Answer Key

Practice C

1. -1; $y = -2$ **2.** $\frac{1}{5}$; x-axis **3.** 7; $y = 4$ **4.** $\frac{1}{75}$; x-axis **5.** 6; $y = 7$ **6.** -3.2×10^{-5}; x-axis **7.** domain: all real numbers; range: $y > 3$

8. domain: all real numbers; range: $y > -2$

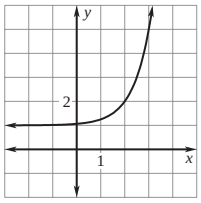
9. domain: all real numbers; range: $y > 4$

10. domain: all real numbers; range: $y > -2$

11. domain: all real numbers; range: $y > 4$

12. domain: all real numbers; range: $y < -3$

13.



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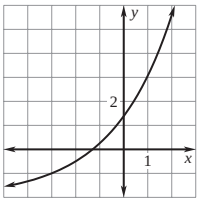
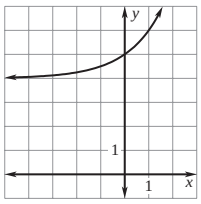


15.



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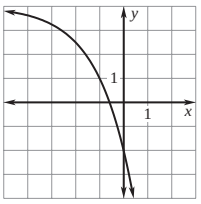

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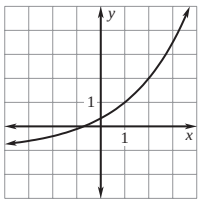

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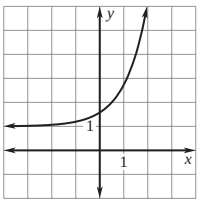

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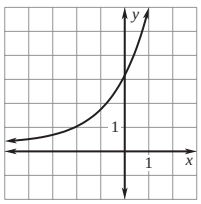

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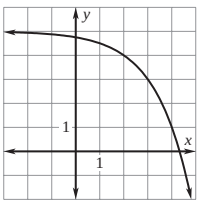

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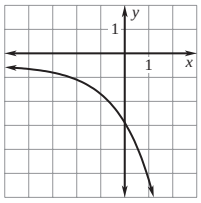
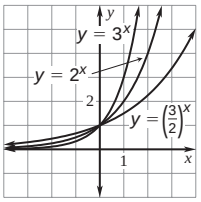


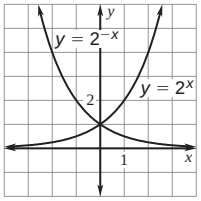
22.



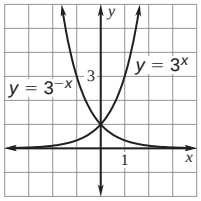
All three graphs have a y-intercept of 1. The larger a is, the steeper the graph.

23.

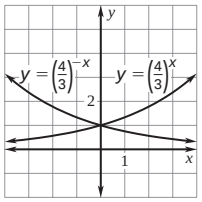
a.



b.



c.



Reflection across the y-axis

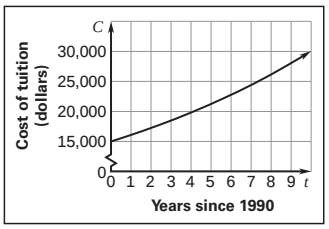
24. a. \$1077.80

b. \$1077.88

c. \$1077.88

25. yes; \$1077.88 **26.** $C = 15{,}000(1.072)^t$

27.



28. 1994 **29.** \$60,254.15

LESSON 8.1

NAME _____ DATE _____

Practice C

For use with pages 465–472

Identify the *y*-intercept and asymptote of the graph of the function.

1. $y = 5^x - 2$ **2.** $y = 5^{x-1}$ **3.** $y = 3(5^x) + 4$

4. $y = \frac{1}{3}(5^{x-2})$ **5.** $y = -3^x + 7$ **6.** $y = -\frac{1}{2}(5^{x-6})$

State the domain and range of the functions.

7. $y = 8^{x+1} + 3$ **8.** $y = 5^{x+3} - 2$ **9.** $y = 6^{x-1} + 4$

10. $y = 7^{x-5} - 2$ **11.** $y = 3(2^x) + 4$ **12.** $y = -2(3^{x+1}) - 3$

Graph the function.

