CHAPTER 14 – CIRCLE RELATIONSHIPS

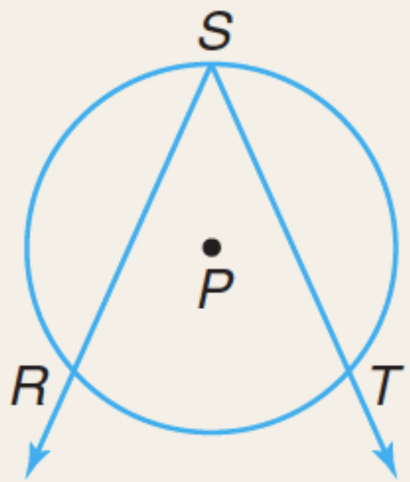
14.1- INSCRIBED ANGLES

Inscribed Angle

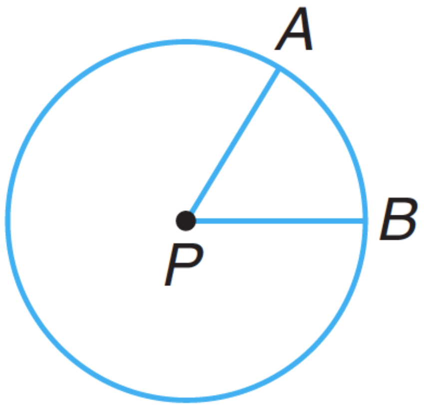
Definition of Inscribed Angle

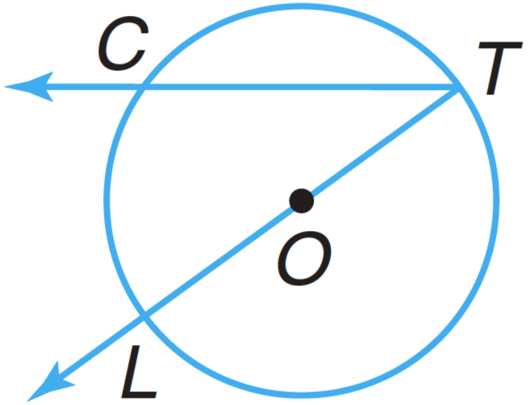
Words: An angle is inscribed if and only if its vertex lies on the circle and its sides contain chords of the circle.

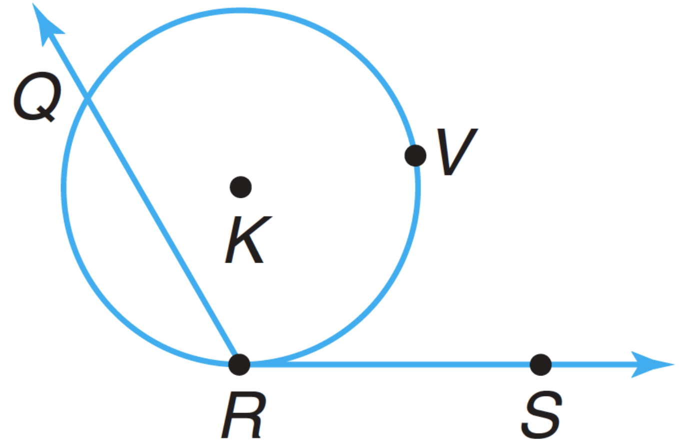
Model:



Symbols:
$\angle RST$ is inscribed in $\odot P$.





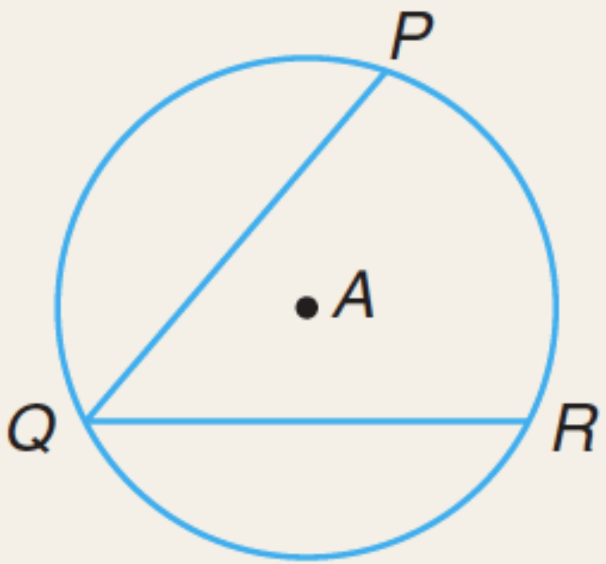


Measure of inscribed angles

Theorem 14–1

Words: The degree measure of an inscribed angle equals one-half the degree measure of its intercepted arc.

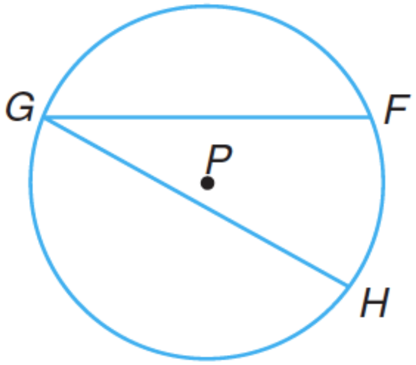
Model:



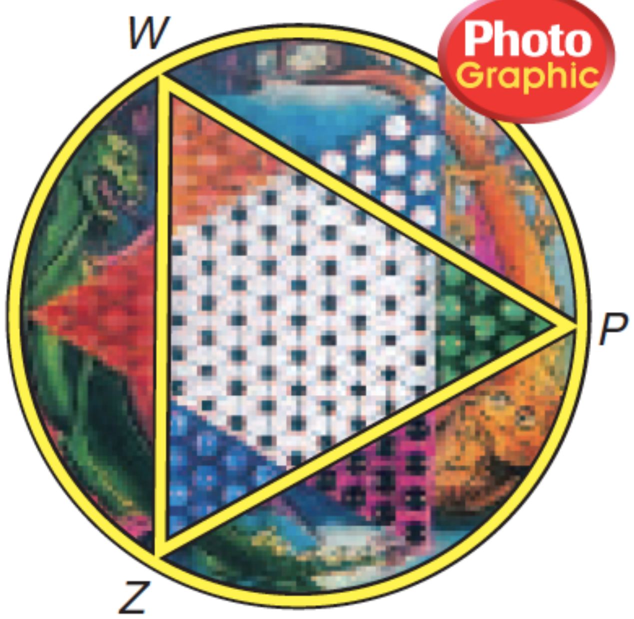
Symbols:

$m\angle PQR = \frac{1}{2} m\overparen{PR}$

If $m\overset{\frown}{FH} = 58$, find $m\angle FGH$.

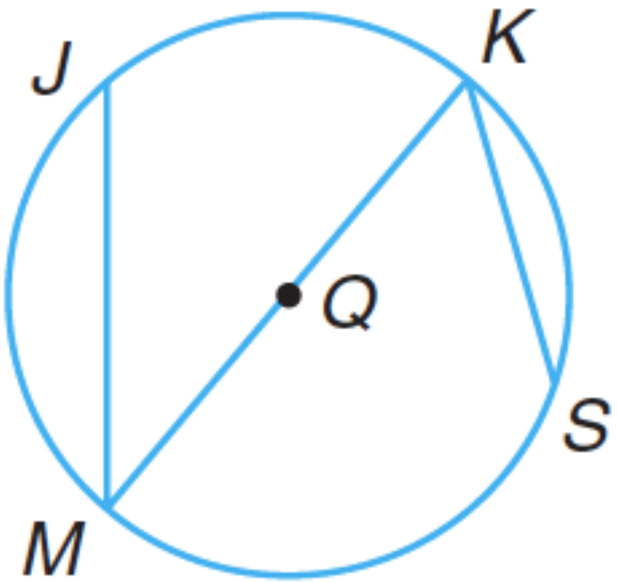


In the game shown at the right, $\triangle WPZ$ is equilateral. Find $m\overset{\frown}{WZ}$.



c. If $m\widehat{JK} = 80$, find $m\angle JMK$.

d. If $m\angle MKS = 56$, find $m\widehat{MS}$.

