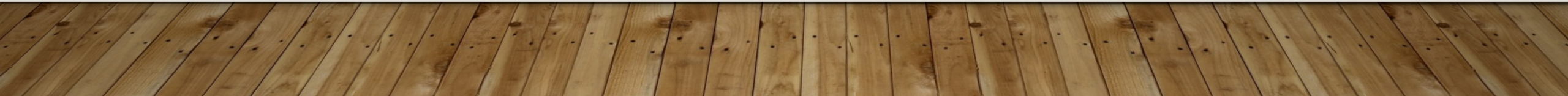# CHAPTER 14: GRAPHING TRIGONOMETRIC FUNCTIONS

What the basic Sine function represents
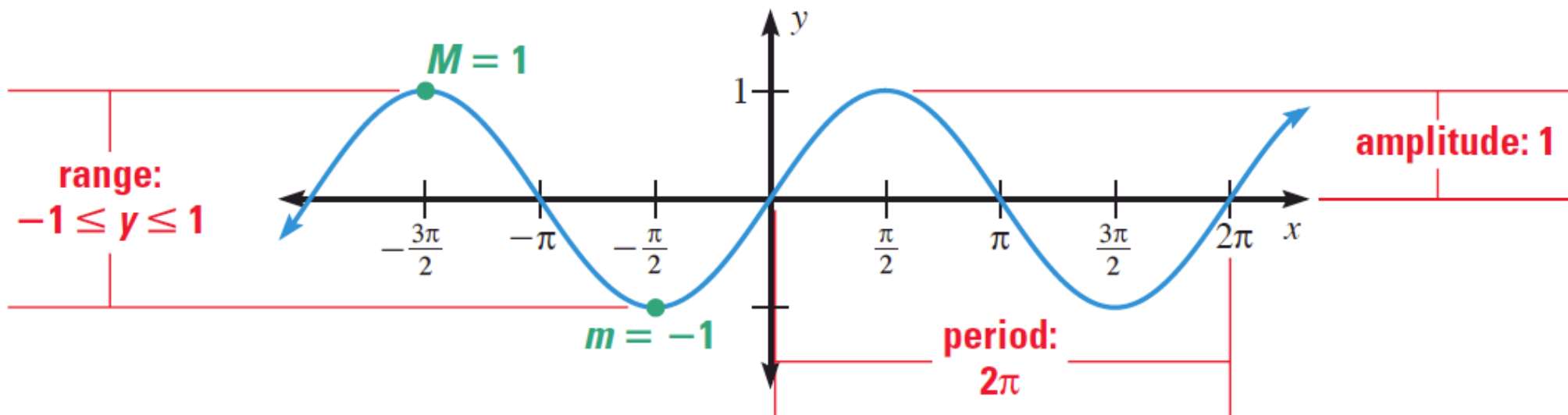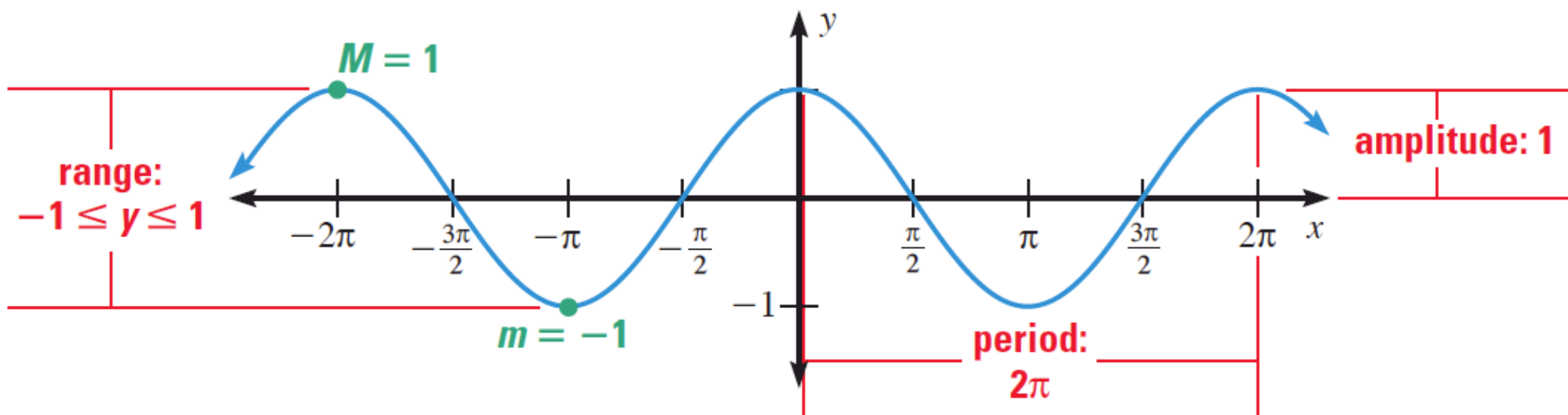
If you are following a point going around on a circle, sine tracks the y-value of the point.

