CHAPTER 7

Powers, Roots and Radicals

7.1 NTH ROOTS AND RATIONAL EXPONENTS

RATIONAL EXPONENTS

RATIONAL EXPONENTS

Let $a^{1/n}$ be an nth root of a, and let m be a positive integer.

- $a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$
- $a^{-m/n} = \dfrac{1}{a^{m/n}} = \dfrac{1}{(a^{1/n})^m} = \dfrac{1}{(\sqrt[n]{a})^m}, a \neq 0$

$\sqrt[n]{a} = a^{1/n}$

$\sqrt{a} = a^{1/2}$

$\sqrt[3]{a} = a^{1/3}$

In addition:

- Even roots of positive numbers have two solutions. $\sqrt{4} = \pm 2$
- Even roots of negative numbers have no solution. $\sqrt{-4} \rightarrow$ no solution
- Odd roots have one solution. $\sqrt[3]{8} = 2$ $\sqrt[3]{-8} = -2$
- Any root of 0 is 0. $\sqrt{0} = 0$ $\sqrt[5]{0} = 0$

Evaluate the expression.

4. $\sqrt[4]{81} = 3$

$x^4 = 81$

$3^4 = 81$

5. $-\left(49^{1/2}\right)$

$-\sqrt{49}$

$-(\pm 7)$

± 7

6. $\left(\sqrt[3]{-8}\right)^5$

$(-2)^5$

-32

7. $3125^{2/5}$

$\left(\sqrt[5]{3125}\right)^2$

$x^5 = 3125$

$5^2 = 25$

Solve the equation.

8. $x^3 = 125$

$\sqrt[3]{x^3} = \sqrt[3]{125}$

$x = 5$

9. $3x^5 = -3$

$\frac{3x^5}{3} = \frac{-3}{3}$

$x^5 = -1$

$x = \sqrt[5]{-1}$

$x = -1$

10. $\sqrt{(x + 4)^2} = \sqrt{0}$

$x + 4 = 0$

$x = -4$

11. $x^4 - 7 = 9993$

$\sqrt[4]{x^4} = \sqrt[4]{10\,000}$

$x = \pm 10$

7.2 PROPERTIES OF RATIONAL EXPONENTS

CONCEPT SUMMARY

PROPERTIES OF RATIONAL EXPONENTS

Let *a* and *b* be real numbers and let *m* and *n* be rational numbers. The following properties have the same names as those listed on page 323, but now apply to rational exponents as illustrated.

	PROPERTY	EXAMPLE
1.	$a^m \cdot a^n = a^{m+n}$	$3^{1/2} \cdot 3^{3/2} = 3^{(1/2 + 3/2)} = 3^2 = 9$
2.	$(a^m)^n = a^{mn}$	$(4^{3/2})^2 = 4^{(3/2 \cdot 2)} = 4^3 = 64$
3.	$(ab)^m = a^m b^m$	$(9 \cdot 4)^{1/2} = 9^{1/2} \cdot 4^{1/2} = 3 \cdot 2 = 6$
4.	$a^{-m} = \frac{1}{a^m},\ a \neq 0$	$25^{-1/2} = \frac{1}{25^{1/2}} = \frac{1}{5}$
5.	$\frac{a^m}{a^n} = a^{m-n},\ a \neq 0$	$\frac{6^{5/2}}{6^{1/2}} = 6^{(5/2 - 1/2)} = 6^2 = 36$
6.	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m},\ b \neq 0$	$\left(\frac{8}{27}\right)^{1/3} = \frac{8^{1/3}}{27^{1/3}} = \frac{2}{3}$