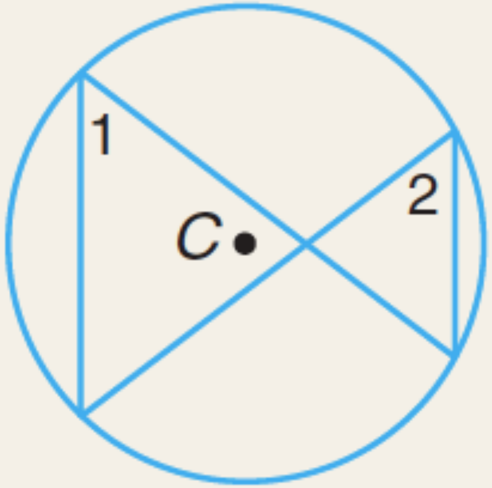


Congruent inscribed angles

Theorem 14–2

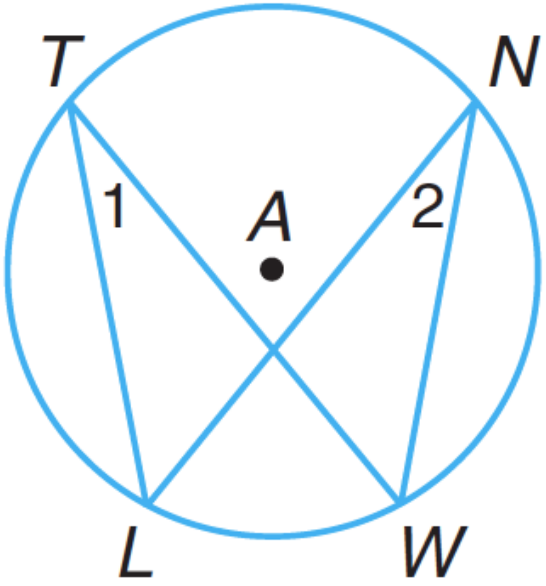
Words: If inscribed angles intercept the same arc or congruent arcs, then the angles are congruent.

Model:

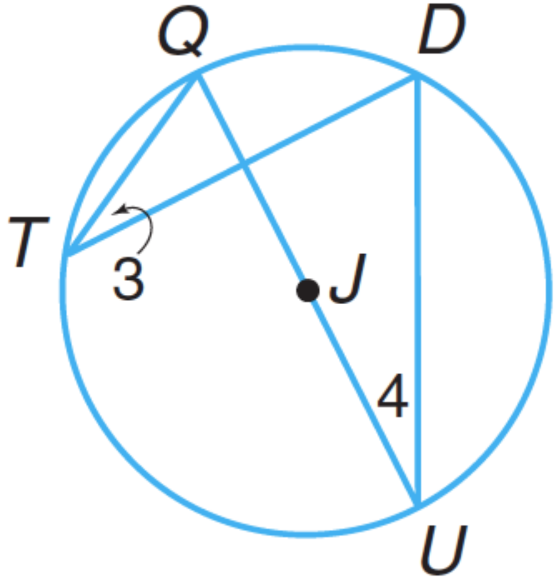


Symbols: $\angle 1 \cong \angle 2$

In $\odot A$, $m\angle 1 = 2x$ and $m\angle 2 = x + 14$. Find the value of x.



In $\odot J$, $m\angle 3 = 3x$ and $m\angle 4 = 2x + 9$. Find the value of x.

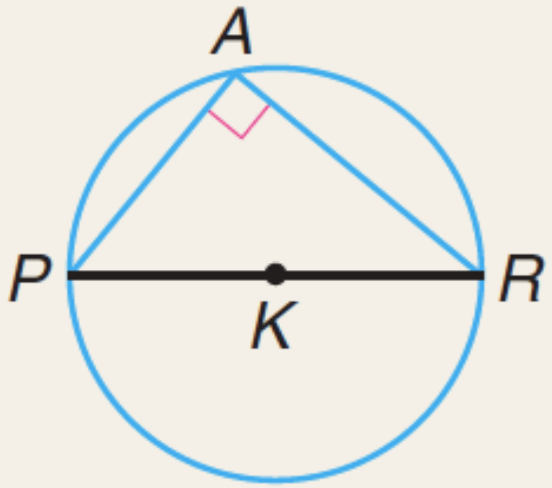


Inscribed right triangles

Theorem 14–3

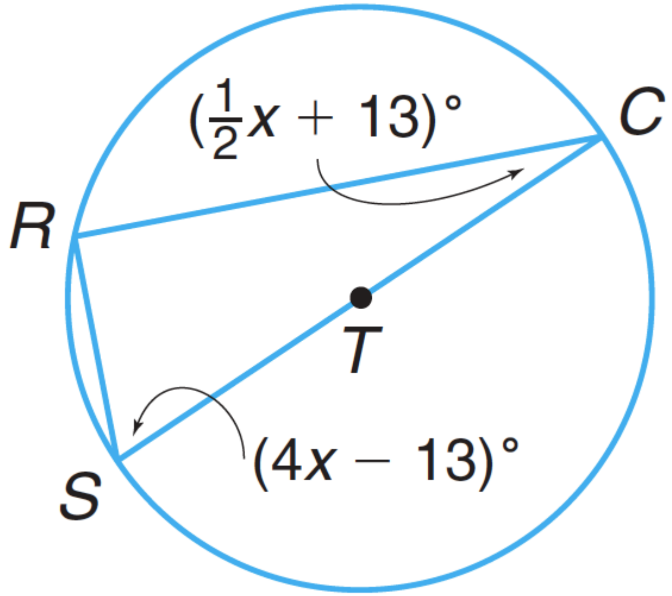
Words: If an inscribed angle of a circle intercepts a semicircle, then the angle is a right angle.

Model:

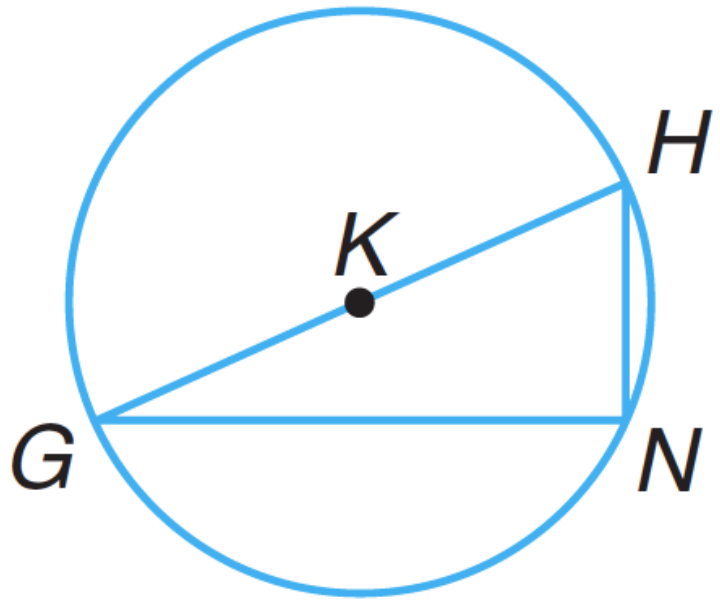


Symbols: $m\angle PAR = 90$

In $\odot T$, $\overline{CS}$ is a diameter. Find the value of x.



In $\odot K$, $\overline{GH}$ is a diameter and $m\angle GNH = 4x - 14$. Find the value of x.

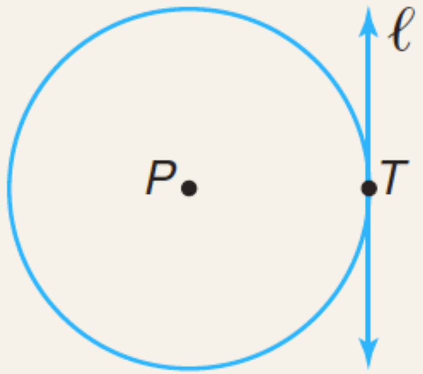


14.1 Recap

- Inscribed angles have their vertex on the circle and sides contained in the circle.
- The inscribed angle measure is half the arc it intercepts.
- Inscribed angles are congruent if they intercept the same arc.
- Inscribed right triangles intercept semicircles.

14.2- TANGENTS TO A CIRCLE

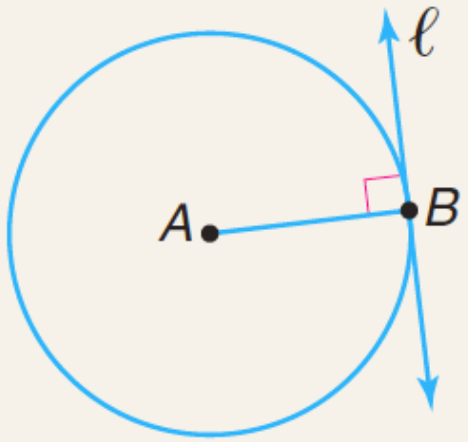
Tangents

Definition of a Tangent	
	Words: In a plane, a line is a tangent if and only if it intersects a circle in exactly one point.
	Model: 
	Symbols: Line ℓ is tangent to $\odot P$. T is called the **point of tangency**.