Demo: https://www.desmos.com/calculator/b8tjkyjesu

What the basic Cosine function represents

If you are following a point going around on a circle, cosine tracks the x-value of the circle.

Demo: https://www.desmos.com/calculator/eouprsrtuc

M = 1

range:
−1 ≤ y ≤ 1

amplitude: 1

m = −1

period:
2π

Graph of y = sin x

M = 1

range:
−1 ≤ y ≤ 1

amplitude: 1

m = −1

period:
2π

Graph of y = cos x

Characteristics of basic sine and cosine functions

- Domain: $\mathbb{R}$

- Range: $-1 \leq y \leq 1$ for basic function.

- Functions are **periodic**, sine and cosine have a period of 2π.

- Has a max of 1 and min of -1.

- The **amplitude** is half the distance between the max and min.

- Sine starts at the midline, cosine starts at a max or min.

Graphing basic Sine and Cosine Functions.

*The period controls what you label the x-axis in. 6

$y = \sin(x)$

Period: 2π

Midline: $y = 0$

Max line: $y = 1$

Min line: $y = -1$

Cycle start: $x = 0$

Cycle End: $x = 2\pi$
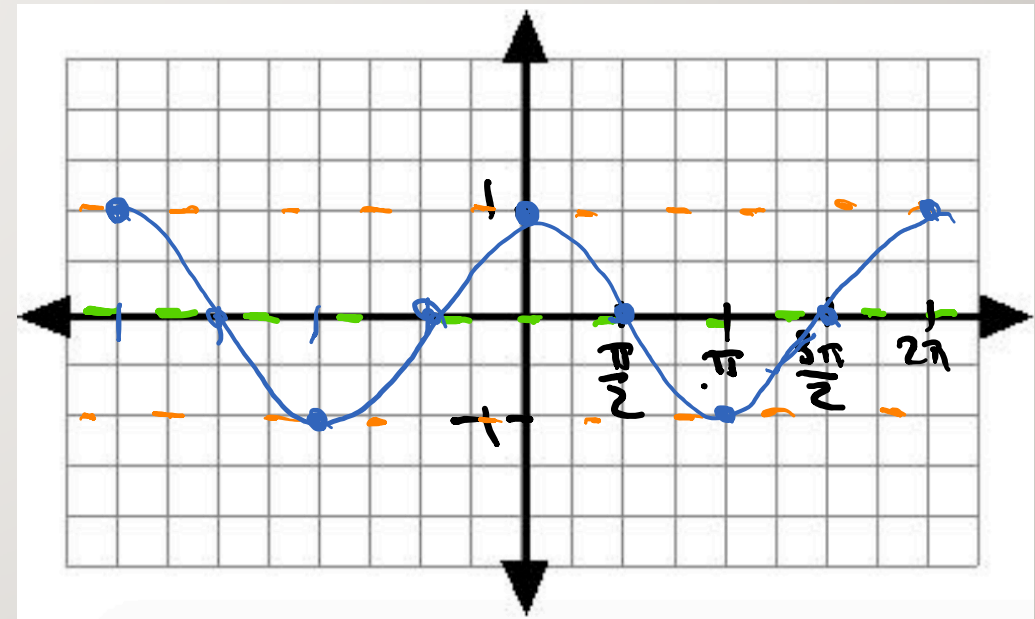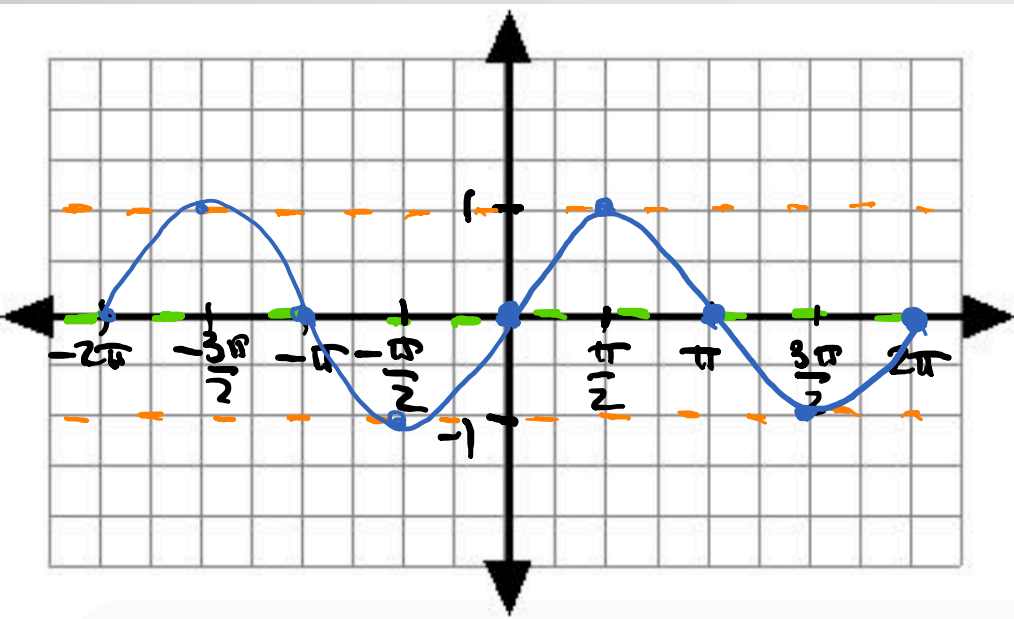
$y = \cos(x)$