13. $y = 4^{x-2} + 1$ **14.** $y = \left(\frac{3}{2}\right)^{x+3} - 2$ **15.** $y = 2(2^{x-1}) + 4$

16. $y = -3(2^{x+1}) + 4$ **17.** $y = 3\left(\frac{3^{x-2}}{2}\right) - 1$ **18.** $y = 3^{x-1/2} + 1$

19. $y = 2^{x+3/2} + \frac{1}{3}$ **20.** $y = -\frac{1}{2}(2^{x-1}) + 5$ **21.** $y = -3\left(2^{x-1/3}\right) - \frac{1}{2}$

22. ***Visual Thinking*** Sketch the graphs of $y = 2^x$, $y = 3^x$, and $y = \left(\frac{3}{2}\right)^x$ on the same coordinate plane. Explain how the value of a in the equation $y = a^x$ affects the graph. Assume that $a > 0$.

23. ***Visual Thinking*** Sketch the following pairs of graphs in the same coordinate plane. Assuming $a > 0$, explain the difference between $y = a^x$ and $y = a^{-x}$.

a. $y = 2^x$ $y = 2^{-x}$ **b.** $y = 3^x$ $y = 3^{-x}$ **c.** $y = \left(\frac{4}{3}\right)^x$ $y = \left(\frac{4}{3}\right)^{-x}$

24. ***Account Balance*** You deposit \$1000 in an account that earns 2.5% annual interest. Find the balance after 3 years if this interest is compounded with the given frequency.

a. monthly **b.** daily **c.** hourly

25. Use your results from Exercise 24 to determine if there is a limit to how much you can earn. If there is a limit, what is the maximum amount?

College Tuition **In Exercises 26–29, use the following information.**

In 1990, the tuition at a private college was \$15,000. During the next 9 years, tuition increased by about 7.2% each year.

26. Write a model giving the cost C of tuition at the college t years after 1990.

27. Graph the model.

28. Estimate the year when the tuition was \$20,000.

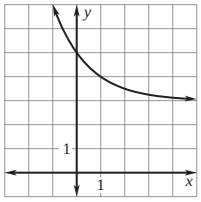
29. Estimate the tuition in 2010.

Answer Key

Practice C

1. exponential decay **2.** exponential growth
3. exponential decay **4.** exponential growth
5. exponential growth **6.** exponential decay
7. 4; $y = 3$ **8.** $\frac{8}{27}$; x-axis **9.** $\frac{3}{16}$; x-axis
10. domain: all real numbers; range: $y > -3$
11. domain: all real numbers; range: $y > 4$
12. domain: all real numbers; range: $y > 1$
13. domain: all real numbers; range: $y > -2$
14. domain: all real numbers; range: $y > 7$
15. domain: all real numbers; range: $y < -4$

16.



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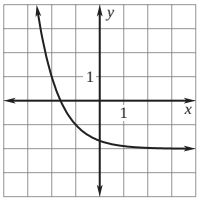

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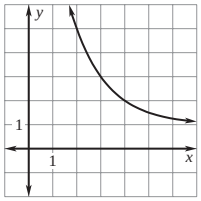

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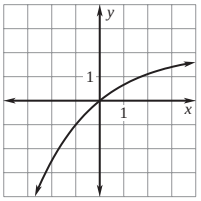
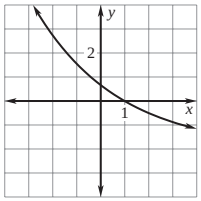


20.



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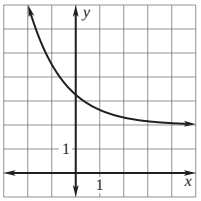

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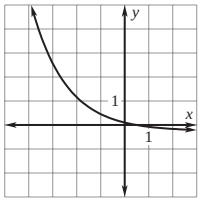
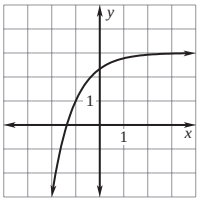
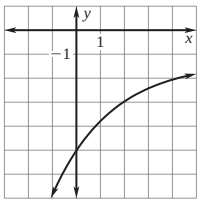


23.

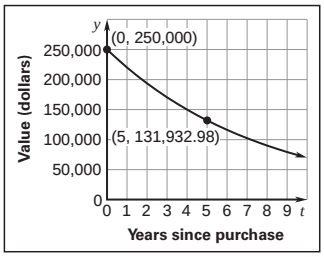


24.