Theorem 14–4

Words: In a plane, if a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

Model:



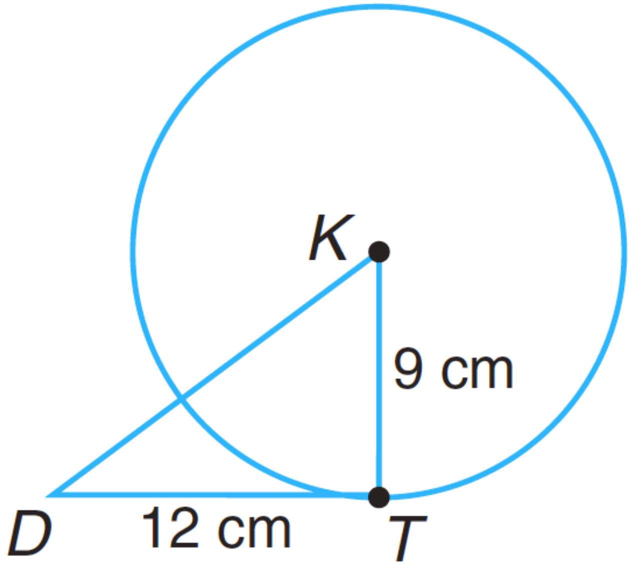
Symbols: If line ℓ is tangent to $\odot A$ at point B, then $\overline{AB} \perp \ell$.

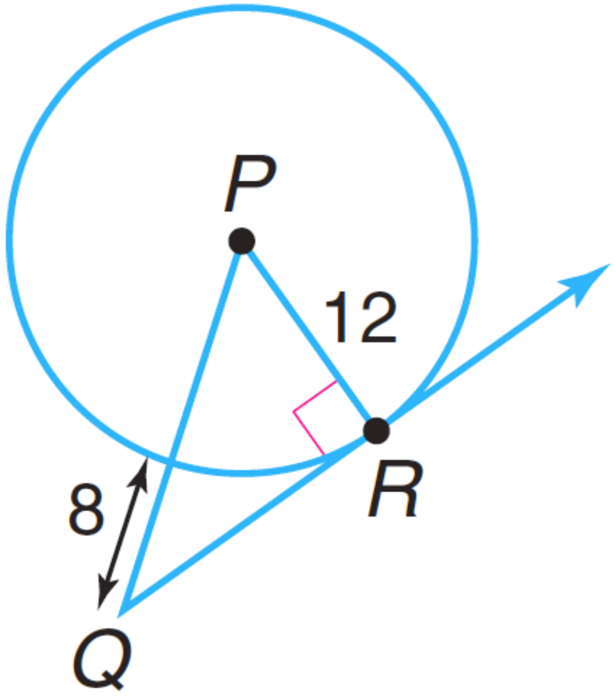
Theorem 14–5

Words: In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is a tangent.

Symbols: If $\overline{AB} \perp \ell$, then ℓ is tangent to $\odot A$ at point B.

$\overline{TD}$ is tangent to $\odot K$ at T. Find KD.



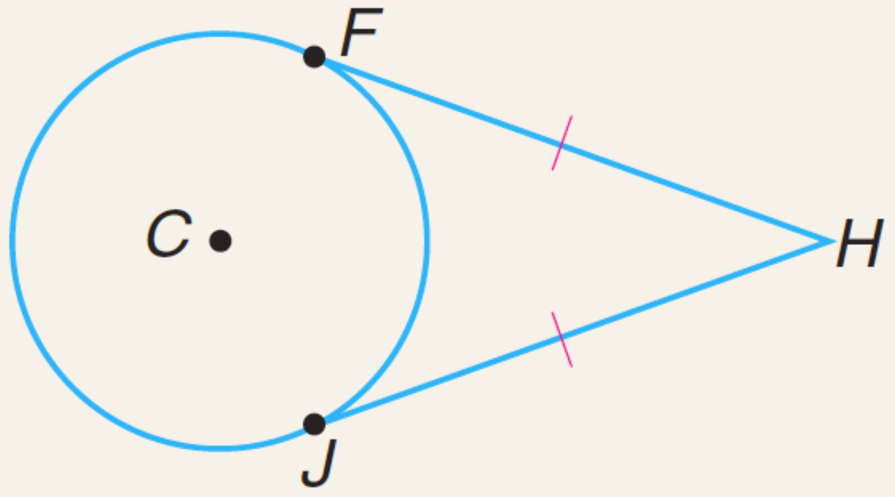


$\overrightarrow{QR}$ is tangent to $\odot P$ at R. Find RQ.

Theorem 14–6

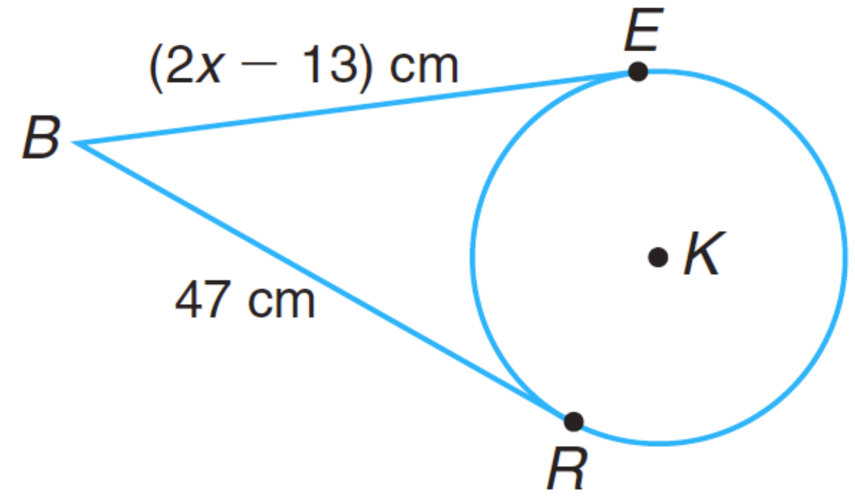
Words: If two segments from the same exterior point are tangent to a circle, then they are congruent.

Model:

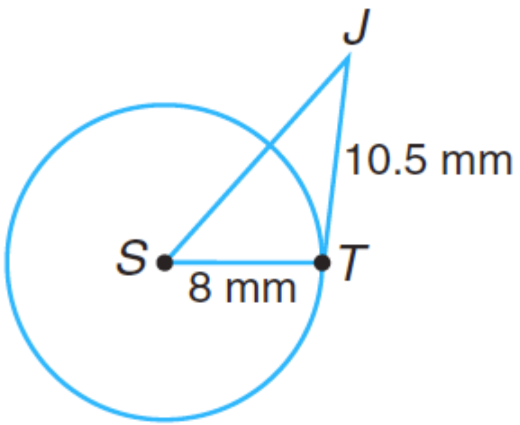


Symbols:
If $\overline{HF}$ and $\overline{HJ}$ are tangent to $\odot C$, then $\overline{HF} \cong \overline{HJ}$.

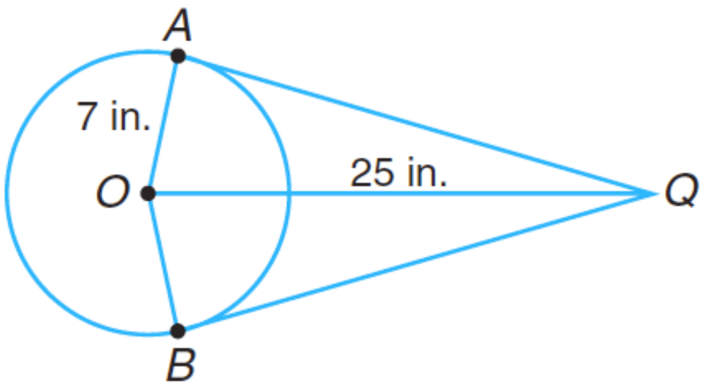
$\overline{BE}$ and $\overline{BR}$ are tangent to $\odot K$. Find the value of x.



6. $\overline{JT}$ is tangent to $\odot S$ at T. Find SJ to the nearest tenth.



7. $\overline{QA}$ and $\overline{QB}$ are tangent to $\odot O$. Find QB.



Recap

- Tangent lines to a circle touch it in exactly one place.
- Tangents are perpendicular to the radius / diameter they intersect. Lines perpendicular to radii are tangents to a circle.
- Two segments that are tangent to a circle and passing through the same point are congruent.

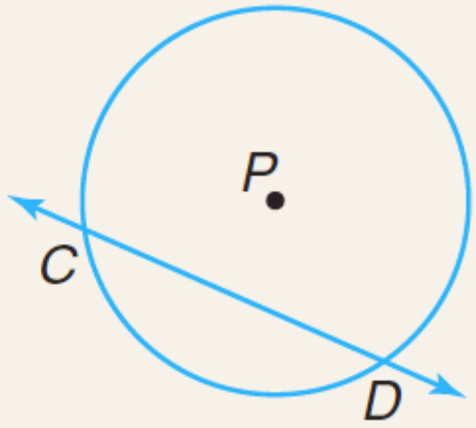
14.3- SECANT ANGLES

Secant segments

Theorem 14–7

Words: A line or line segment is a secant to a circle if and only if it intersects the circle in two points.

