CHAPTER 7

Powers, Roots and Radicals

7.1 NTH ROOTS AND RATIONAL EXPONENTS

RATIONAL EXPONENTS

RATIONAL EXPONENTS

Let $a^{1/n}$ be an nth root of a, and let m be a positive integer.

- $a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$
- $a^{-m/n} = \dfrac{1}{a^{m/n}} = \dfrac{1}{(a^{1/n})^m} = \dfrac{1}{(\sqrt[n]{a})^m}, \; a \neq 0$

In addition:

- Even roots of positive numbers have two solutions.

- Even roots of negative numbers have no solution.

- Odd roots have one solution.

- Any root of 0 is 0.

Evaluate the expression.

4. $\sqrt[4]{81}$

5. $-(49^{1/2})$

6. $(\sqrt[3]{-8})^5$

7. $3125^{2/5}$

Solve the equation.

8. $x^3 = 125$

9. $3x^5 = -3$

10. $(x + 4)^2 = 0$

11. $x^4 - 7 = 9993$

7.2 PROPERTIES OF RATIONAL EXPONENTS

CONCEPT SUMMARY

PROPERTIES OF RATIONAL EXPONENTS

Let *a* and *b* be real numbers and let *m* and *n* be rational numbers. The following properties have the same names as those listed on page 323, but now apply to rational exponents as illustrated.

	PROPERTY	EXAMPLE
1.	$a^m \cdot a^n = a^{m+n}$	$3^{1/2} \cdot 3^{3/2} = 3^{(1/2 + 3/2)} = 3^2 = 9$
2.	$(a^m)^n = a^{mn}$	$(4^{3/2})^2 = 4^{(3/2 \cdot 2)} = 4^3 = 64$
3.	$(ab)^m = a^m b^m$	$(9 \cdot 4)^{1/2} = 9^{1/2} \cdot 4^{1/2} = 3 \cdot 2 = 6$
4.	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$25^{-1/2} = \frac{1}{25^{1/2}} = \frac{1}{5}$
5.	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{6^{5/2}}{6^{1/2}} = 6^{(5/2 - 1/2)} = 6^2 = 36$
6.	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{8}{27}\right)^{1/3} = \frac{8^{1/3}}{27^{1/3}} = \frac{2}{3}$

5. $3^{1/4} \cdot 3^{3/4}$

6. $(5^{1/3})^6$

7. $\sqrt[3]{16} \cdot \sqrt[3]{4}$

8. $4^{-1/2}$

9. $\sqrt[4]{\dfrac{16}{81}}$

10. $\sqrt[3]{\dfrac{1}{4}}$

11. $8^{1/7} + 2\left(8^{1/7}\right)$

12. $\sqrt{200} - 3\sqrt{2}$

13. $x^{2/3} \cdot x^{4/3}$

14. $(y^{1/6})^3$

15. $\sqrt{4a^6}$

16. $b^{-1/3}$

17. $\sqrt[5]{\dfrac{x^{10}}{y^5}}$ **18.** $\sqrt[3]{\dfrac{x^2}{z}}$ **19.** $2a^{1/5} - 6a^{1/5}$ **20.** $x\sqrt[3]{y^6} + y^2\sqrt[3]{x^3}$

7.3 POWER FUNCTIONS AND FUNCTION OPERATIONS

Concept Summary — OPERATIONS ON FUNCTIONS

Let f and g be any two functions. A new function h can be defined by performing any of the four basic operations (addition, subtraction, multiplication, and division) on f and g.

Operation	Definition	Example: $f(x) = 2x$, $g(x) = x + 1$
ADDITION	$h(x) = f(x) + g(x)$	$h(x) = 2x + (x + 1) = 3x + 1$
SUBTRACTION	$h(x) = f(x) - g(x)$	$h(x) = 2x - (x + 1) = x - 1$
MULTIPLICATION	$h(x) = f(x) \cdot g(x)$	$h(x) = (2x)(x + 1) = 2x^2 + 2x$
DIVISION	$h(x) = \dfrac{f(x)}{g(x)}$	$h(x) = \dfrac{2x}{x + 1}$

The domain of h consists of the x-values that are in the domains of both f and g. Additionally, the domain of a quotient does not include x-values for which $g(x) = 0$.