25. $y = 250{,}000(0.88)^t$
26. \$131,932.98

27.



28. 10 years

29. $y = \begin{cases} 830, & 0 \le t \le \frac{1}{4} \\ 830(0.87)^{t-1/4}, & t > \frac{1}{4} \end{cases}$

30. \$747.68

LESSON 8.2

NAME _____ DATE _____

Practice C

For use with pages 474–479

Tell whether the function represents *exponential growth* or *exponential decay.*

1. $f(x) = \left(\frac{2}{3}\right)^x$ **2.** $f(x) = \left(\frac{3}{2}\right)^x$ **3.** $f(x) = \left(\frac{3}{2}\right)^{-x}$

4. $f(x) = \left(\frac{2}{3}\right)^{-x}$ **5.** $f(x) = -\left(\frac{3}{2}\right)^{-x}$ **6.** $f(x) = -\left(\frac{2}{3}\right)^{-x}$

Identify the *y*-intercept and asymptote of the graph of the function.

7. $y = \left(\frac{1}{2}\right)^x + 3$ **8.** $y = \left(\frac{2}{3}\right)^{x+3}$ **9.** $y = \frac{1}{4}\left(\frac{3}{4}\right)^{x+1}$

State the domain and range of the function.

10. $y = \left(\frac{1}{2}\right)^{x+1} - 3$ **11.** $y = \left(\frac{1}{3}\right)^{x-2} + 4$ **12.** $y = \left(\frac{2}{5}\right)^{x+4} + 1$

13. $y = \left(\frac{3}{5}\right)^{x-3} - 2$ **14.** $y = 3^{-x} + 7$ **15.** $y = -2(3^{-x}) - 4$

Graph the function.

16. $y = \left(\frac{1}{2}\right)^{x-1} + 3$ **17.** $y = \left(\frac{1}{3}\right)^{x+1} - 2$ **18.** $y = 2\left(\frac{1}{2}\right)^{x-3} + 1$

19. $y = -3\left(\frac{2}{3}\right)^{x+1} + 2$ **20.** $y = 2\left(\frac{3}{4}\right)^{x-1} - 2$ **21.** $y = \left(\frac{1}{2}\right)^{x-1/3} + 2$

22. $y = \left(\frac{1}{2}\right)^{x+3/2} - \frac{1}{4}$ **23.** $y = -2\left(\frac{1}{3}\right)^{x+1} + 3$ **24.** $y = -3\left(\frac{2}{3}\right)^{x-1/2} - \frac{4}{3}$

Equipment Depreciation **In Exercises 25–28, use the following information.**

A tool and die business purchases a piece of equipment for $250,000. The value of the equipment depreciates at a rate of 12% each year.

25. Write an exponential decay model for the value of the equipment.

26. What is the value of the equipment after 5 years?

27. Graph the model.

28. Use the model to estimate when the equipment will have a value of $70,000.

Stereo System **In Exercises 29 and 30, use the following information.**

You purchase a stereo system for $830. After a 3 month trial period, the value of the stereo system decreases 13% each year.

29. Write an exponential decay model for the value of the stereo system in terms of the number of years since the purchase.

30. What was the value of the system after 1 year?

Answer Key

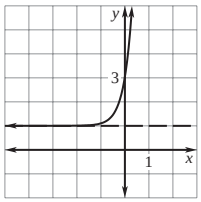
Practice C

1. 5.652 **2.** 0.074 **3.** 0.493 **4.** 15.154

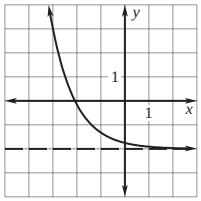
5. $8e^{14}$ **6.** $81e^{8}$ **7.** $\frac{8}{e^{6}}$ **8.** $\frac{1}{4096e^{3x}}$

9. $\frac{e^{6x-2}}{4}$ **10.** $2e^{4x}$ **11.** $y = -1$ **12.** $y = 4$

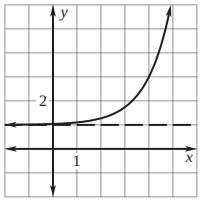
13. $y = 0$

14. 