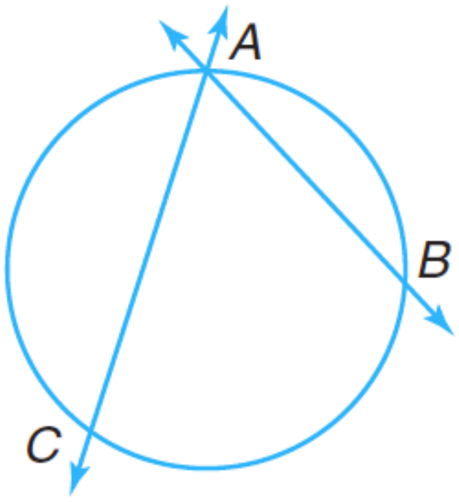
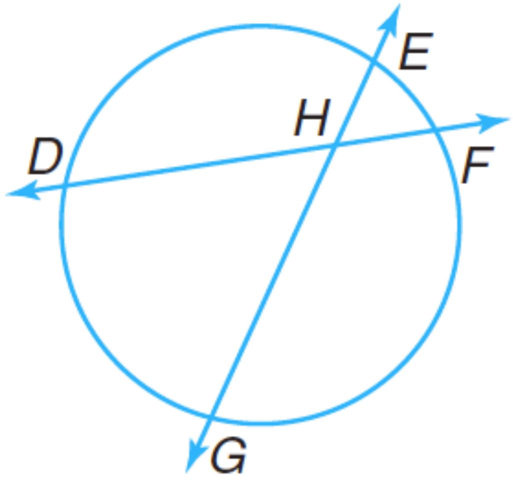
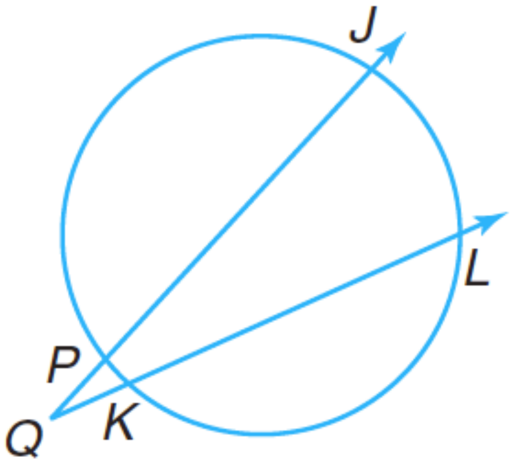
Model:



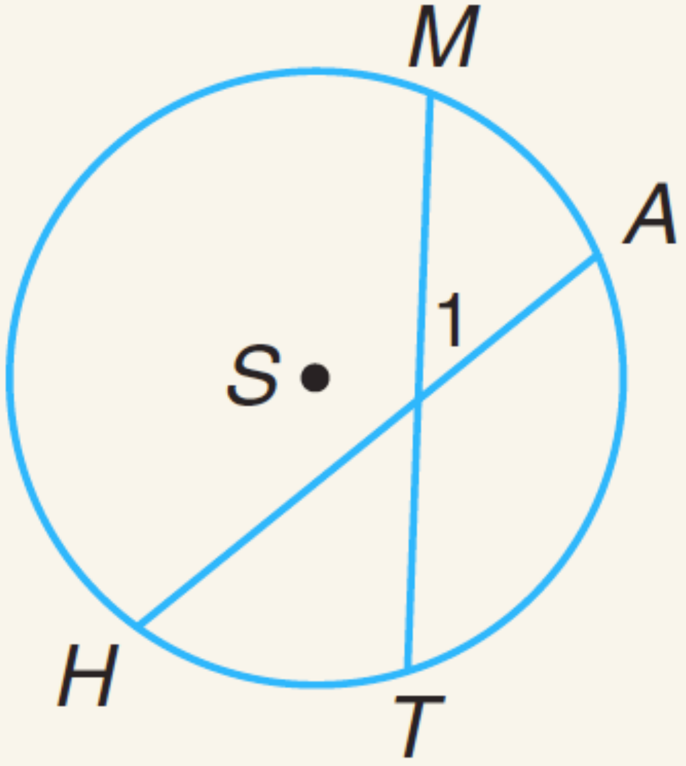
Symbols:
$\overleftrightarrow{CD}$ is a secant of $\odot P$.
Chord CD is a secant segment.

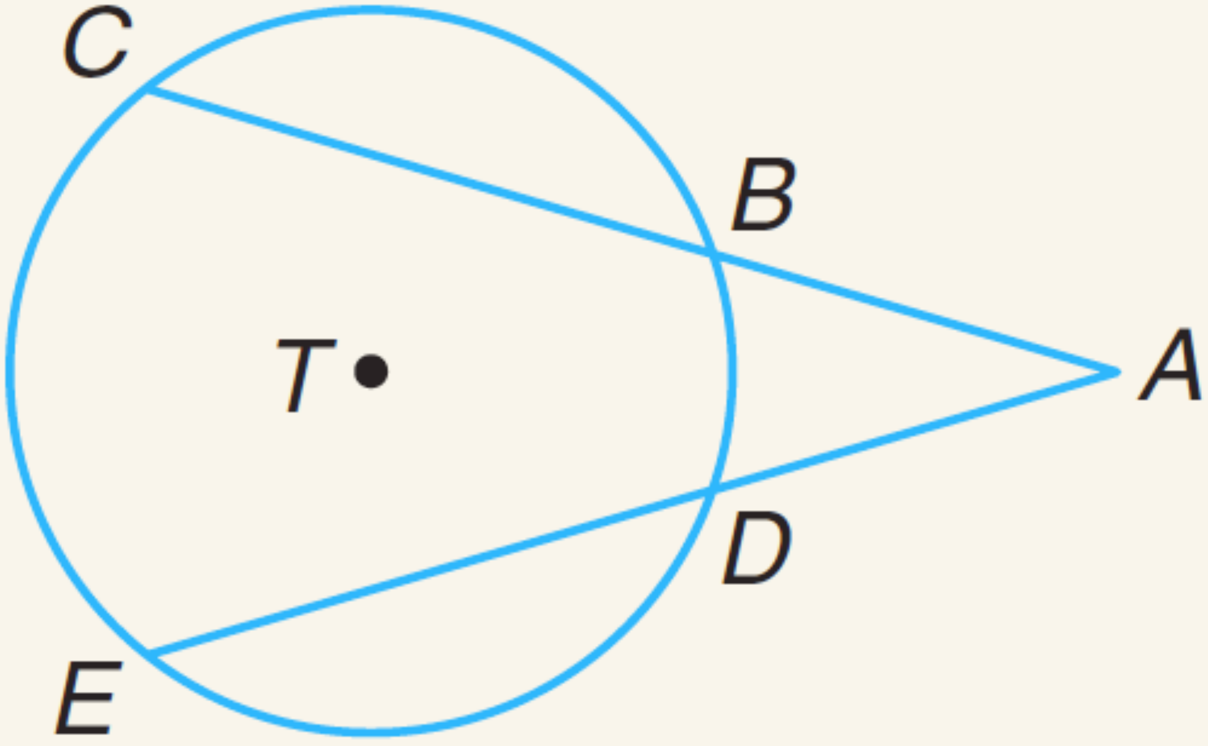
Secant Angles

- Secant Angles are formed when two or more secants segments intersect.

Case 1 Vertex On the Circle	Case 2 Vertex Inside the Circle	Case 3 Vertex Outside the Circle
		
Secant angle *CAB* intercepts $\overset{\frown}{BC}$ and is an inscribed angle.	Secant angle *DHG* intercepts $\overset{\frown}{DG}$, and its vertical angle intercepts $\overset{\frown}{EF}$.	Secant angle *JQL* intercepts $\overset{\frown}{JL}$ and $\overset{\frown}{PK}$.