COMPOSITION OF FUNCTIONS

A composition of functions, written $f \circ g$ or $f(g(x))$, occurs when you input a function into another function.

The result of a composition is a function.

FINDING THE DOMAIN OF $f \circ g$

$f \circ g$ or $f(g(x))$

1) Find the domain of $f(x)$.

2) Set $\mathrm{g}(x)$ equal to the value(s) from step 1 and solve.

Let $f(x) = 3x^{-1}$ and $g(x) = 2x - 1$. Find the following.

a. $f(g(x))$ **b.** $g(f(x))$ **c.** $f(f(x))$ **d.** the domain of each composition

7.4 INVERSE FUNCTIONS

DEFINITION: INVERSE FUNCTION

INVERSE FUNCTIONS

Functions f and g are inverses of each other provided:

$$f(g(x)) = x \qquad \text{and} \qquad g(f(x)) = x$$

The function g is denoted by f^{-1}, read as "f inverse."

The domain of the inverse is the range of the original function.

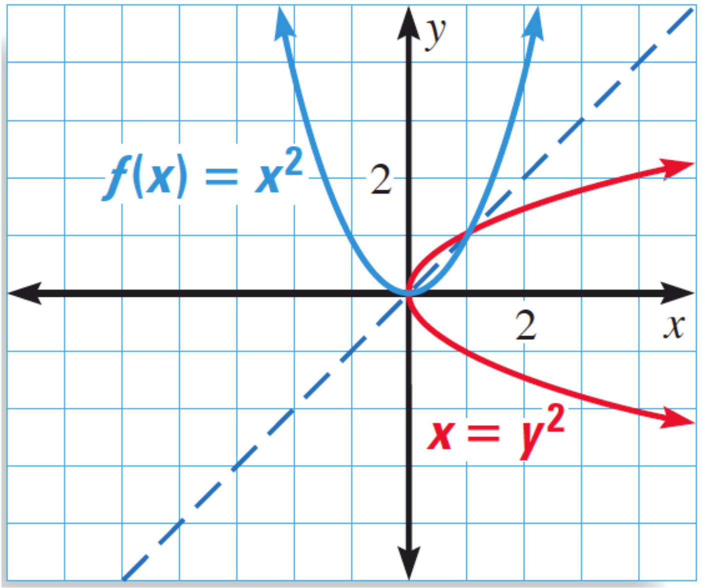
METHODS TO GET THE INVERSE

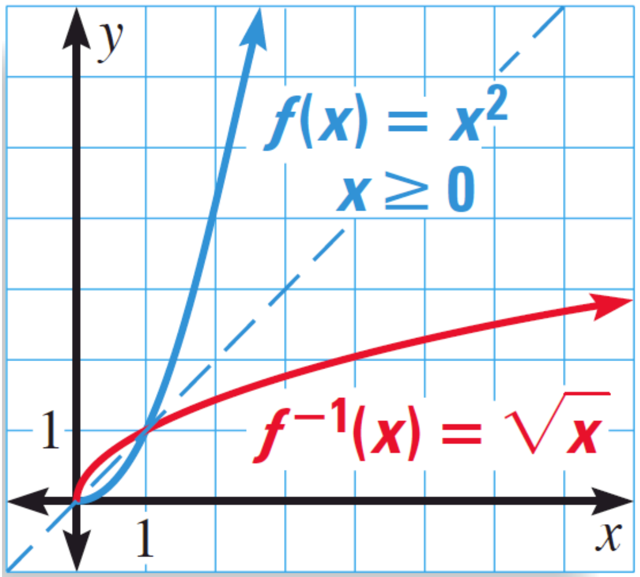
- Graphically.
- From coordinates.
- Algebraically.

FINDING THE INVERSE GRAPHICALLY

1) Draw the lines $y = x$.

2) Draw the reflection of the function with respect to lines $y = x$.





FINDING THE INVERSE FROM COORDINATES

Switch the x and lines y coordinates.