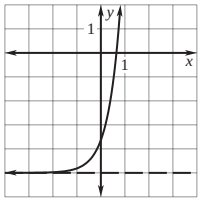
Domain:
All real numbers;
Range: $y > 1$

15. 

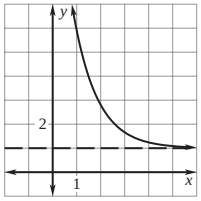
Domain:
All real numbers;
Range: $y > -2$

16. 

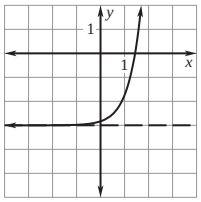
Domain:
All real numbers;
Range: $y > 1$

17. 

Domain:
All real numbers;
Range: $y > -5$

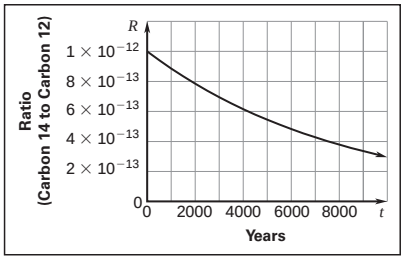
18. 

Domain:
All real numbers;
Range: $y > 1$

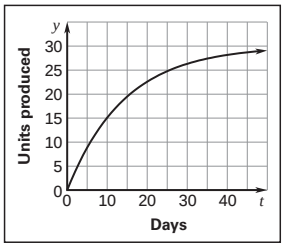
19. 

Domain:
All real numbers;
Range: $y > -3$

20. exponential decay

21. 

22. 10,000 years **23.** exponential growth

24. 

25. 13 units
26. 26 days

LESSON 8.3

NAME _____ DATE _____

Practice C

For use with pages 480–485

Use a calculator to evaluate the expression. Round the result to three decimal places.

1. $e^{\sqrt{3}}$ **2.** $e^{-2.6}$ **3.** $e^{-\frac{1}{\sqrt{2}}}$ **4.** e^{e}

Simplify the expression.

5. $e^2(2e^4)^3$ **6.** $\left(\frac{1}{3}e^{-2}\right)^{-4}$ **7.** $\left(\frac{e^2}{2}\right)^{-3}$

8. $(4e^{0.5x})^{-6}$ **9.** $\left(\frac{e^{3x}}{2e}\right)^2$ **10.** $\sqrt[3]{8e^{12x}}$

Identify the horizontal asymptote of the function.

11. $f(x) = 3e^{2x} - 1$ **12.** $f(x) = \frac{1}{2}e^{3x+1} + 4$ **13.** $f(x) = 245e^{-0.023x}$

Graph the function. State the domain and range.

14. $f(x) = 2e^{3x} + 1$ **15.** $f(x) = \frac{1}{4}e^{-x} - 2$ **16.** $f(x) = 2e^{x-4} + 1$

17. $f(x) = \frac{1}{2}e^{2x+1} - 5$ **18.** $f(x) = \frac{2}{3}e^{3-x} + 1$ **19.** $f(x) = \frac{5}{4}e^{2(x-1)} - 3$

Carbon Dating **In Exercises 20–22, use the following information.**

Carbon dating is a process to estimate the age of organic material. In carbon dating the formula used is

$$R = \frac{1}{10^{12}}e^{-t/8233}$$

where R is the ratio of Carbon 14 to Carbon 12 and t is time in years.

20. Is the model an example of exponential growth or exponential decay?

21. Graph the function.

22. Use the graph to estimate the age of a fossil whose Carbon 14 to Carbon 12 ratio is 3×10^{-13}.

Learning Curve **In Exercises 23–26, use the following information.**

The management at a factory has determined that a worker can produce a maximum of 30 units per day. The model $y = 30 - 30e^{-0.07t}$ indicates the number of units y that a new employee can produce per day after t days on the job.