5. $3^{1/4} \cdot 3^{3/4}$

$3^{\frac{4}{4}}$

3^{1}

6. $(5^{1/3})^6$

$5^{\frac{6}{3}}$

5^{2}

25

7. $\sqrt[3]{16} \cdot \sqrt[3]{4}$

$16^{1/3}\, 4^{1/3}$

$(16 \cdot 4)^{1/3}$

$(64)^{1/3}$

4

8. $4^{-1/2}$

$\frac{1}{4^{1/2}}$

$\frac{1}{\sqrt{4}}$

$\pm\frac{1}{2}$

9. $\sqrt[4]{\frac{16}{81}}$

$\frac{\sqrt[4]{16}}{\sqrt[4]{81}}$

$\pm\frac{2}{3}$

10. $\sqrt[3]{\frac{1}{4}}$

$\frac{1}{\sqrt[3]{4}}$

$\frac{1}{4^{1/3}}$

11. $8^{1/7} + 2(8^{1/7})$

$8^{1/7}(1+2)$

$3 \cdot 8^{1/7}$

$\sqrt[7]{8} + 2\sqrt[7]{8}$

2 different methods

$3\sqrt[7]{8}$

12. $\sqrt{200} - 3\sqrt{2}$

$\sqrt{100}\sqrt{2} - 3\sqrt{2}$

$10\sqrt{2} - 3\sqrt{2}$

$7\sqrt{2}$

13. $x^{2/3} \cdot x^{4/3}$

$x^{\frac{2}{3}+\frac{4}{3}}$

$x^{\frac{6}{3}}$

x^{2}

14. $(y^{1/6})^3$

$y^{3/6}$

$y^{1/2}$

$\sqrt{y}$

15. $\sqrt{4a^6}$

$\sqrt{4}\,(a^6)^{1/2}$

$2a^{6\cdot\frac{1}{2}}$

$2a^3$

16. $b^{-1/3}$

$\frac{1}{b^{1/3}}$

$\frac{1}{\sqrt[3]{b}}$

17. $\sqrt[5]{\frac{x^{10}}{y^5}}$

$\left(\frac{x^{10}}{y^5}\right)^{1/5}$

$\frac{x^{10 \cdot 1/5}}{y^{5 \cdot 1/5}}$

$\frac{x^2}{y}$

18. $\sqrt[3]{\frac{x^2}{z}}$

$\left(\frac{x^2}{z}\right)^{1/3}$

$\frac{x^{2/3}}{z^{1/3}}$

19. $2a^{1/5} - 6a^{1/5}$

$-4a^{1/5}$

$-4\sqrt[5]{a}$

20. $x\sqrt[3]{y^6} + y^2\sqrt[3]{x^3}$

$x(y^6)^{1/3} + y^2(x^3)^{1/3}$

$xy^2 + y^2x$

$2xy^2$

7.3 POWER FUNCTIONS AND FUNCTION OPERATIONS

CONCEPT SUMMARY

OPERATIONS ON FUNCTIONS

Let f and g be any two functions. A new function h can be defined by performing any of the four basic operations (addition, subtraction, multiplication, and division) on f and g.

Operation	Definition	Example: $f(x) = 2x$, $g(x) = x + 1$
ADDITION	$h(x) = f(x) + g(x)$	$h(x) = 2x + (x + 1) = 3x + 1$
SUBTRACTION	$h(x) = f(x) - g(x)$	$h(x) = 2x - (x + 1) = x - 1$
MULTIPLICATION	$h(x) = f(x) \cdot g(x)$	$h(x) = (2x)(x + 1) = 2x^2 + 2x$
DIVISION	$h(x) = \frac{f(x)}{g(x)}$	$h(x) = \frac{2x}{x + 1}$ $x \neq -1$

The domain of h consists of the x-values that are in the domains of both f and g. Additionally, the domain of a quotient does not include x-values for which $g(x) = 0$.

Let $f(x) = 4x$ and $g(x) = x - 1$. Perform the indicated operation and state the domain.

5. $f(x) + g(x)$

$h(x) = 4x + (x - 1)$

$h(x) = 5x - 1$

Dom: $\mathbb{R}$

6. $f(x) - g(x)$

$h(x) = 4x - (x - 1)$

$h(x) = 3x + 1$

Dom: $\mathbb{R}$

7. $f(x) \cdot g(x)$

$h(x) = 4x(x - 1)$

$h(x) = 4x^2 - 4x$

Dom: $\mathbb{R}$

8. $\frac{f(x)}{g(x)}$

$h(x) = \frac{4x}{x - 1}$

Dom: $x \neq 1$

COMPOSITION OF FUNCTIONS

A composition of functions, written $f \circ g$ or $f(g(x))$, occurs when you input a function into another function.

The result of a composition is a function. Its domain is made up of the values that belong to the range of f and the domain of g.

FINDING THE DOMAIN OF $f \circ g$

$f \circ g$ or $f(g(x))$

1) Find the domain of $f(x)$.

2) Set $g(x)$ equal to the value(s) from step 1 and solve.

3) The domain of $f(g(x))$ is the set of values that was allowed through by $g(x)$ and is allowed through by the function resulting from $f(g(x))$.