Secant Angle-Arc Relationships

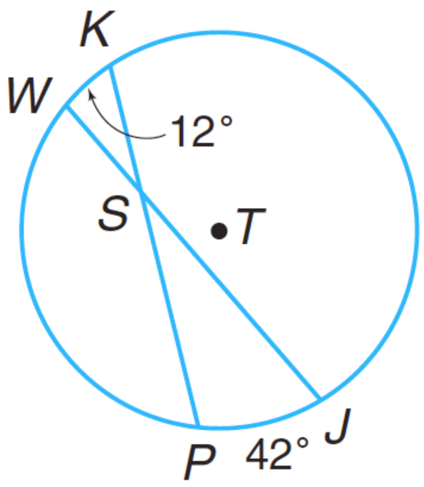




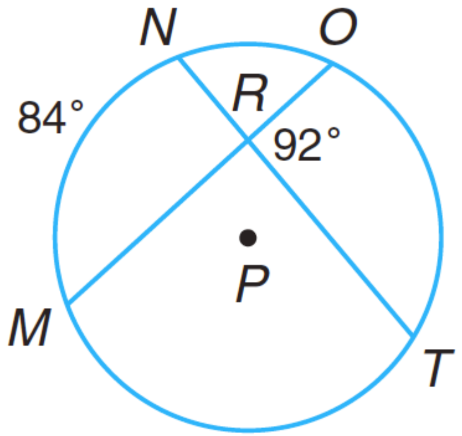
$$m\angle 1 = \frac{m\widehat{AM}+m\widehat{HT}}{2}$$

$$m\angle A = \frac{m\widehat{CE}-m\widehat{BD}}{2}$$

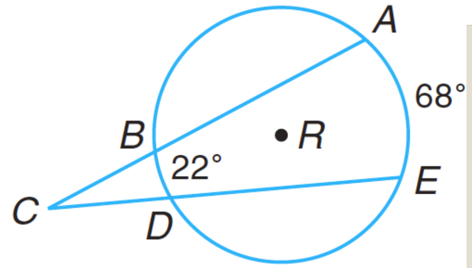
Find $m\angle WSK$.



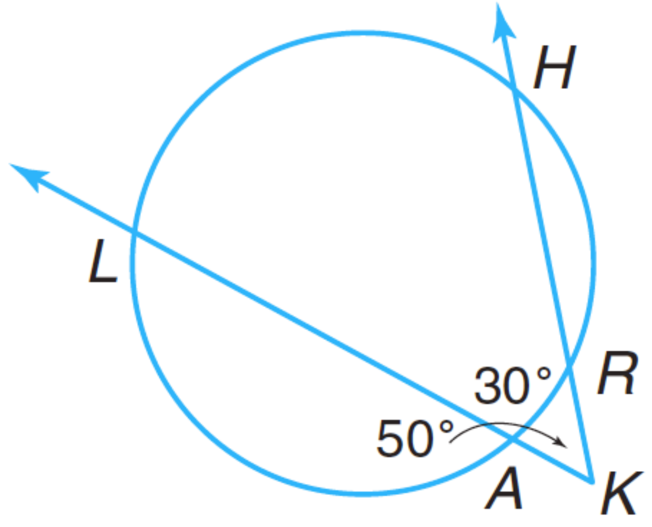
Find $m\widehat{OT}$.



Find $m\angle C$.



$m\widehat{LH}$



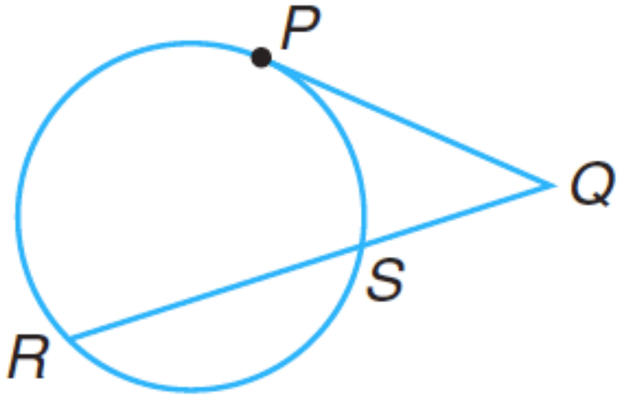
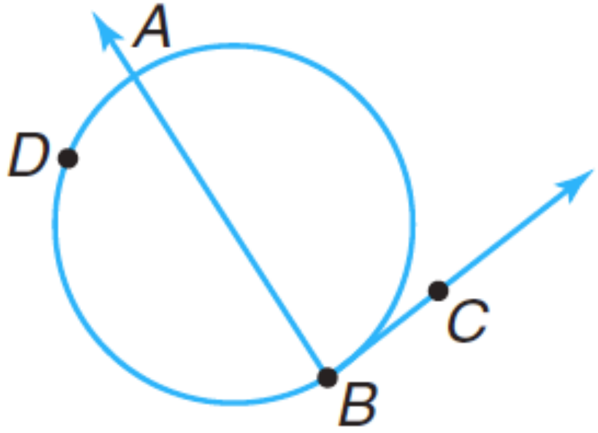
Recap

- Secant angles are formed when secants intersect in a circle.
- There is a relationship between the angle measures and the measures of the intercepted arcs (see previous slide for equations).