23. Is the model an example of exponential growth or exponential decay?

24. Graph the function.

25. How many units can be produced per day by an employee who has been on the job 8 days?

26. Use the graph to estimate how many days of employment are required for a worker to produce 25 units per day.

Answer Key

Practice C

1. $5^3 = 125$ **2.** $8^{1/3} = 2$ **3.** $3^{-3} = \frac{1}{27}$

4. 2.099 **5.** 0.092 **6.** -1.199 **7.** -5

8. -3 **9.** $\frac{2}{3}$ **10.** $\frac{3}{4}$ **11.** $-\frac{2}{3}$ **12.** $-\frac{3}{2}$

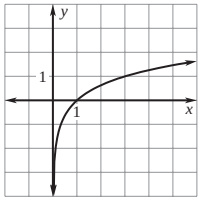
13. $f^{-1}(x) = 4^x$ **14.** $f^{-1}(x) = \dfrac{2^x}{7}$

15. $f^{-1}(x) = \dfrac{10^x - 2}{3}$ **16.** $f^{-1}(x) = e^{x+3}$

17. $f^{-1}(x) = e^{x-1} + 2$

18. $f^{-1}(x) = \dfrac{x - 4}{2}$ or $f^{-1}(x) = \dfrac{1}{2}x - 2$

19.



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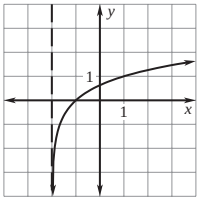

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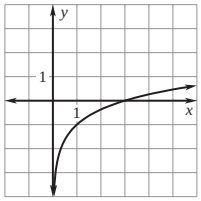
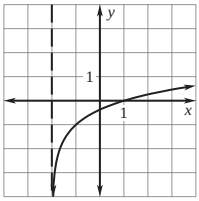
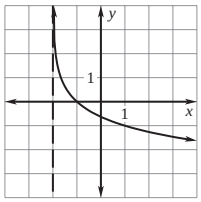


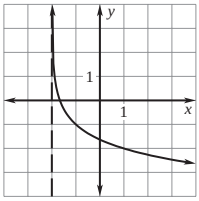
22.



23.



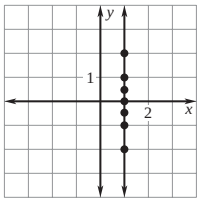
24.



25.

x	1	1	1	1	1	1	1
y	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2

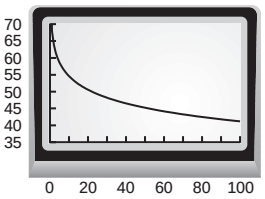
26.



27. no **28.** no

29. 41.9 seconds

30.



31. 41.2 seconds

LESSON 8.4

NAME _____ DATE _____

Practice C

For use with pages 486–492

Rewrite the equation in exponential form.

1. $\log_5 125 = 3$ **2.** $\log_8 2 = \frac{1}{3}$ **3.** $\log_3 \frac{1}{27} = -3$

Use a calculator to evaluate the expression. Round the result to three decimal places.

4. $\ln 3 + 1$ **5.** $\frac{\ln 2.5}{10}$ **6.** $\frac{\log 4 - 3}{2}$

Evaluate the expression without using a calculator.

7. $\log_2 \frac{1}{32}$ **8.** $\log \frac{1}{1000}$ **9.** $\log_8 4$

10. $\log_{16} 8$ **11.** $\log_{27} \frac{1}{9}$ **12.** $\log_{100} \frac{1}{1000}$

Find the inverse of the function.

13. $f(x) = \log_4 x$ **14.** $f(x) = \log_2 (7x)$ **15.** $f(x) = \log (3x + 2)$

16. $f(x) = \ln x - 3$ **17.** $f(x) = \ln (x - 2) + 1$ **18.** $f(x) = \log 100^{x+2}$

Graph the function.

19. $f(x) = \log_3 x$ **20.** $f(x) = \log_3 (x + 2)$ **21.** $f(x) = \log_3 x - 1$

22. $f(x) = \log_3 (x + 2) - 1$ **23.** $f(x) = -\log_3 (x + 2)$ **24.** $f(x) = -\log_3 (x + 2) - 1$

Critical Thinking **In Exercises 25–28, use the following information.**

By definition of a logarithm, the base b of a logarithmic function must be a positive number and $b \neq 1$.

25. Assuming that $b = 1$, the "logarithmic function' would be written $y = \log_1 x$. Complete the table of values for this "logarithmic function."

x							
y	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2

26. Use the data to sketch a graph.

27. Does the graph look like a typical logarithmic graph?

28. Is the relation a function?

400-Meter Relay **In Exercises 29–31, use the following information.**

The winning time (in seconds) in the women's 400-meter relay at the Olympic Games from 1928 to 1996 can be modeled by the function $f(t) = 67.99 - 5.82 \ln t$, where t is the number of years since 1900.