Let $f(x) = 3x^{-1}$ and $g(x) = 2x - 1$. Find the following.

a. $f(g(x))$ **b.** $g(f(x))$ **c.** $f(f(x))$ **d.** the domain of each composition

a) $f(g(x)) = 3(2x-1)^{-1} = \frac{3}{2x-1}$

b) $g(f(x)) = 2(3x^{-1}) - 1 = 6x^{-1} - 1 = \frac{6}{x} - 1$

c) $3(3x^{-1})^{-1} = \frac{3}{3x^{-1}} = x$

Let $f(x) = 3x^{-1}$ and $g(x) = 2x - 1$. Find the following.

a. $f(g(x))$ **b.** $g(f(x))$ **c.** $f(f(x))$ **d.** the domain of each composition

a) $f(g(x)) = 3(2x-1)^{-1} = \frac{3}{2x-1}$

Domain:

① $f(x) = \frac{3}{x} \rightarrow x \neq 0 \quad g(x) \neq 0$

② $2x - 1 \neq 0$

$2x \neq 1$

$x \neq \frac{1}{2}$

Dom $f(g(x))$ is all real numbers except $x = \frac{1}{2}$

Let $f(x) = 3x^{-1}$ and $g(x) = 2x - 1$. Find the following.

a. $f(g(x))$ **b.** $g(f(x))$ **c.** $f(f(x))$ **d.** the domain of each composition

b) $g(f(x)) = 2(3x^{-1}) - 1 = 6x^{-1} - 1 = \frac{6}{x} - 1$

Domain: no restrictions for g

f(x) has $x \neq 0$ as a restriction

Let $f(x) = 3x^{-1}$ and $g(x) = 2x - 1$. Find the following.

a. $f(g(x))$ **b.** $g(f(x))$ **c.** $f(f(x))$ **d.** the domain of each composition

c) $3(3x^{-1})^{-1} = \frac{3}{3x^{-1}} = x$

Domain:

The domain of f(x) excludes x=0, so the domain of f(f(x)) also excludes x=0.

7.4 INVERSE FUNCTIONS

DEFINITION: INVERSE FUNCTION

INVERSE FUNCTIONS

Functions f and g are inverses of each other provided:

$$f(g(x)) = x \quad \text{and} \quad g(f(x)) = x$$

The function g is denoted by f^{-1}, read as "f inverse."

The domain of the inverse is the range of the original function.