$f(x)$

x	1	-2	3	10
y	5	2	7	-6

g(x).   (1, 5) (-2, 2) (3, 7)

$f^{-1}(x)$

x				
y				

$g^{-1}(x)$

FINDING THE INVERSE ALGEBRAICALLY

1) Switch the x and y.

2) Solve for y.

Find an equation for the inverse of the relation $y = 2x - 4$

find the inverse $f(x) = \frac{1}{2}x^3 - 2$

VERIFYING THE INVERSE

$f^{-1}(g(x)) = g^{-1}(f(x)) = x$

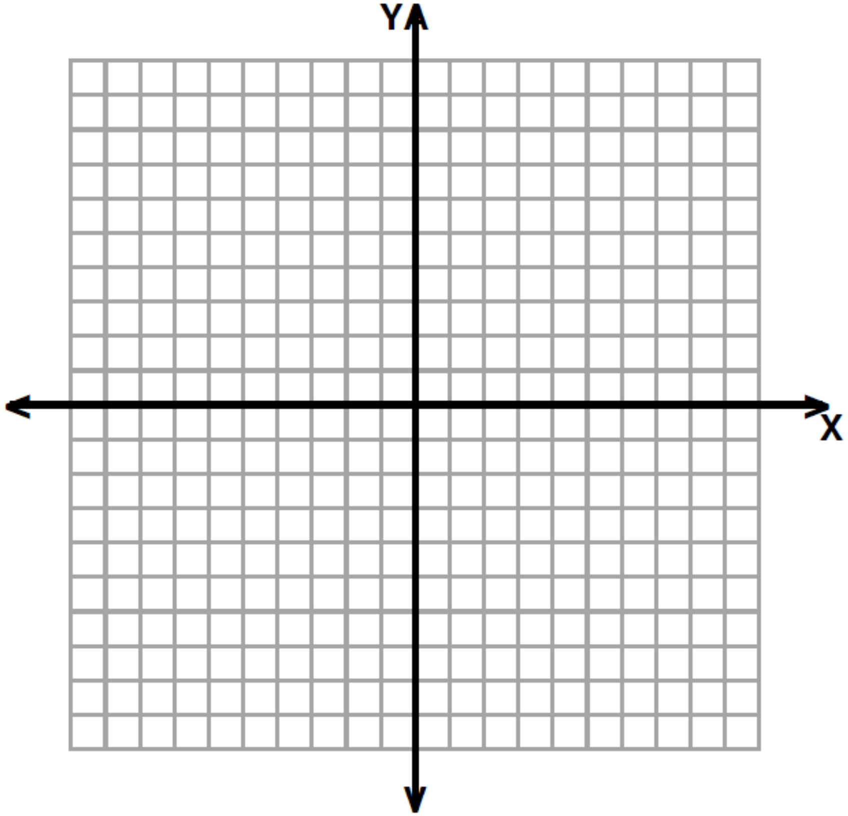
To verify that a function is the inverse of another, find $f \circ g$. If is equals x, the functions are inverses of each other.

Verify that $f(x) = 2x - 4$ and $f^{-1}(x) = \frac{1}{2}x + 2$ are inverses.