29. In 1988 the United States team won the 400-meter relay. What was its winning time?

30. Use a graphing calculator to graph the model.

31. Use the graph to approximate the winning time in the 2000 Olympic Games.

Practice C

1. $\ln 2 + \ln 3 \approx 1.792$

2. $\ln 2 + \ln 5 - \ln 3 \approx 1.203$

3. $\ln 2 + \ln 3 + \ln 5 \approx 3.401$

4. $2 \ln 2 + \ln 3 \approx 2.485$

5. $\ln 2 - \ln 5 \approx -0.916$

6. $\ln 5 - \ln 3 - \ln 2 \approx -0.183$

7. $\log 8 + \log x$ **8.** $\log_3 x + \log_3 y + \log_3 z$

9. $\frac{1}{2} + \log_4 x + \log_4 y - \log_4 z$

10. $\ln x - \ln y - \ln z$

11. $\frac{1}{2}(\log 3 + \log x + \log y)$

12. $\frac{1}{2}\log_5 x - \log_5 y$ **13.** $\ln 3 + \ln y - \frac{1}{4}\ln x$

14. $3(\log 3 + \log x + \log y + 2 \log z)$

15. $4(\log_2 x + \log_2 y) - 2 \log_2 z$

16. $\log \dfrac{3}{28}$ **17.** $\ln \dfrac{3xz}{y}$ **18.** $\ln \dfrac{x^3}{y^2z^4}$

19. $\log_2 \dfrac{(x-4)(x+1)^5}{(x-1)^3}$

20. $\log_2 \dfrac{\sqrt{x+5}}{x^2} + \ln y$

21. $\ln\left[\dfrac{(x-2)(x+1)^2}{(x+2)(x-1)^5}\right]^3$

22. $y = \dfrac{\log x}{\log 3}$ or $y = \dfrac{\ln x}{\ln 3}$

23. $y = \dfrac{\log(x+3)}{\log 6}$ or $y = \dfrac{\ln(x+3)}{\ln 6}$

24. $y = \dfrac{\log(x-1)}{\log 2} + 3$ or $y = \dfrac{\ln(x-1)}{\ln 2} + 3$

25. $t = \dfrac{\ln(Sr + Pn) - \ln P - \ln n}{n(\ln(n+r) - \ln n)}$

26. ≈ 19.7 years

NAME _____ DATE _____

Practice C

For use with pages 493–499

Use the properties of logarithms to rewrite the expression in terms of ln 2, ln 3, and ln 5. Then use ln 2 ≈ 0.693, ln 3 ≈ 1.099, and ln 5 ≈ 1.609 to approximate the expression.

1. $\ln 6$ **2.** $\ln\left(\frac{10}{3}\right)$ **3.** $\ln 30$

4. $\ln 12$ **5.** $\ln\left(\frac{2}{5}\right)$ **6.** $\ln\left(\frac{5}{6}\right)$

Expand the expression.

7. $\log (8x)$ **8.** $\log_3 xyz$ **9.** $\log_4 \frac{2xy}{z}$

10. $\ln \frac{x}{yz}$ **11.** $\log \sqrt{3xy}$ **12.** $\log_5 \frac{\sqrt{x}}{y}$

13. $\ln \frac{3y}{\sqrt[4]{x}}$ **14.** $\log(3xyz^2)^3$ **15.** $\log_2 \frac{(xy)^4}{z^2}$

Condense the expression.

16. $\log 3 - \log 4 - \log 7$ **17.** $\ln x - \ln y + \ln z + \ln 3$

18. $3 \ln x - 2 \ln y - 4 \ln z$ **19.** $\log_2(x - 4) + 5 \log_2(x + 1) - 3 \log_2(x - 1)$

20. $\frac{1}{2}\log(x + 5) - 2 \log x + \ln y$ **21.** $3[\ln(x - 2) + 2 \ln(x + 1) - \ln(x + 2) - 5 \ln(x - 1)]$

Use the change-of-base formula to rewrite the function in terms of common (base 10) or natural (base ln) logarithms.