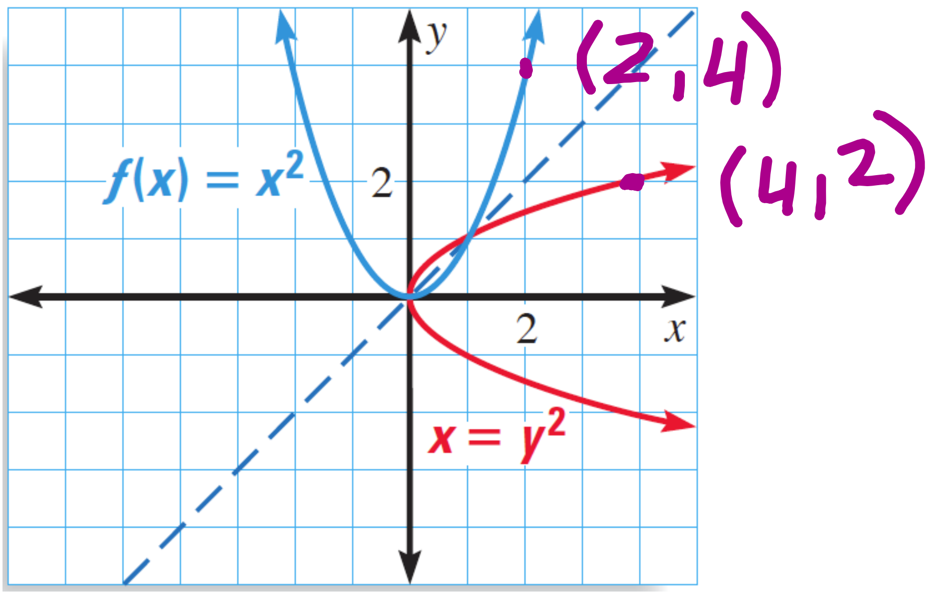
METHODS TO GET THE INVERSE

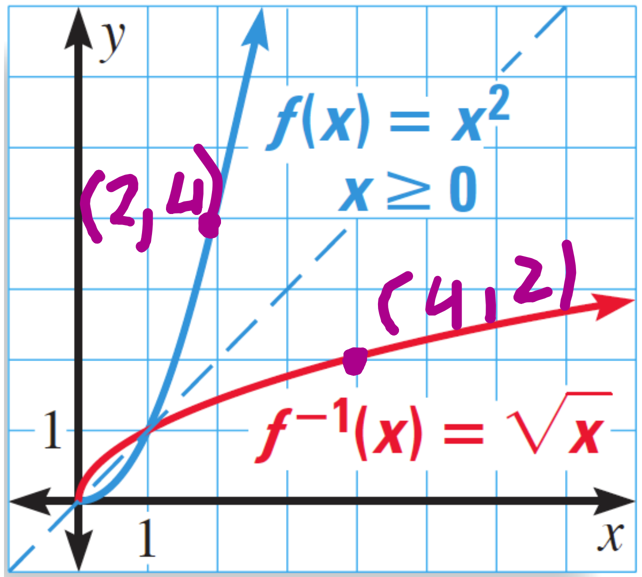
- Graphically.
- From coordinates.
- Algebraically.

FINDING THE INVERSE GRAPHICALLY

1) Draw the lines $y = x$.

2) Draw the reflection of the function with respect to lines $y = x$.





FINDING THE INVERSE FROM COORDINATES

Switch the x and lines y coordinates.

$f(x)$

x	1	-2	3	10
y	5	2	7	-6

$f^{-1}(x)$

x	5	2	7	-6
y	1	-2	3	10

g(x). (1, 5) (-2, 2) (3, 7)

$g^{-1}(x)$ (5, 1) (2, -2) (7, 3)

FINDING THE INVERSE ALGEBRAICALLY

1) Switch the x and y.

2) Solve for y.

Find an equation for the inverse of the relation $y = 2x - 4$

$x = 2y - 4$

$\frac{x+4}{2} = \frac{2y}{2}$

$\frac{x+4}{2} = y \quad y = \frac{x}{2} + 2$

find the inverse $f(x) = \frac{1}{2}x^3 - 2$

$$x = \frac{1}{2}y^3 - 2$$

$$x = \frac{y^3}{2} - 2$$

$$2 \cdot (x+2) = \frac{y^3}{2} \cdot 2$$

$$\sqrt[3]{2x+4} = \sqrt[3]{y^3}$$

$$y = \sqrt[3]{2x+4}$$

VERIFYING THE INVERSE

$$f^{-1}(f(x)) = f(f^{-1}(x)) = x$$

To verify that a function is the inverse of another, find $f \circ f^{-1}$. If is equals x, the functions are inverses of each other.

Verify that $f(x) = 2x - 4$ and $f^{-1}(x) = \frac{1}{2}x + 2$ are inverses.

$$f(f^{-1}(x)) = 2\left(\frac{1}{2}x+2\right)-4$$
$$= x+4-4$$
$$= x$$

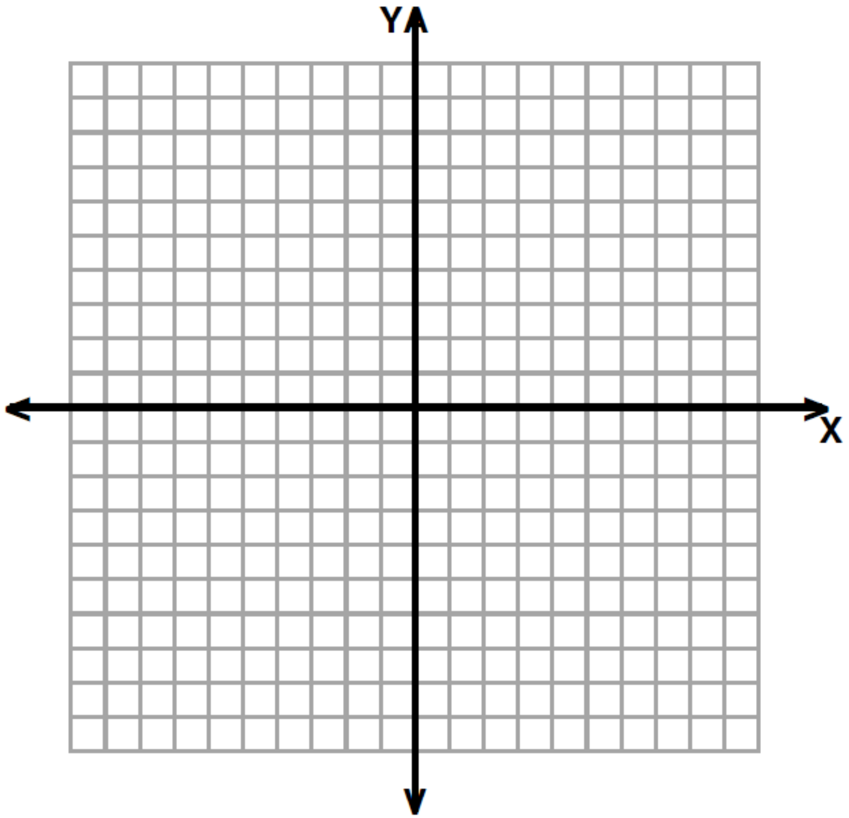
$$f^{-1}(f(x)) = \frac{1}{2}(2x-4)+2$$
$$= x-2+2$$
$$= x$$

They are inverses.

