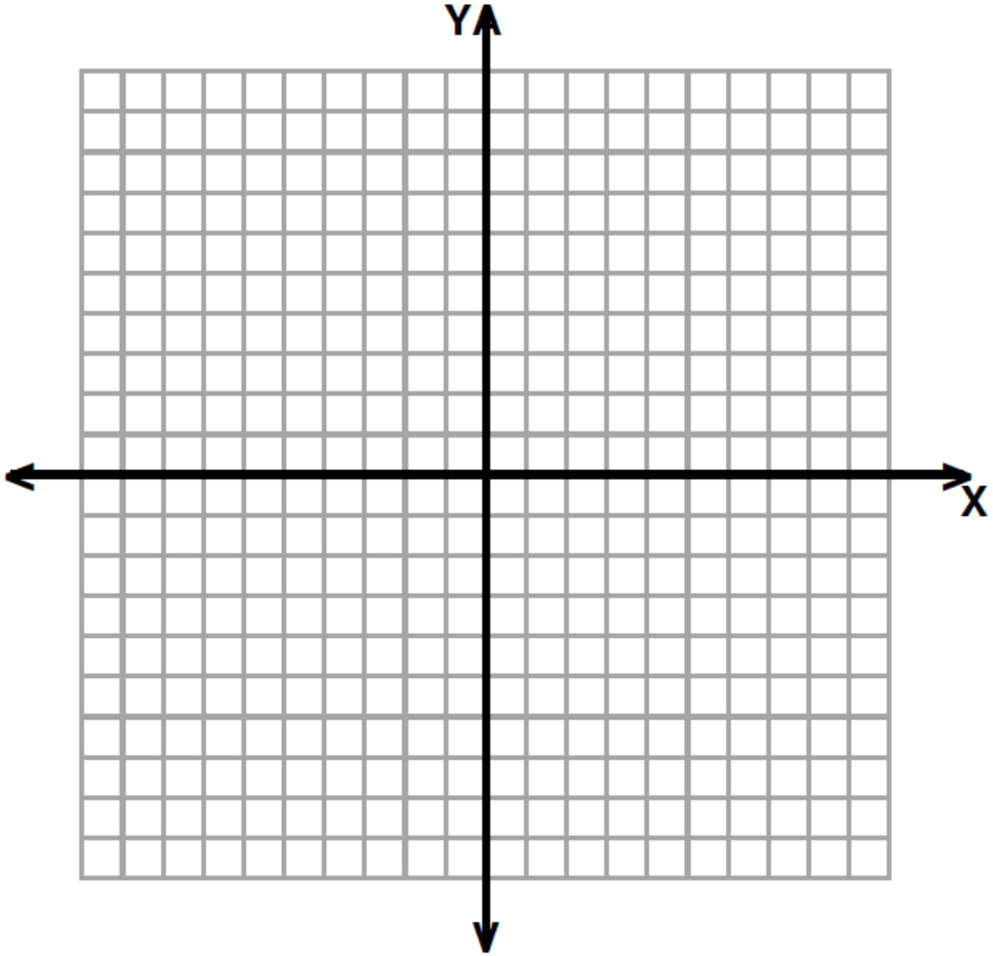
Graph $y = \dfrac{4}{x^2 + 1}$. State the domain and range.



Graph $y = \dfrac{3x^2}{x^2 - 4}$.



Graph $y = \dfrac{x^2 - 2x - 3}{x + 4}$.



9.4 – MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

SIMPLIFYING A RATIONAL EXPRESSION

Simplify: $\dfrac{x^2 - 4x - 12}{x^2 - 4}$

MULTIPLYING RATIONAL EXPRESSIONS

Multiply: $\dfrac{4x - 4x^2}{x^2 + 2x - 3} \cdot \dfrac{x^2 + x - 6}{4x}$

MULTIPLYING RATIONAL EXPRESSIONS BY A POLYNOMIAL

Multiply: $\frac{x + 3}{8x^3 - 1} \cdot (4x^2 + 2x + 1)$

DIVIDING RATIONAL EXPRESSIONS

Divide: $\dfrac{5x}{3x - 12} \div \dfrac{x^2 - 2x}{x^2 - 6x + 8}$

MULTIPLYING AND DIVIDING

Divide: $\dfrac{6x^2 + 7x - 3}{6x^2} \div (2x^2 + 3x)$

9.5 – ADDITION, SUBTRACTION AND COMPLEX FRACTIONS

ADDING / SUBTRACTING RATIONAL EXPRESSIONS WITH SAME DENOMINATORS

Perform the indicated operation.

$$\frac{2x}{x+3} - \frac{4}{x+3}$$

ADDING RATIONAL EXPRESSIONS WITH DIFFERENT DENOMINATORS

Add: $\dfrac{5}{6x^2} + \dfrac{x}{4x^2 - 12x}$

SUBTRACTING RATIONAL EXPRESSIONS WITH DIFFERENT DENOMINATORS

Subtract: $\dfrac{x+1}{x^2+4x+4} - \dfrac{2}{x^2-4}$

SIMPLIFYING COMPLEX FRACTIONS

Simplify: $$\frac{\dfrac{2}{x+2}}{\dfrac{1}{x+2}+\dfrac{2}{x}}$$

COMPLEX FRACTIONS "CHEAT SHEET"

- $\dfrac{\frac{a}{b}}{c} = \dfrac{a}{bc}$
- $\dfrac{a}{\frac{b}{c}} = \dfrac{ac}{b}$
- $\dfrac{\frac{a}{b}}{\frac{c}{d}} = \dfrac{ad}{bc}$

9.6 – SOLVING RATIONAL EQUATIONS

STEPS TO SOLVING

- 1) Find the least common denominator of all the rational parts.
- 2) Multiply both sides by the LDC.
- 3) Solve the remaining equation.
- 4) Check for extraneous solutions.

EQUATION WITH ONE SOLUTION

Solve: $\frac{4}{x} + \frac{5}{2} = -\frac{11}{x}$

EQUATION WITH AN EXTRANEOUS SOLUTION

Solve: $\frac{5x}{x-2} = 7 + \frac{10}{x-2}$

EQUATION WITH TWO SOLUTIONS

Solve: $\dfrac{4x + 1}{x + 1} = \dfrac{12}{x^2 - 1} + 3$

SOLVING BY CROSS-MULTIPLYING

Solve: $\dfrac{2}{x^2 - x} = \dfrac{1}{x - 1}$