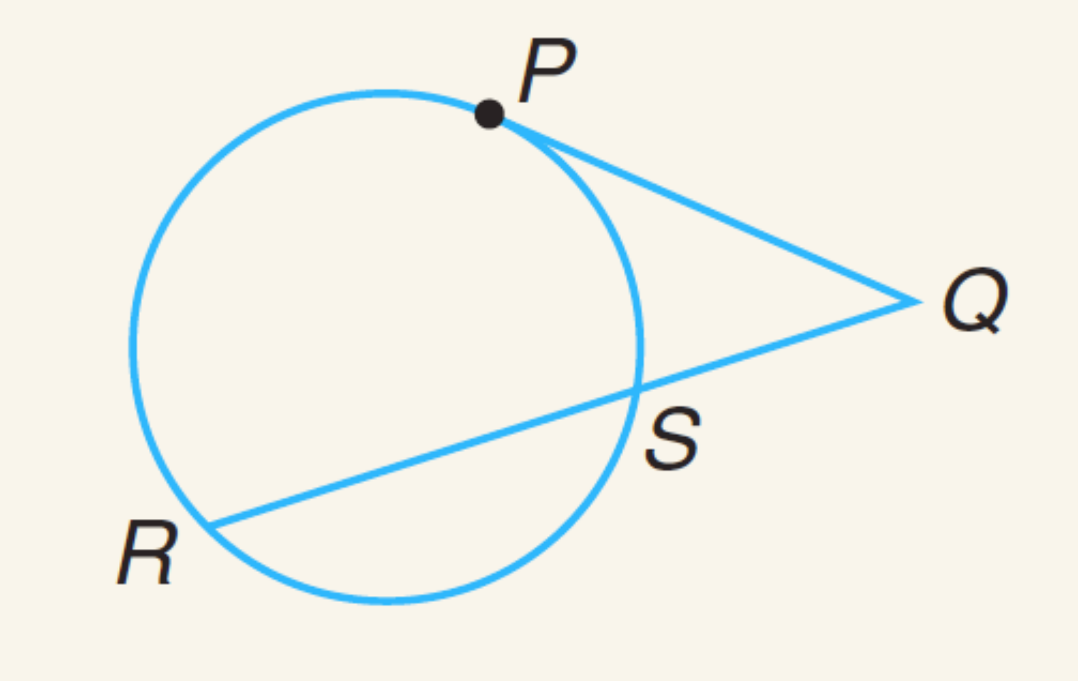
14.4- SECANT TANGENT ANGLES

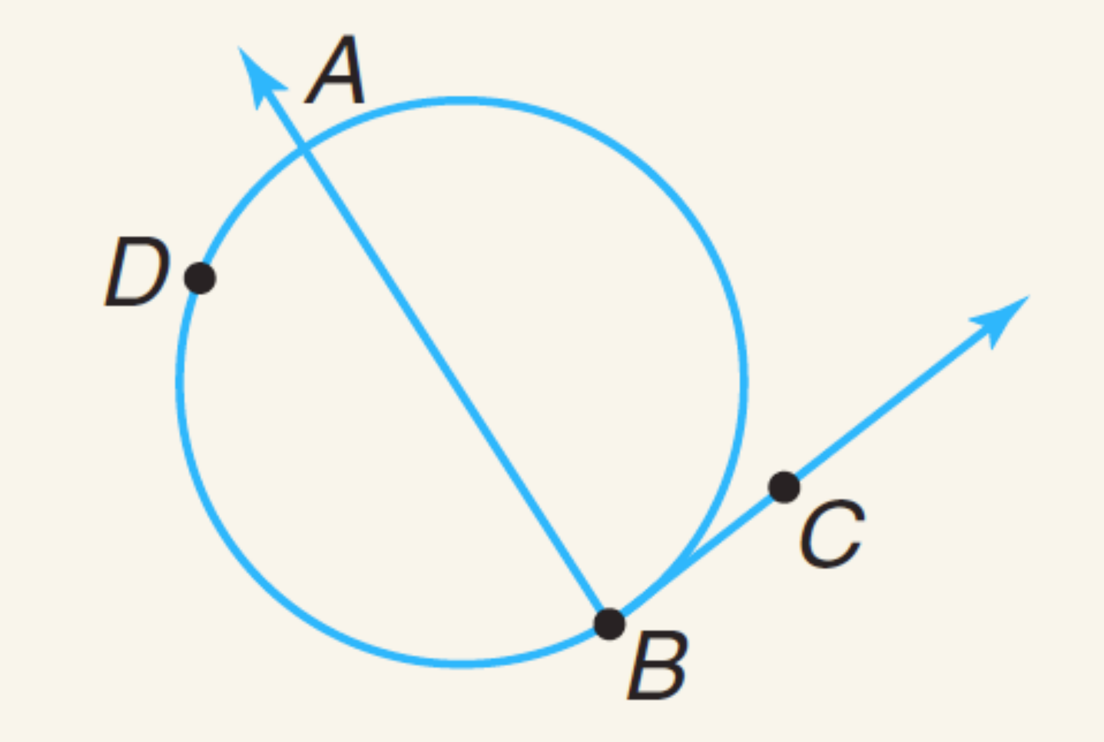
Secant-Tangent Angles

◦ Secant-Tangent Angles are formed when a secant segment and a tangent intersect.

Case 1 Vertex Outside the Circle	Case 2 Vertex On the Circle
	
Secant-tangent angle *PQR* intercepts $\overset{\frown}{PR}$ and $\overset{\frown}{PS}$.	Secant-tangent angle *ABC* intercepts $\overset{\frown}{AB}$.

Secant-Tangent Angle-Arc Relationships

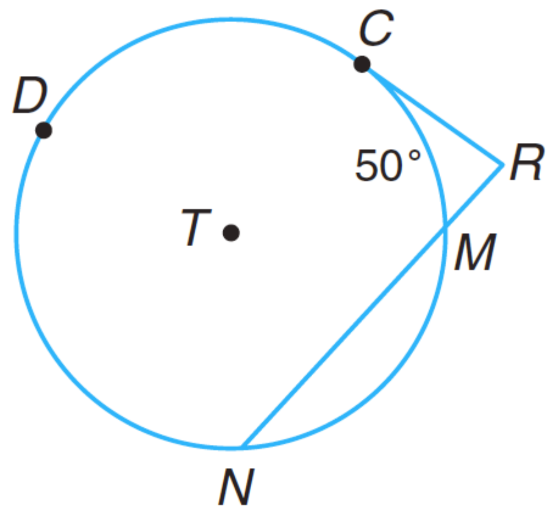




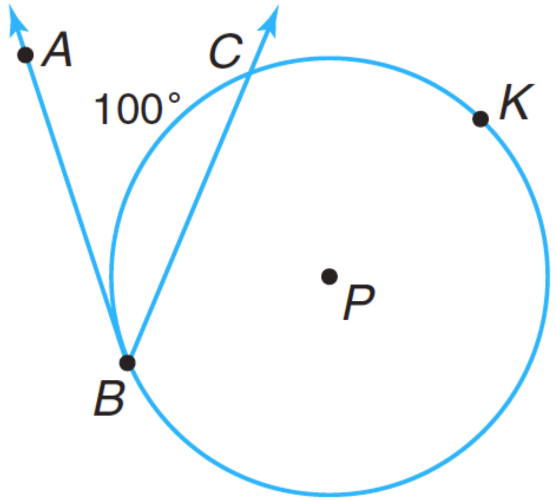
$$m\angle PQR = \frac{m\widehat{PR} - m\widehat{PS}}{2}$$

$$m\angle ABC = \frac{m\widehat{AB}}{2}$$

$\overline{CR}$ is tangent to $\odot T$ at C. If $m\overset{\frown}{CDN} = 200$, find $m\angle R$.



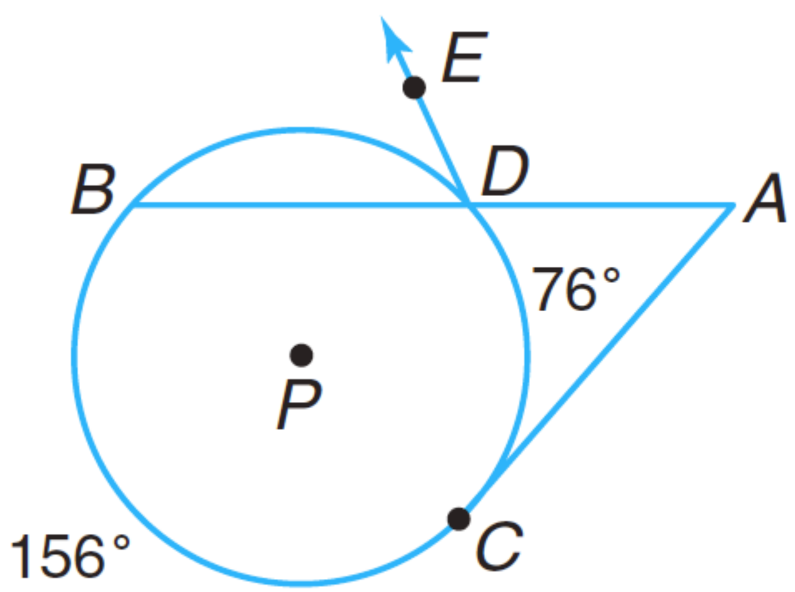
$\overrightarrow{BA}$ is tangent to $\odot P$ at B. Find $m\angle ABC$.



$\overline{AC}$ **is tangent to** $\odot P$ **at** C **and** $\overrightarrow{DE}$ **is tangent to** $\odot P$ **at** D**.**

a. Find $m\angle A$.

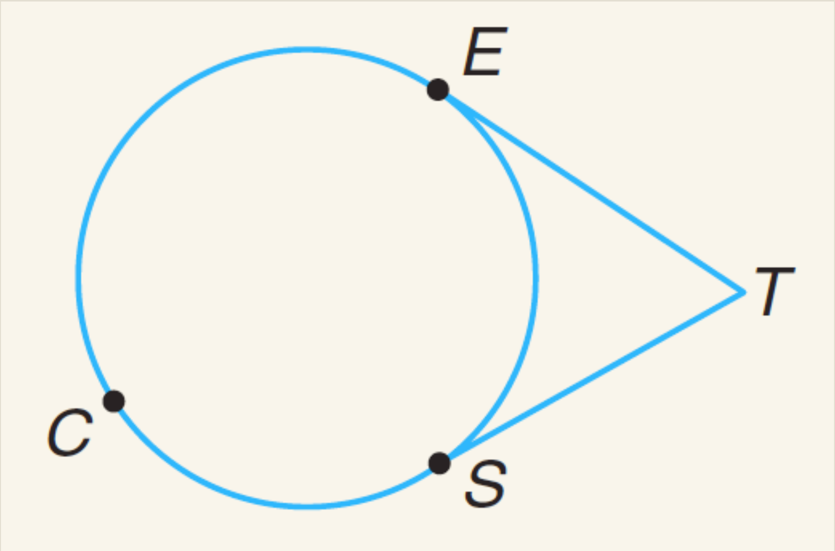
b. Find $m\angle BDE$.



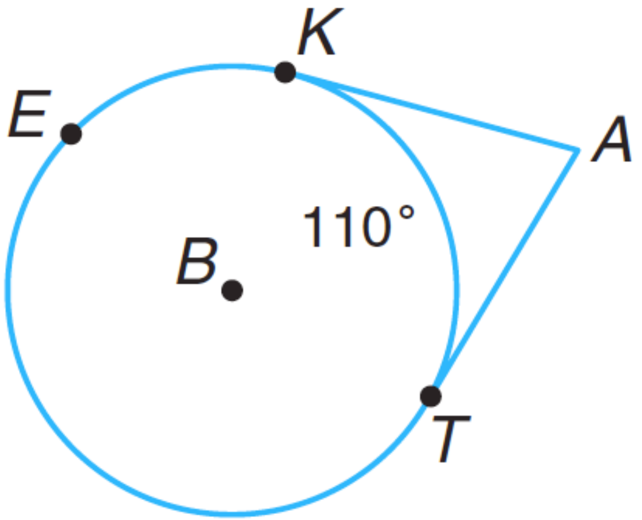
Tangent-Tangent Angles

- Tangent-Tangent Angles are formed when two tangents intersect.