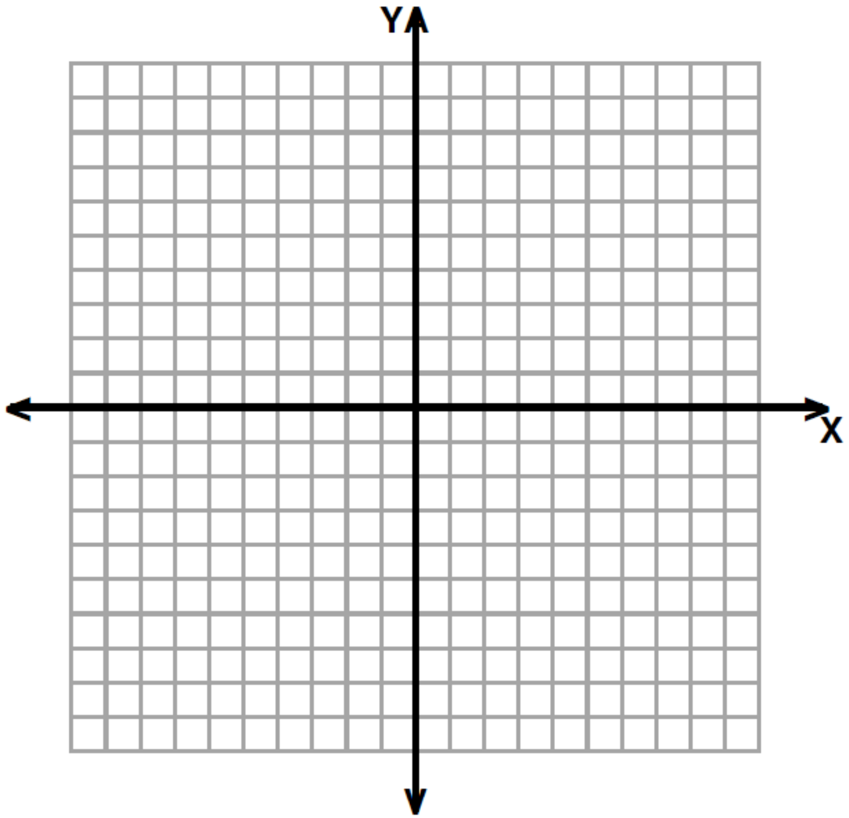
7.5 GRAPHING SQUARE ROOT AND CUBE ROOT FUNCTIONS