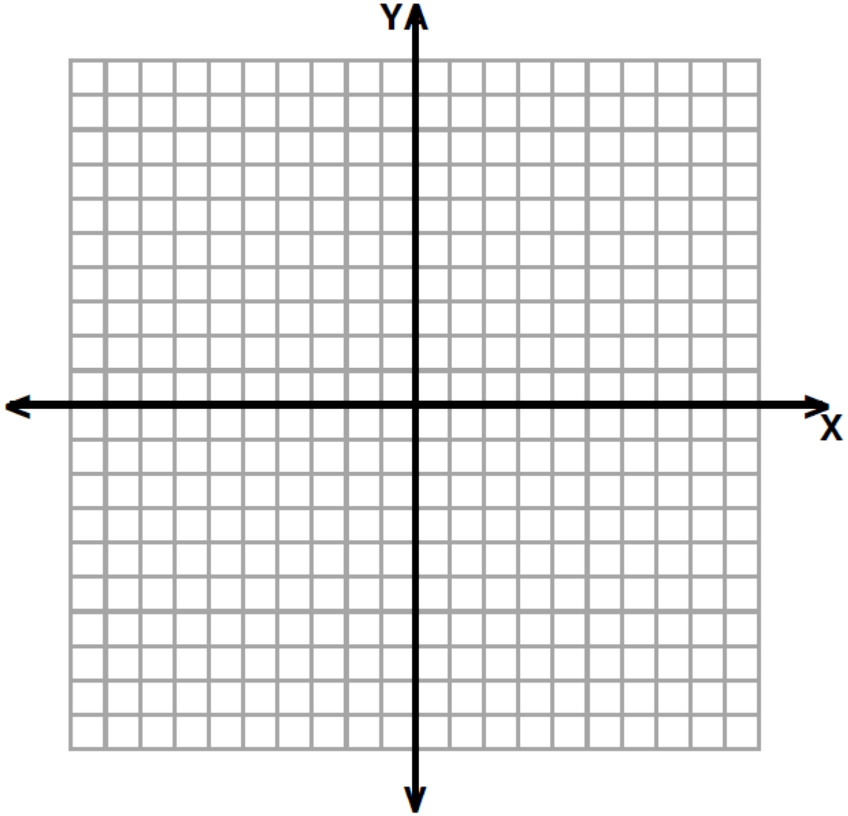
7.5 GRAPHING SQUARE ROOT AND CUBE ROOT FUNCTIONS