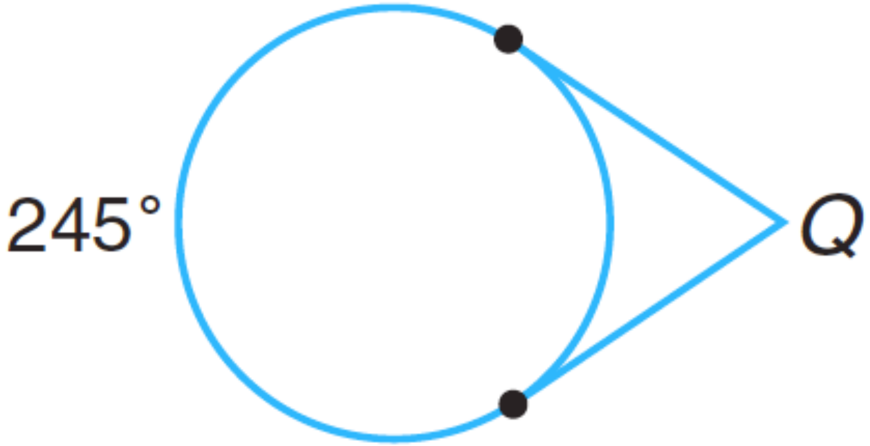
$$m\angle ETS = \frac{m\widehat{ECS} - m\widehat{ES}}{2}$$



Find $m\angle A$.



$\angle Q$



Recap

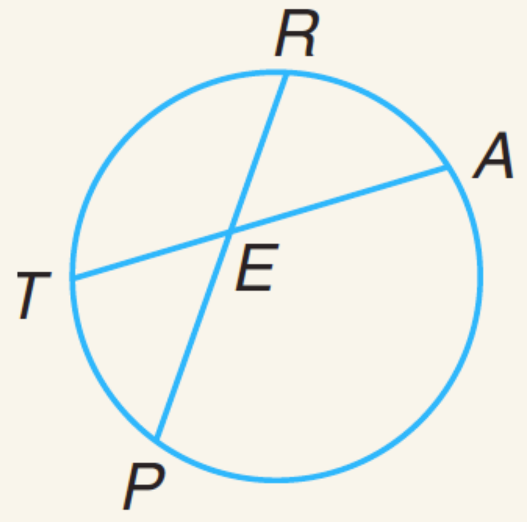
- Secant-tangent angles are formed when a secant and a tangent intersect on or outside a circle.
- There is a relationship between the angle measures and the measures of the intercepted arcs (see previous slide for equations).

Recap

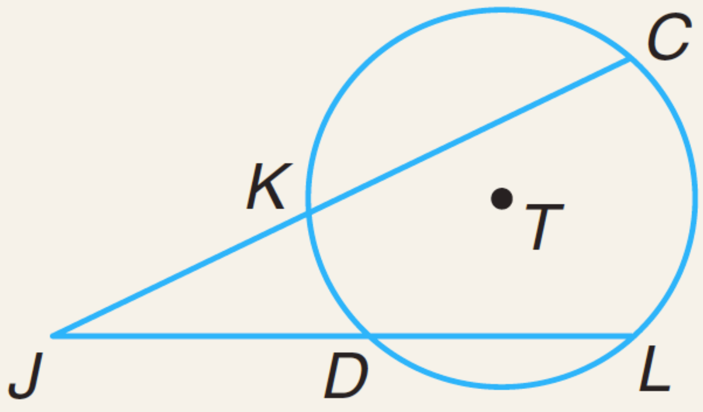
- Fill in the angle and Arc Relationships in Circles table

14.5- SEGMENT MEASURES

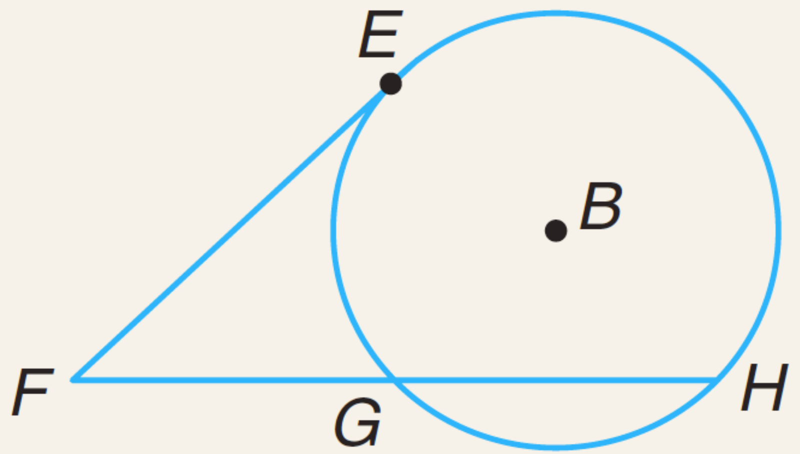
Segment Measures Relationships



$TE \cdot EA = RE \cdot EP$

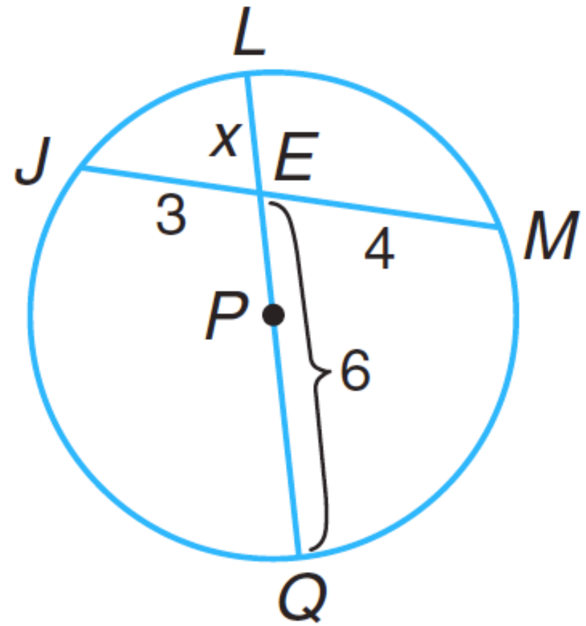


$JC \cdot JK = JL \cdot JD$

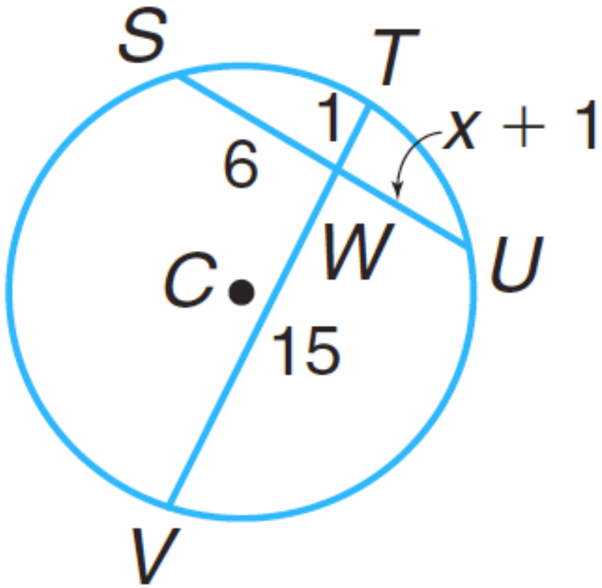


$(FE)^2 = FH \cdot FG$

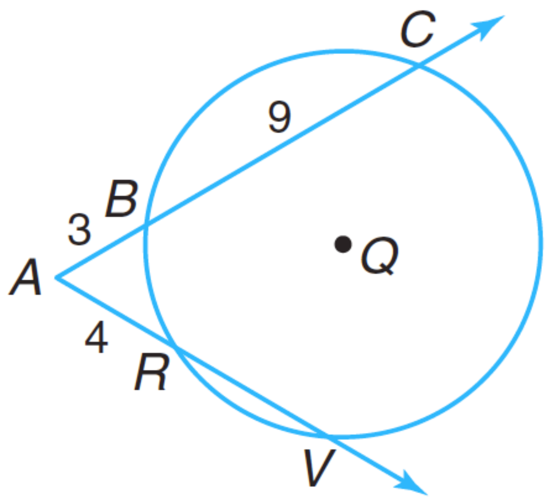
In ⊙P, find the value of x.



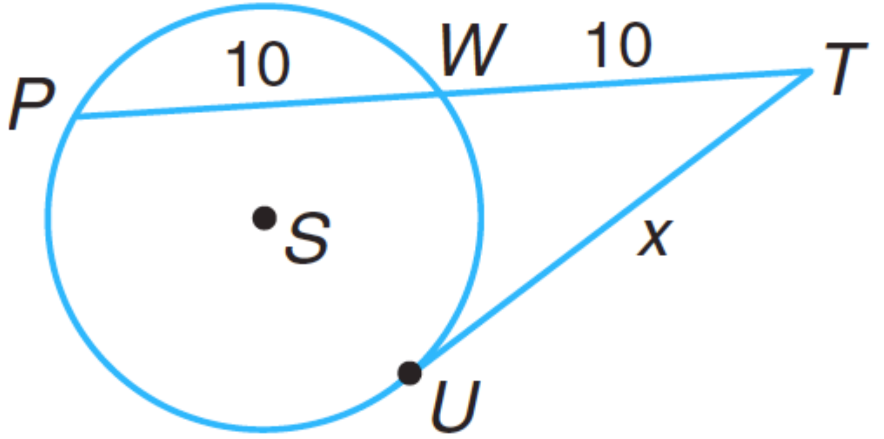
In ⊙C, find UW.