BASE FUNCTION

$f(x) = \sqrt{x}$

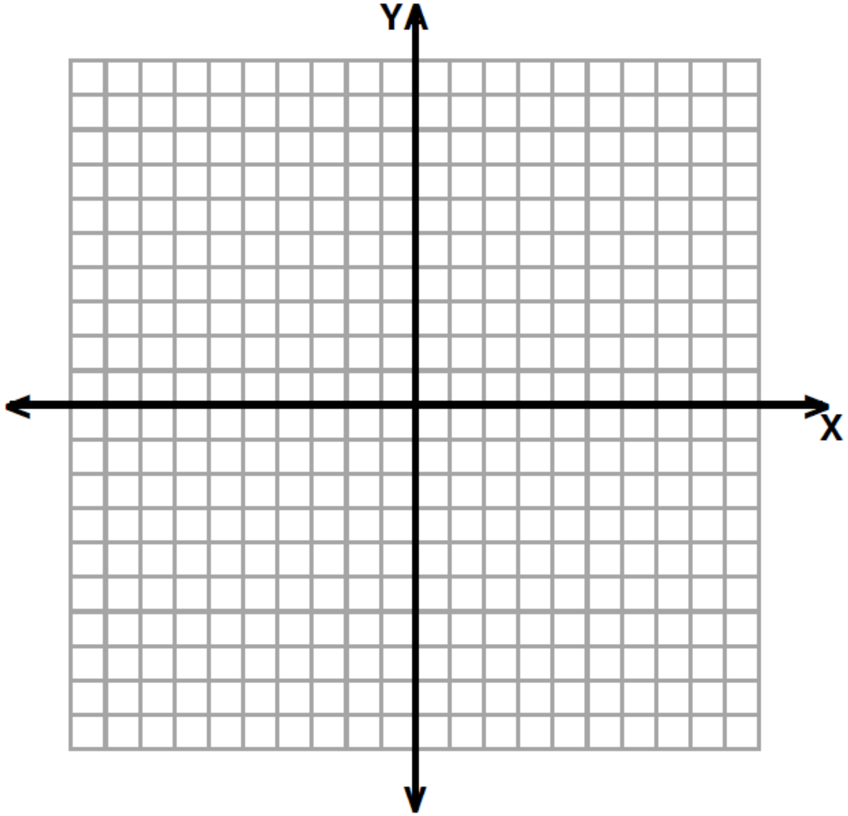


$g(x) = \sqrt[3]{x}$

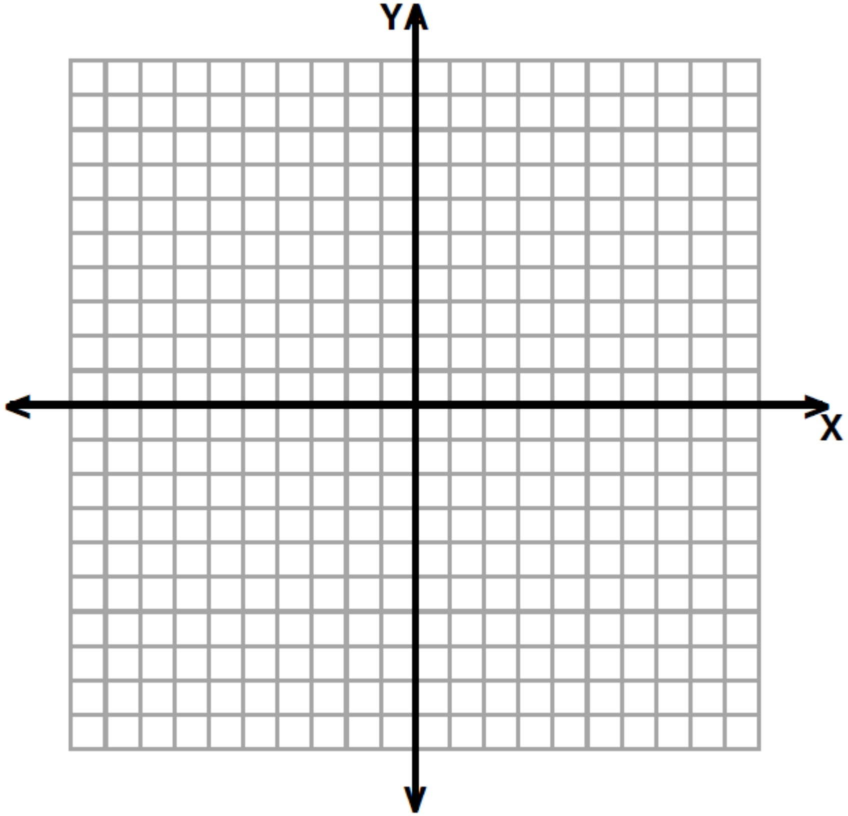


NEGATIVE "A"