Period: 2π

Midline: $y = 0$

Max line: $y = 1$

Min line: $y = -1$

Cycle start: $x = 0$

Cycle End: $x = 2\pi$

Transformed Sine and Cosine Functions

 # h is always the opposite of what you see.

$$y = a \sin b(x - h) + k \qquad\qquad y = a \cos b(x - h) + k$$

(make sure that "b" is factored.

a: amplitude
→ max and min line
→ start by going up
  or down.

b: affects the period.

$$P = \frac{2\pi}{|b|}$$

h: horizontal shift /
   phase shift → where
              you start

k: vertical shift
   → midline

Steps to Graph Transformed Sine and Cosine Functions

1) Determine the period: $P = \frac{2\pi}{|b|}$. <u>Use it to label the x-axis.</u>

2) Determine the midline: $y = k$ and draw it.

3) Determine the max and min lines: $y = k + a$ and $y = k - a$ and draw them.

Start on midline

4) Determine starting point: $(0 + h, k)$ for sine and $(0 + h, k \pm a)$ for cos (do this visually).

start on a max or min

5) Draw the point one period later.

6) Figure out if you are going up or down first. Draw the mid-period and quarter /3-quarter period points.

7) Draw the curve.

Graphing Sine and Cosine functions with amplitude change

$y = 2\sin(x)$

$a = 2$
$b = 1$
$h = 0$
$k = 0$

Period: 2π

Midline: $y = 0$

Max line: $y = 2$

Min line: $y = -2$

Cycle start: $x = 0$

Cycle End: $x = 2\pi$

$y = -2\cos(x)$

$a = -2$
$b = 1$
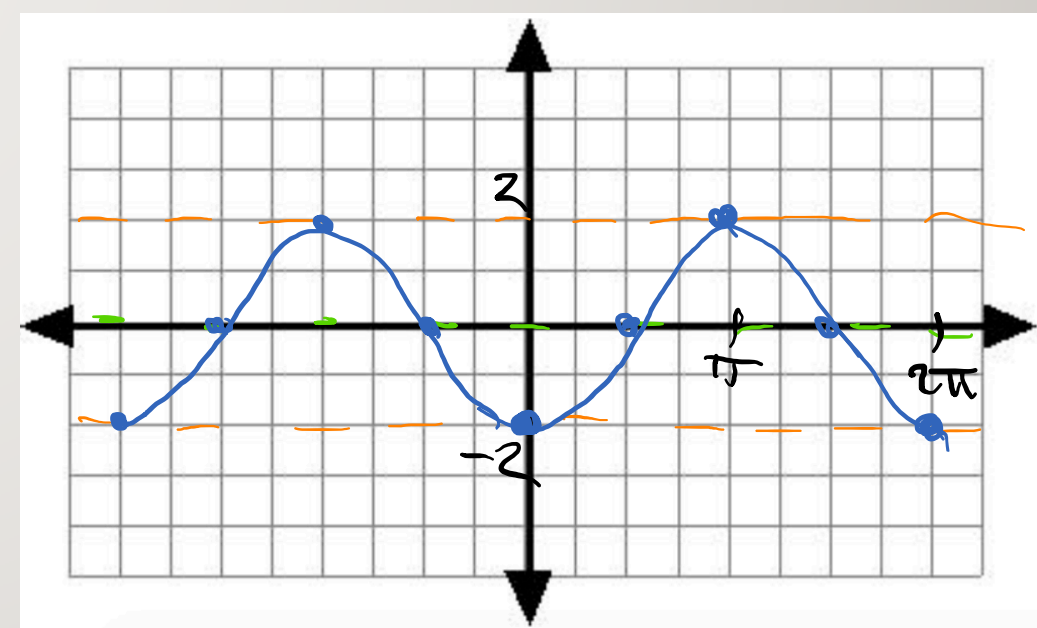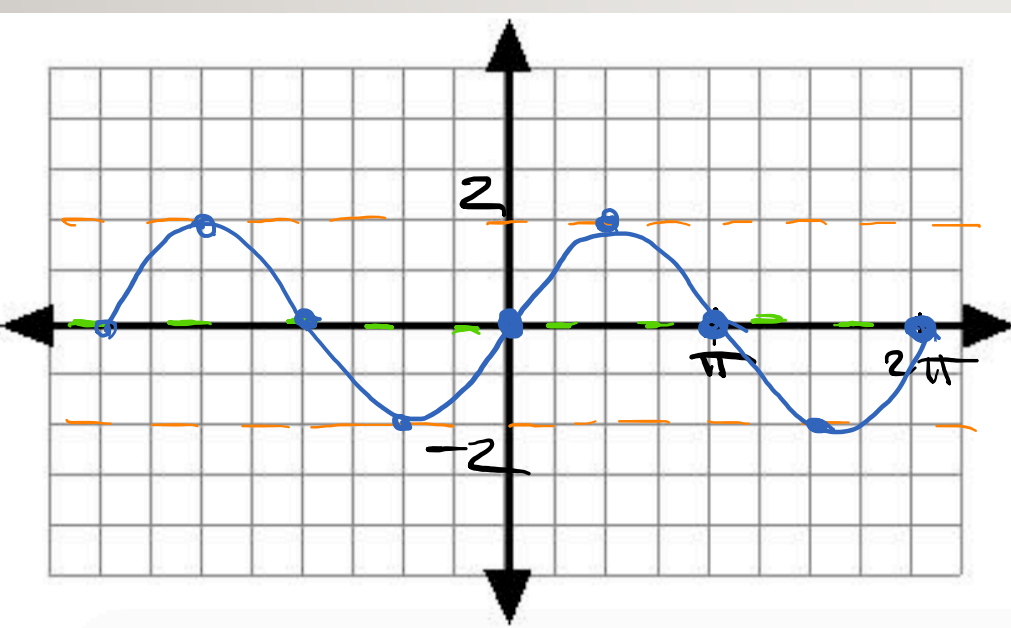$h = 0$
$k = 0$

Period: 2π

Midline: $y = 0$

Max line: $y = 2$

Min line: $y = -2$

Cycle start: $x = 0$

Cycle End: $x = 2\pi$

start at a min
(because of
the negative)