Find *AV* and *RV*.

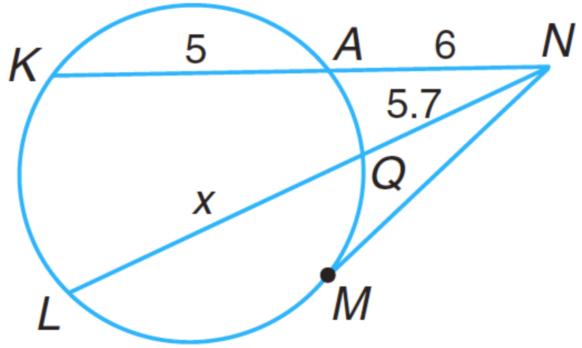


Find the value of *x* to the nearest tenth.



b. Find the value of x to the nearest tenth.

c. Find *MN* to the nearest tenth.

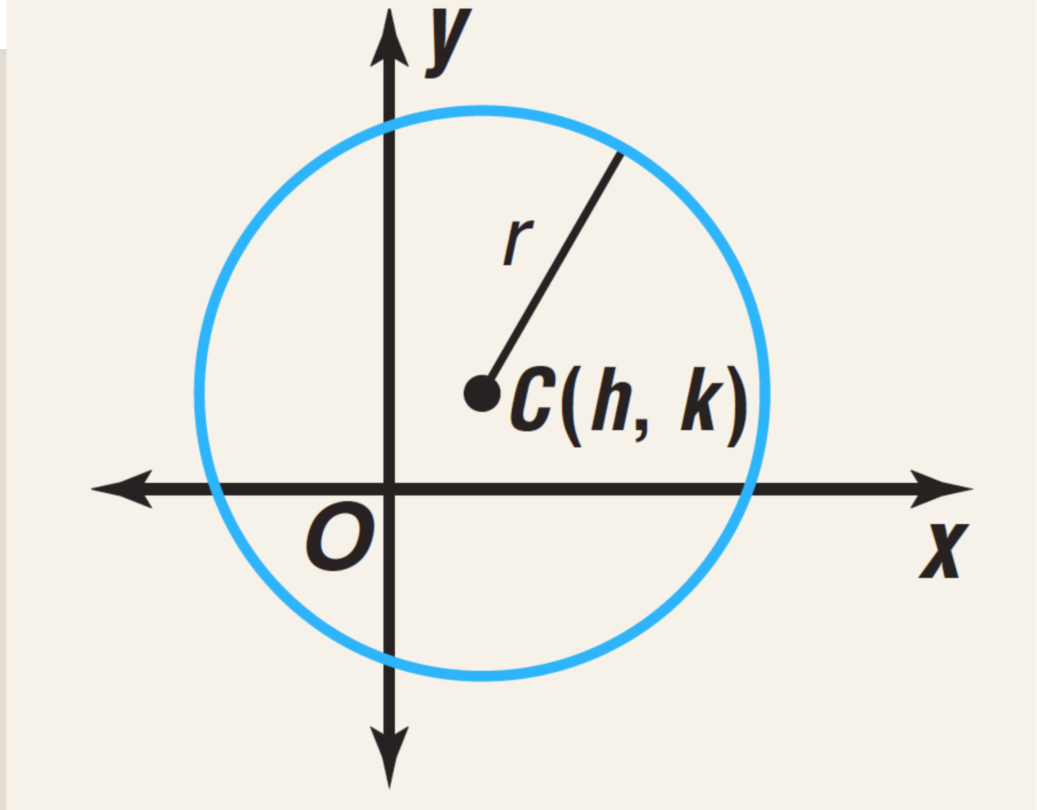


Recap

- There are relationships between the measures of segments created when secants and tangents intersect in or outside a circle.
- See previous slide for equations.

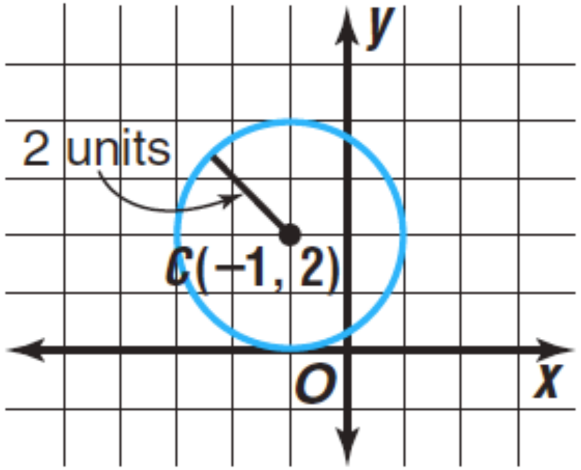
14.6- EQUATIONS OF CIRCLES

Equation of a circle



$$(x-h)^2+(y-k)^2=r^2$$

Write an equation of a circle with center $C(-1, 2)$ and a radius of 2 units.



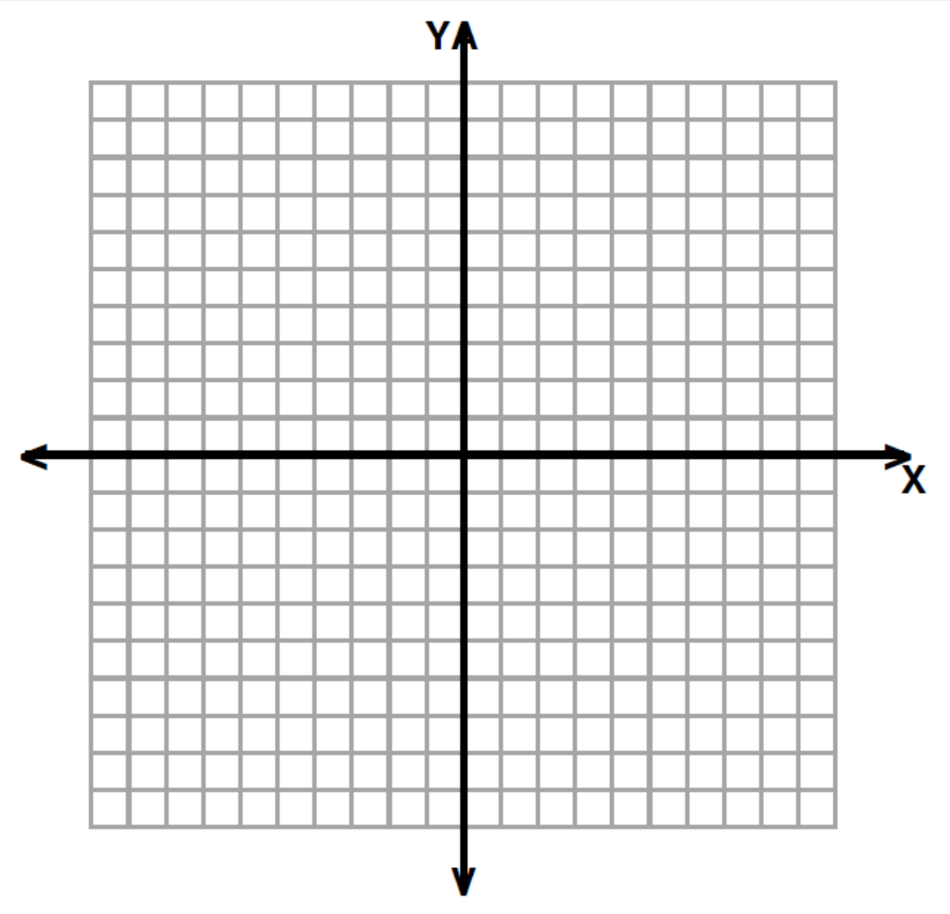
Write an equation of a circle with center at $(3, -2)$ and a diameter of 8 units.

Find the coordinates of the center and the measure of the radius of a circle whose equation is $x^2 + \left(y - \frac{3}{4}\right)^2 = \frac{25}{4}$.

$(x - 7)^2 + (y + 5)^2 = 4$

Graph each equation on a coordinate plane.

26. $(x + 5)^2 + (y - 2)^2 = 4$



27. $x^2 + (y - 3)^2 = 16$