$f(x) = -\sqrt{x}$

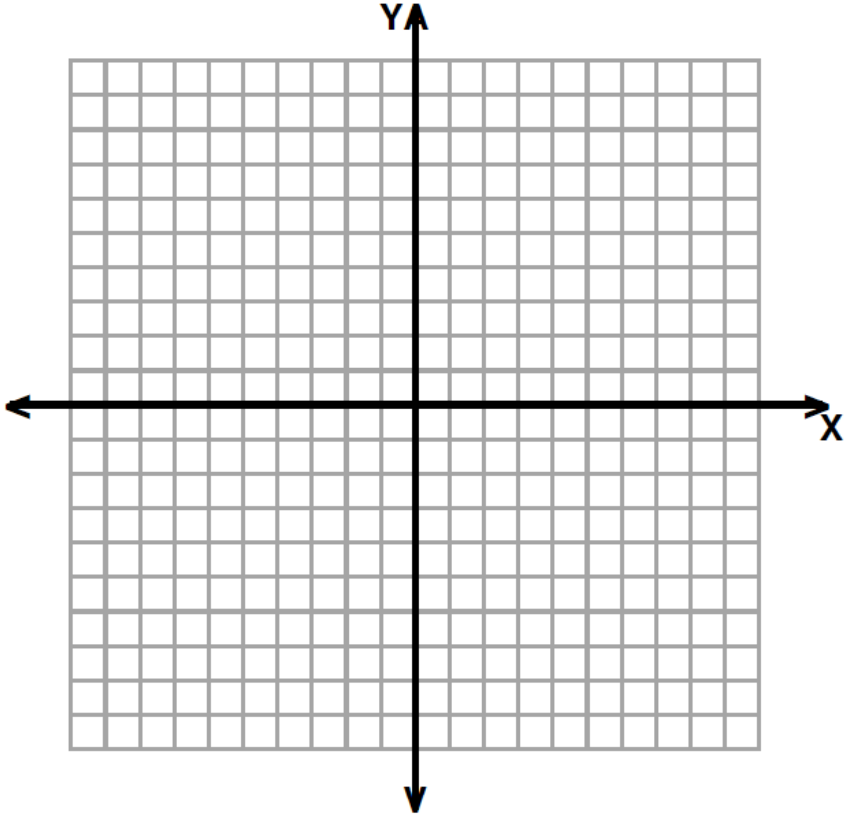


$g(x) = -\sqrt[3]{x}$

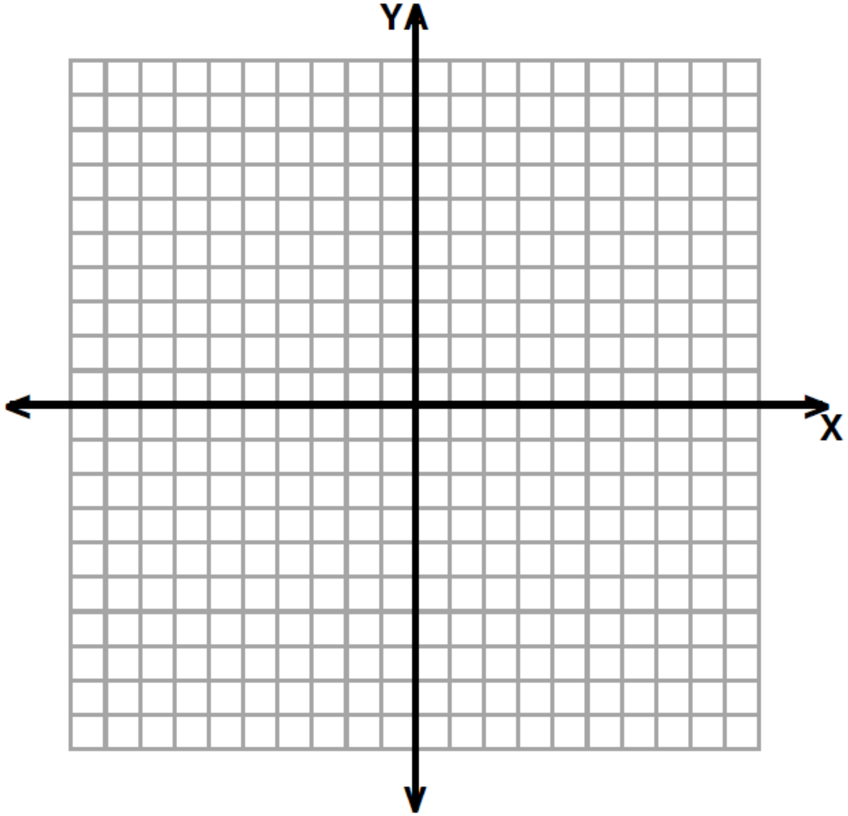


EFFECT OF H

$f(x) = \sqrt{x - h}$

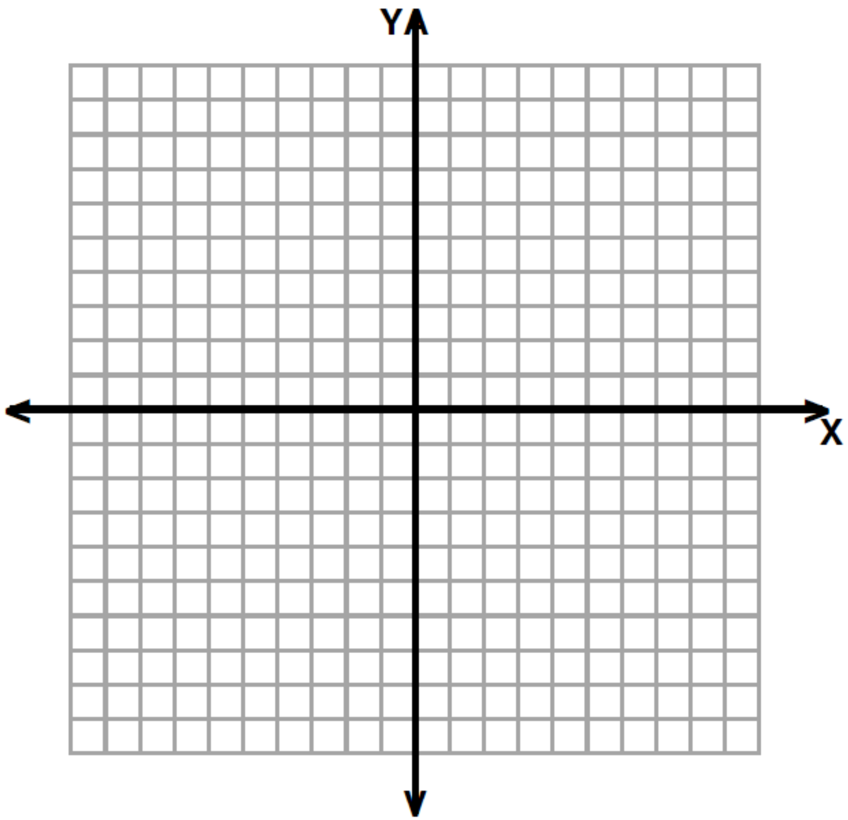


$g(x) = \sqrt[3]{x - h}$

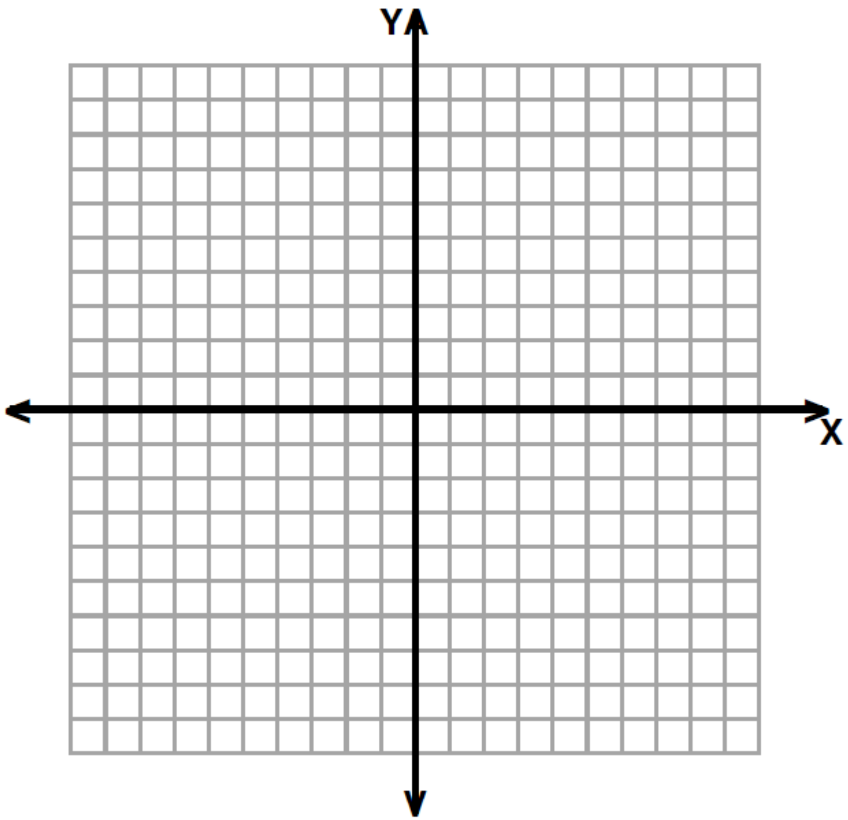


EFFECT OF K

$f(x) = \sqrt{x} + k$



$g(x) = \sqrt[3]{x} + k$



DOMAIN AND RANGE

$f(x) = \sqrt{x}$

$g(x) = \sqrt[3]{x}$

7.6 SOLVING RADICAL EQUATIONS

STEPS TO SOLVING RADICAL EQUATIONS

1) Solve the equation.

2) Plug solution(s) into original equation to check for extraneous solution.

Solve $\sqrt[3]{x} - 4 = 0$.

Solve $2x^{3/2} = 250$.

Solve $\sqrt{4x - 7} + 2 = 5$.

Solve $\sqrt{3x + 2} - 2\sqrt{x} = 0$.

Solve $x - 4 = \sqrt{2x}$.