Graphing Sine and Cosine functions with period change

$y = \sin(2x)$

$a = 1$
$b = 2$
$h = 0$
$k = 0$

Period: $\frac{2\pi}{2} = \pi$

Midline: $y = 0$

Max line: $y = 1$

Min line: $y = -1$

Cycle start: $x = 0$

Cycle End: $x = \pi$

$y = \cos(\pi x)$

$a = 1$
$b = \pi$
$h = 0$
$k = 0$

Period: $\frac{2\pi}{\pi} = 2$

Midline: $y = 0$

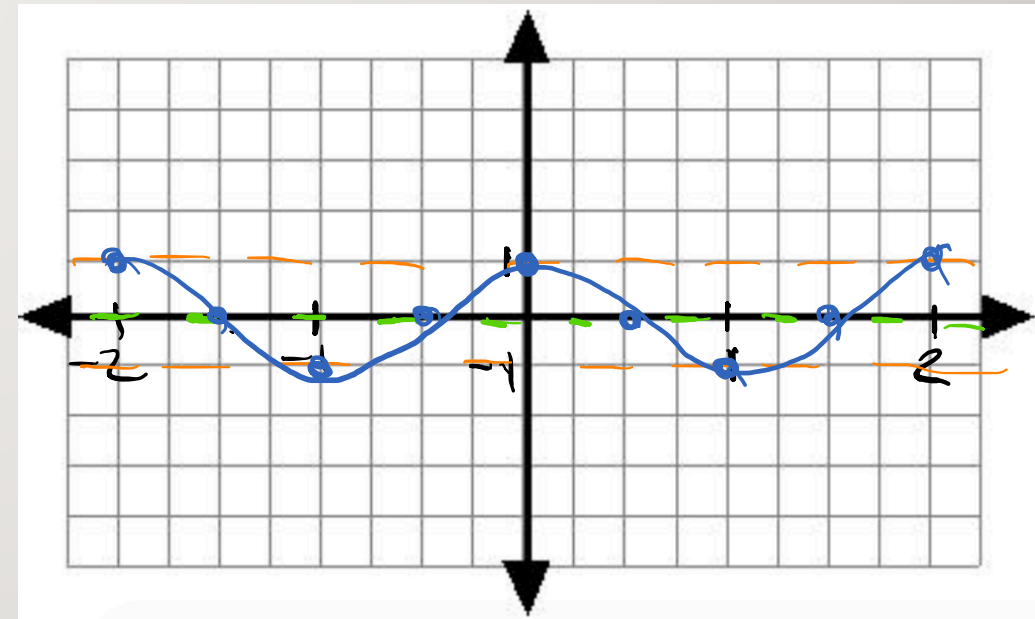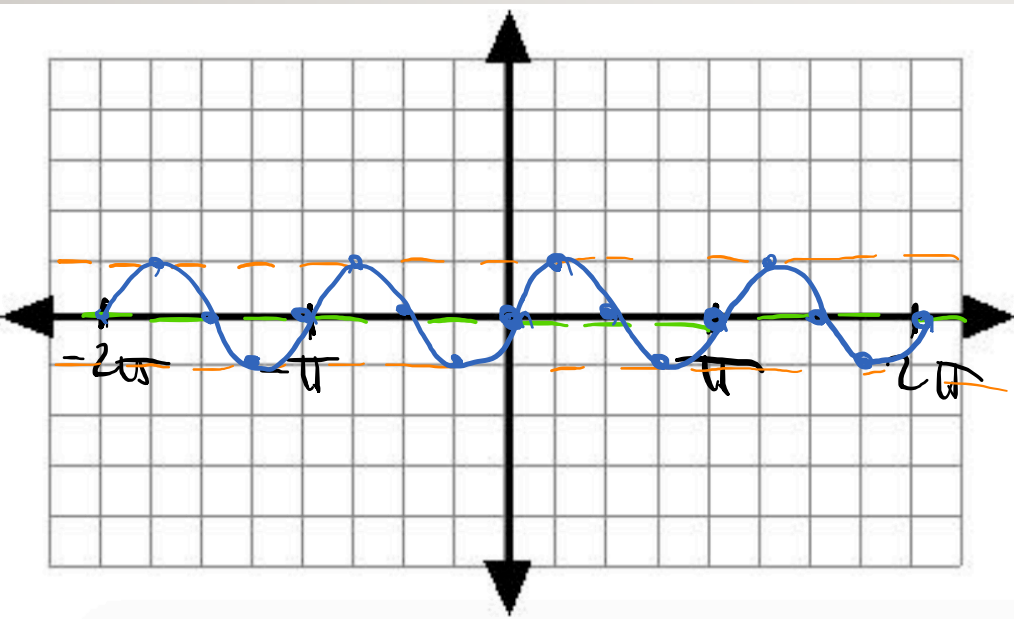Max line: $y = 1$

Min line: $y = -1$

Cycle start: $x = 0$ (max)

Cycle End: $x = 2$

Graphing Sine and Cosine functions with vertical shift

$$y = \sin(x) + 3$$

$a = 1$
$b = 1$
$h = 0$
$k = 3$

Period: 2π
Midline: $y = 3$
Max line: $y = 4$
Min line: $y = 2$
Cycle start: $x = 0$ (midline)
Cycle End: $x = 2\pi$