BASE FUNCTION

$f(x) = \sqrt{x}$

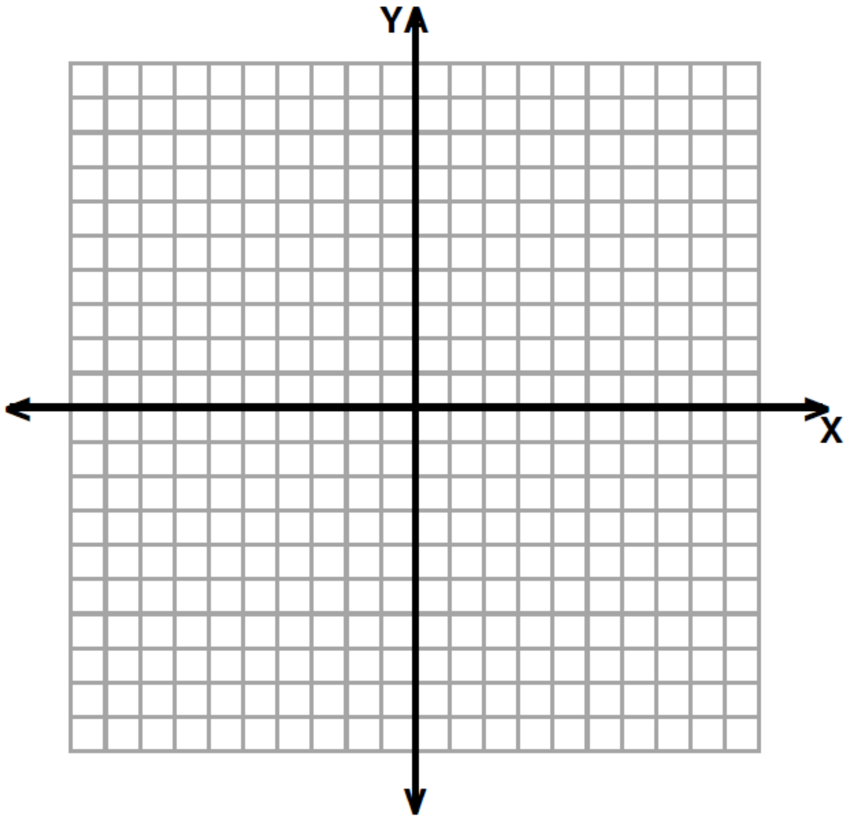


$g(x) = \sqrt[3]{x}$

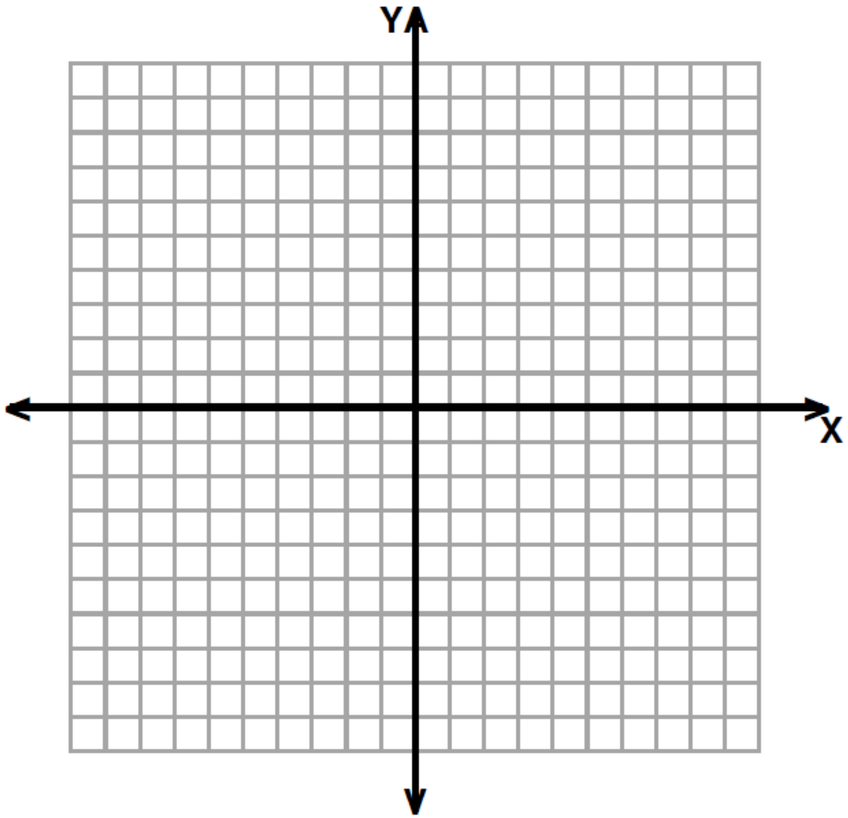


NEGATIVE "A"