22. $y = \log_3 x$ **23.** $y = \log_6(x + 3)$ **24.** $y = \log_2(x - 1) + 3$

Annuities **In Exercises 25 and 26, use the following information.**

An ordinary annuity is an account in which you make a fixed deposit at the end of each compounding period. You want to use an annuity to help you save money for college. The formula

$$t = \frac{\ln\left[\frac{Sr + Pn}{Pn}\right]}{n \ln\left[\frac{n + r}{n}\right]}$$

gives the time t (in years) required to have S dollars in the annuity if your periodic payments P (in dollars) are made n times a year and the annual interest rate is r (in decimal form).

25. Expand the right side of the formula.

26. How long will it take you to save \$20,000 in annuity that earns an annual interest rate of 5% if you make monthly payments of \$50?

Practice C

1. 2.197 **2.** 0.333 **3.** 3.386 **4.** 0.349
5. 1.436 **6.** 6.447 **7.** 0.376 **8.** 1.269
9. 0.258 **10.** 0 **11.** $-1, 2$ **12.** $-1, 0.667$
13. 4.5 **14.** 22,023.466 **15.** 11 **16.** 181.939
17. 2.414 **18.** 1 **19.** 3 **20.** 0.143 **21.** 3.333
22. no solution **23.** $-2, 3$ **24.** 7 **25.** 4, 6
26. no solution **27.** 0.461 **28.** 3.697
29. -8.266 **30.** no solution **31.** 5.303
32. 7.193 **33.** 5.2 years **34.** 30 years
35. \$211,320 **36.** \$131,320

LESSON 8.6

NAME _____ DATE _____

Practice C

For use with pages 501–508

Solve the exponential equation. Round the result to three decimal places if necessary.

1. $e^x = 9$

2. $2^{3x+1} = 4$

3. $3^{2x-5} = 7$

4. $e^{4x+1} - 3 = 8$

5. $e^{5-3x} + 4 = 6$

6. $3^{0.4x} - 7 = 10$

7. $\frac{2}{3}e^{4x} + 5 = 8$

8. $\frac{1}{4}(2^{3x+1}) - 2 = 5$

9. $\frac{5}{3}e^{1-x} + 1 = \frac{9}{2}$

10. $e^{x^2} + 3 = 4$

11. $e^{x^2+1} = e^{x+3}$

12. $2^{3x+1} = 2^{2/x}$

Solve the logarithmic equation. Round the result to three decimal places if necessary.

13. $\log(2x + 1) = 1$

14. $\ln(x + 3) - 2 = 8$

15. $\log_3(x - 2) + 5 = 7$

16. $\ln(6x + 5) = 7$

17. $\ln(x - 2) + \ln x = 0$

18. $\log_2 x + \log_2(x + 1) = 1$

19. $\log_3 x + \log_3(x - 2) = 1$

20. $\log_2(x + 1) - \log_2 x = 3$

21. $\log_4(x + 2) - \log_4(x - 3) = 2$

22. $\log(3x + 2) = \log(2x - 1)$

23. $\log(x^2 - 1) = \log(x + 5)$

24. $\log(x + 2) + \log(x - 3) = \log(x + 29)$

25. $\log_2 x + \log_2(x - 2) - \log_2(x - 3) = 3$

26. $\log_2(-x - 3) - \log_2(x - 1) - \log_2(x + 3) = 1$

Solve the exponential equation. Round the result to three decimal places.

27. $2^{x+1} = 3^{2x}$

28. $e^{x-3} = 10^{4-x}$

29. $5^{2x+1} = 2^{4x-3}$

Solve the logarithmic equation. Round the result to three decimal places.

30. $\log_2(x + 1) = \log_4(2x - 3)$

31. $\log_3(x - 3) = \log_9 x$

32. $\log(x - 4) = \log_{100}(x + 3)$

33. ***Compound Interest*** You deposit \$2500 into an account that pays 3.5% annual interest compounded daily. How long will it take for the balance to reach \$3000?

Loan Repayment **In Exercises 34–36, use the following information.**

The formula $L = P\left[\dfrac{1 - \left(1 + \dfrac{r}{n}\right)^{-nt}}{\dfrac{r}{n}}\right]$ gives the amount of a loan L in terms of the amount of each payment P, the interest rate r, the number of payments per year n, and the number of years t.

34. When purchasing a home, you need a loan for \$80,000. The interest rate of the loan is 8% and you are required to make monthly payments of \$587. How long will it take you to pay off the loan?

35. When the loan is paid off, how much money will you have paid the bank?

36. How much did you pay in interest?