$$y = \cos(x) - 2$$

$a = 1$
$b = 1$
$h = 0$
$k = -2$

Period: 2π
Midline: $y = -2$
Max line: $y = -1$
Min line: $y = -3$
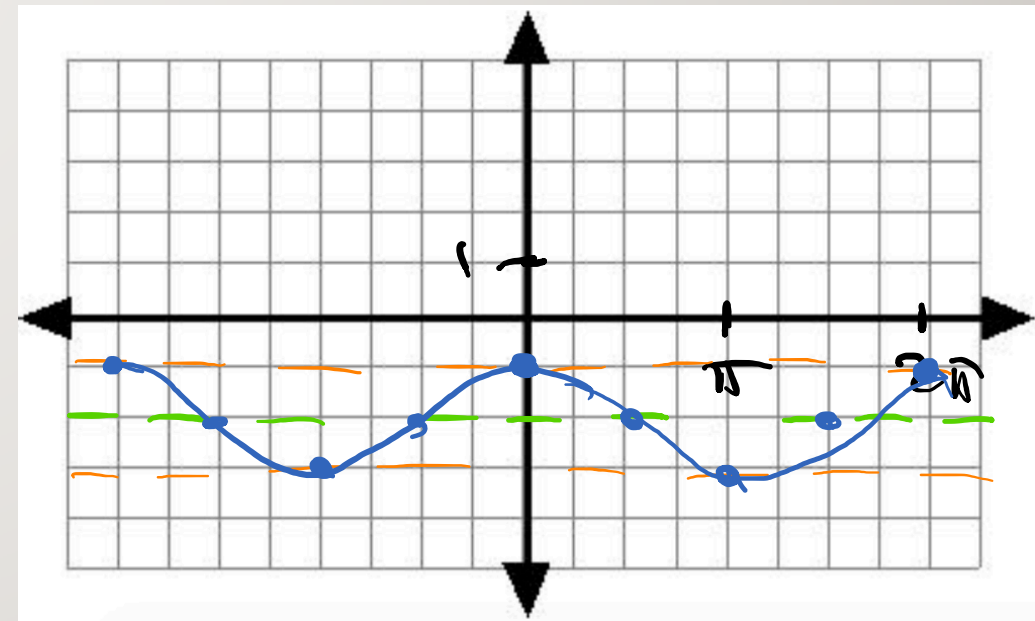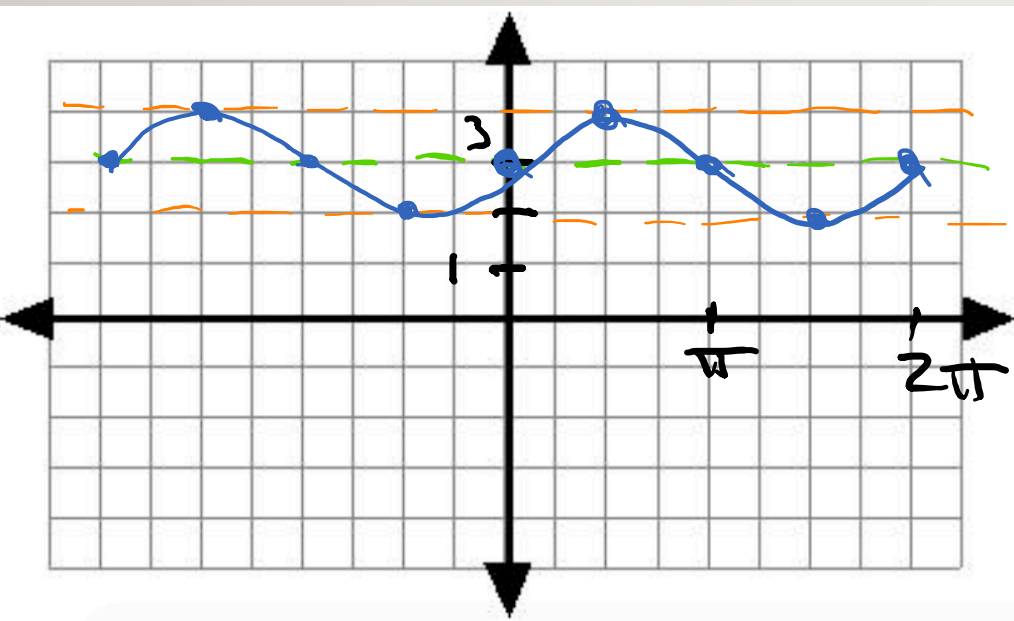Cycle start: $x = 0$ (max)
Cycle End: $x = 2\pi$

$$y = \sin(x - \frac{\pi}{2})$$

$a = 1$
$b = 1$
$h = \frac{\pi}{2}$
$k = 0$

Period: 2π

Midline: $y = 0$

Max line: $y = 1$

Min line: $y = -1$

Cycle start: $x = \frac{\pi}{2}$ (midline)

Cycle End: $2\pi + \frac{\pi}{2}$

$$y = \cos(x + \pi)$$

$a = 1$
$b = 1$
$h = -\pi$
$k = 0$

Period: 2π

Midline: $y = 0$

Max line: $y = 1$

Min line: $y = -1$

Cycle start: $x = -\pi$ (max)