$f(x) = -\sqrt{x}$

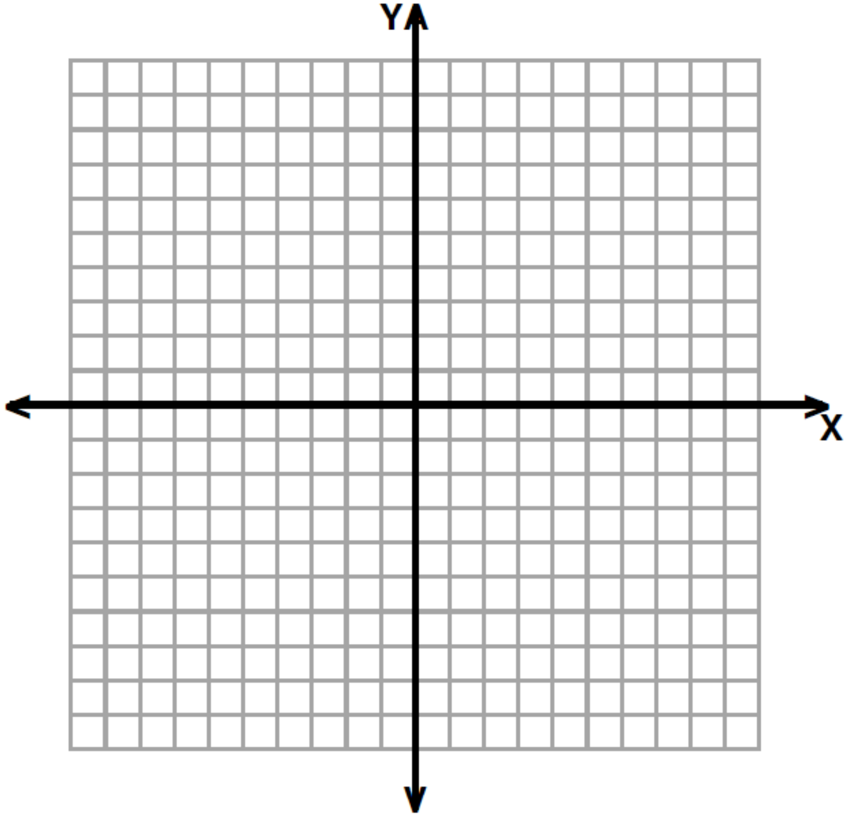


$g(x) = -\sqrt[3]{x}$

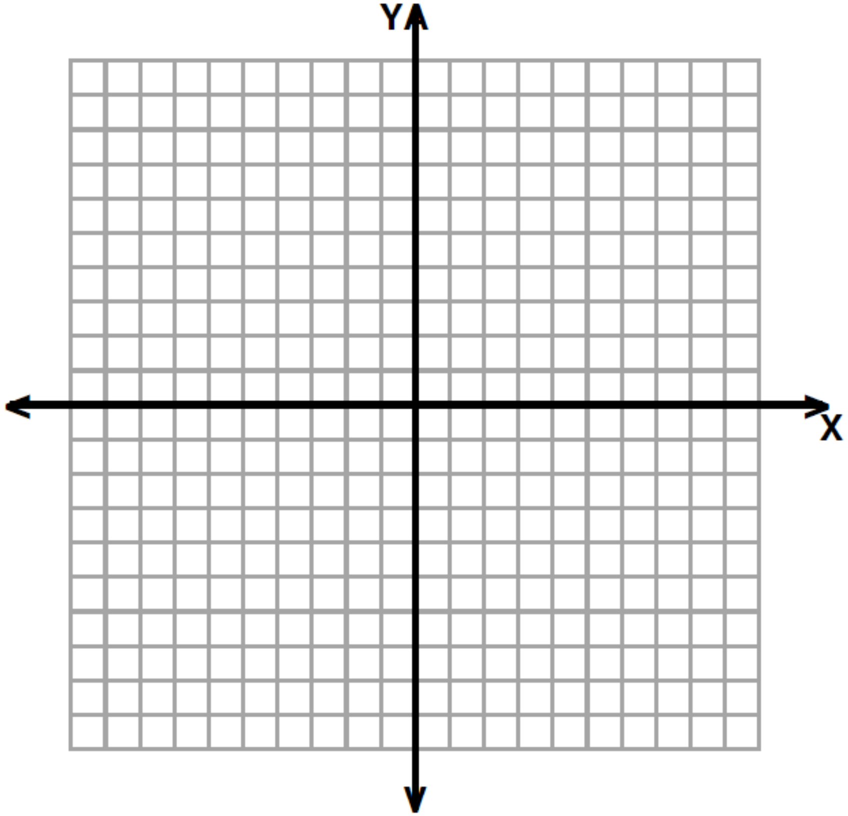


EFFECT OF H

$f(x) = \sqrt{x - h}$

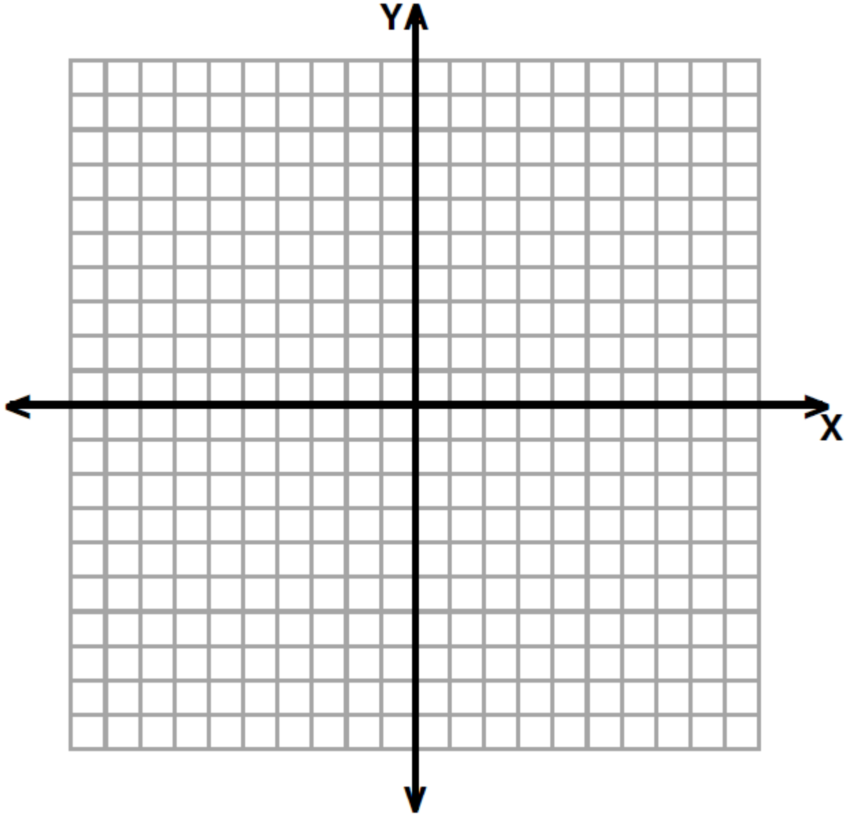


$g(x) = \sqrt[3]{x - h}$

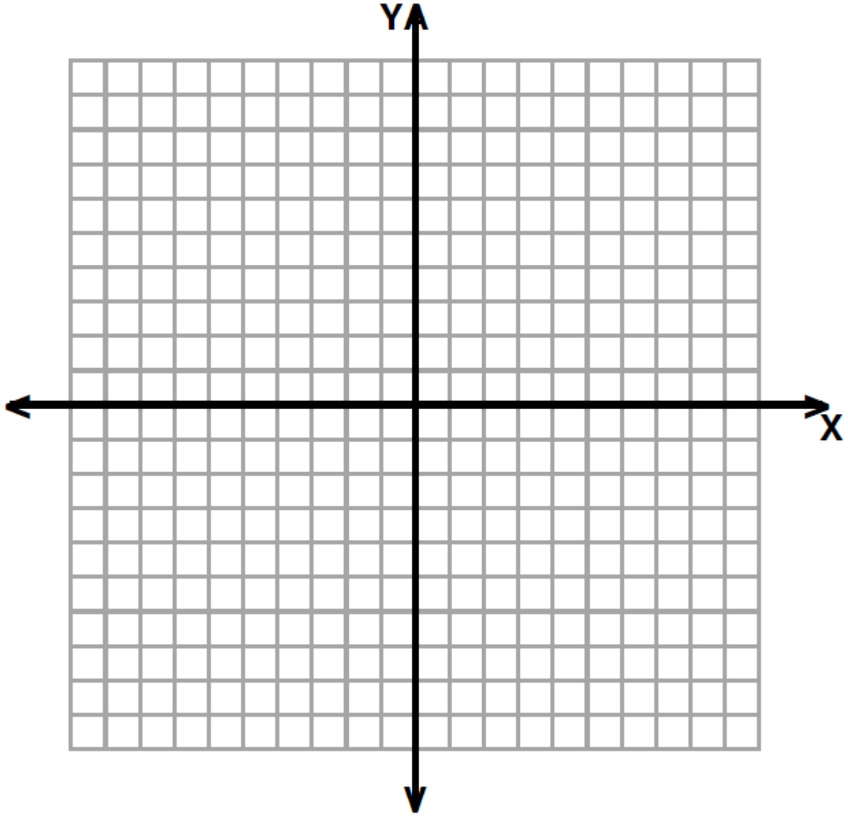


EFFECT OF K

$f(x) = \sqrt{x} + k$



$g(x) = \sqrt[3]{x} + k$



DOMAIN AND RANGE

$f(x) = \sqrt{x}$

$g(x) = \sqrt[3]{x}$

7.6 SOLVING RADICAL EQUATIONS

STEPS TO SOLVING RADICAL EQUATIONS

1) Solve the equation.

2) Plug solution(s) into original equation to check for extraneous solution.

Solve $\sqrt[3]{x} - 4 = 0$.

Solve $2x^{3/2} = 250$.

Solve $\sqrt{4x - 7} + 2 = 5$.

Solve $\sqrt{3x + 2} - 2\sqrt{x} = 0$.

Solve $x - 4 = \sqrt{2x}$.