Cycle End: $x = \pi$
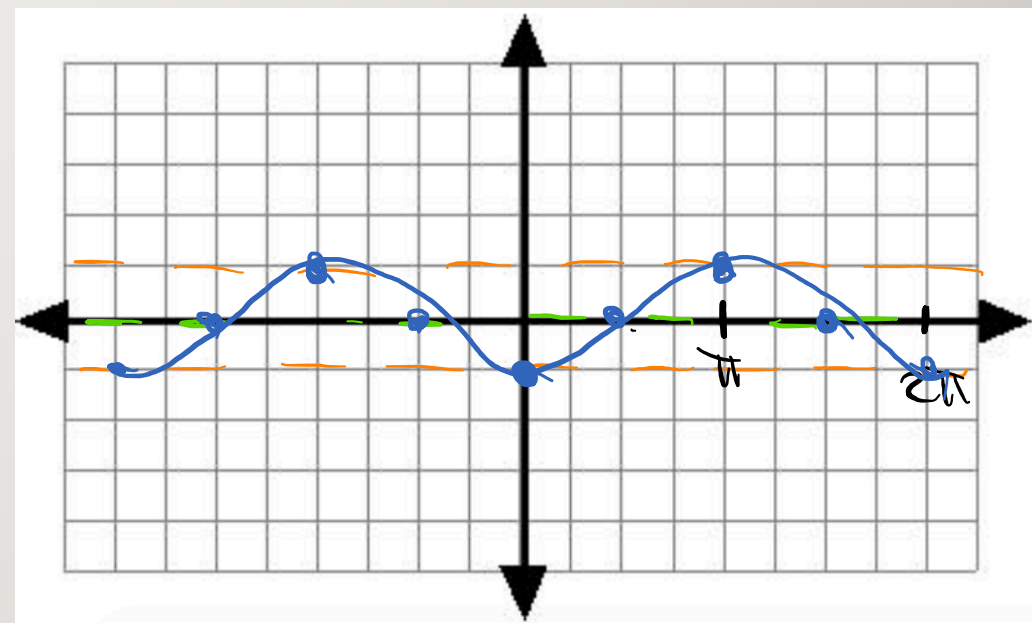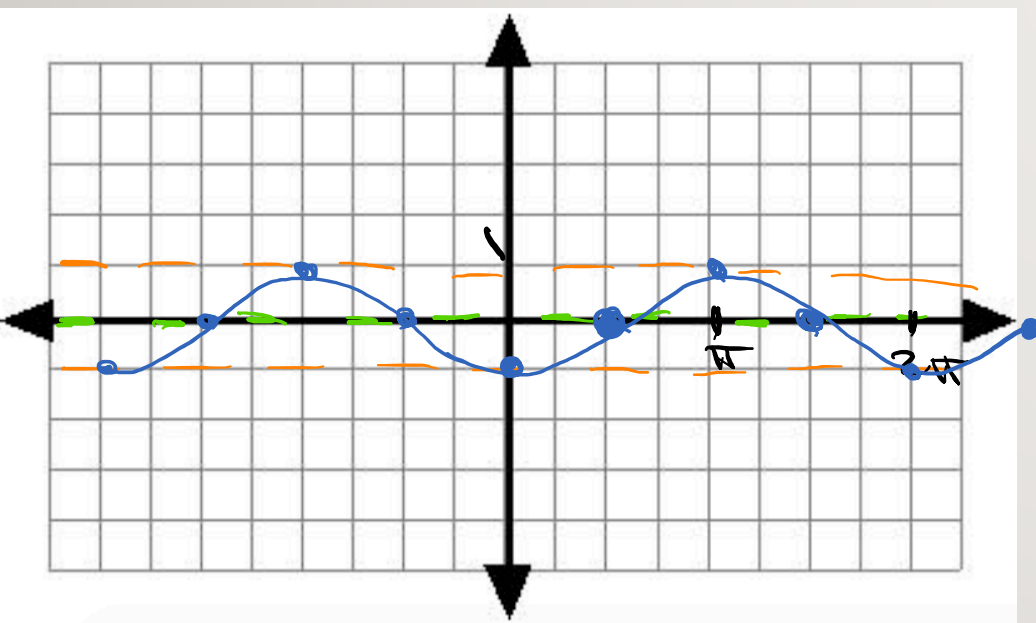
$$y = -3 \sin 0.5\left(x - \frac{\pi}{2}\right) + 1$$

$a = -3$
$b = 0.5$
$h = \frac{\pi}{2}$
$k = 1$

Period: $\frac{2\pi}{0.5} = 4\pi$

Midline: $y = 1$

Max line: $y = 4$

Min line: $y = -2$

Cycle start: $\frac{\pi}{2}$ (midline, start going down)

Cycle End: $4\pi + \frac{\pi}{2}$

$$y = +\cos \pi(x + 2) - 2$$

$a = 1$
$b = \pi$
$h = -2$
$k = -2$
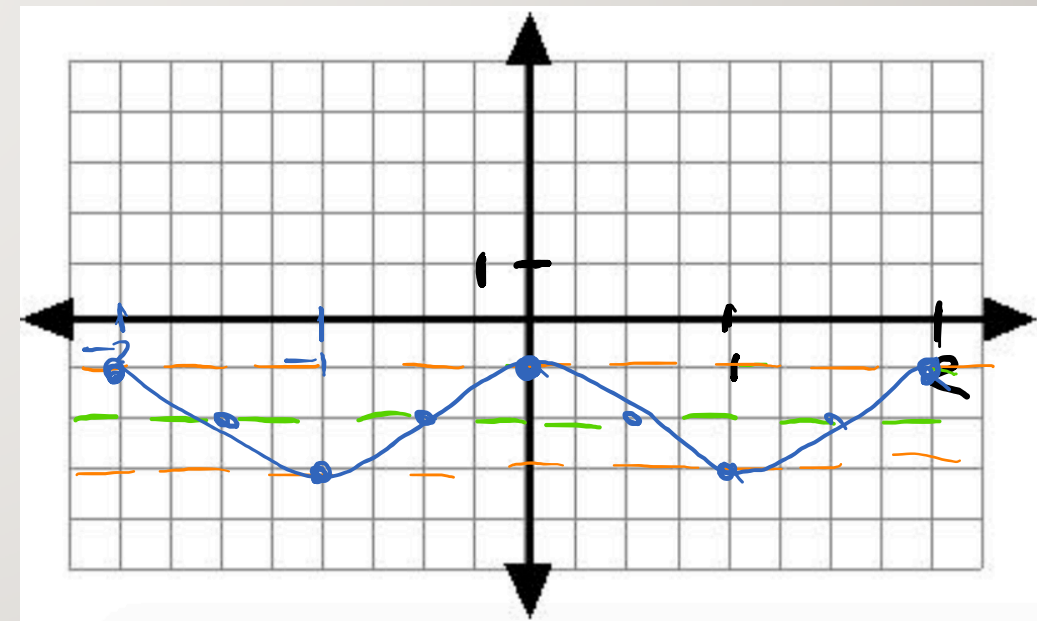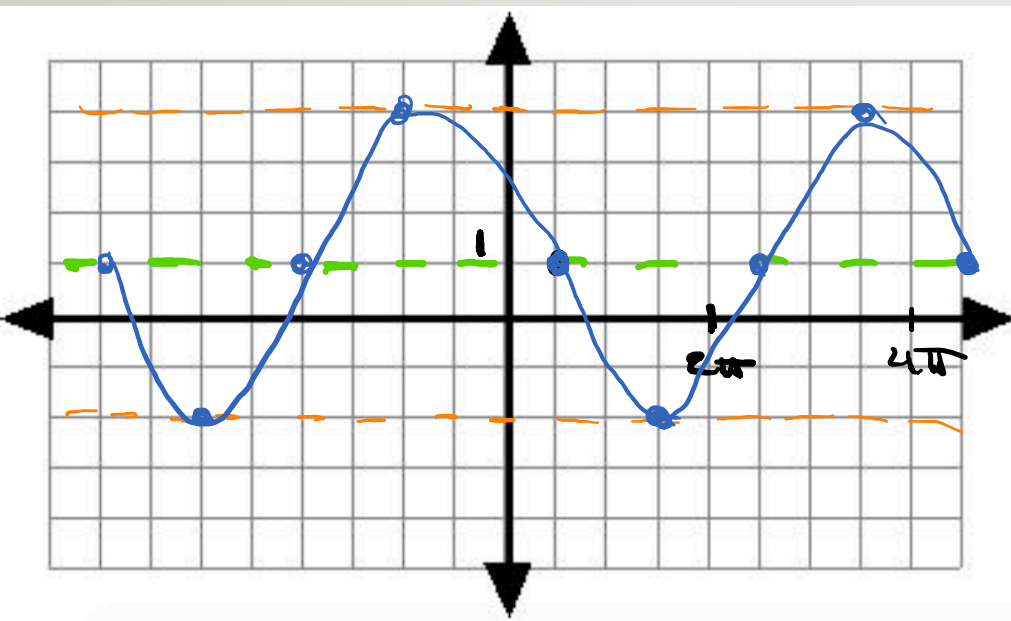
Period: $\frac{2\pi}{\pi} = 2$

Midline: $y = -2$

Max line: $y = -1$

Min line: $y = -3$

Cycle start: -2 (max)

Cycle End: 0

What the basic Tangent function represents

If you are following a point going around on a circle, tangent tracks the ratio of the y-value over the x-value of the point.

Demo: https://www.desmos.com/calculator/n9aeestsd8

Characteristics of sine and cosine functions

$$y = a \tan b(x-h) + k$$

- Period is $\dfrac{2\pi}{|b|}$

- Vertical asymptotes at odd multiples of $\dfrac{\pi}{2|b|}$.

$\cos \dfrac{\pi}{2} = 0$

→ those values are not allowed, b/c I would be dividing by zero