CHAPTER 14 – CIRCLE RELATIONSHIPS

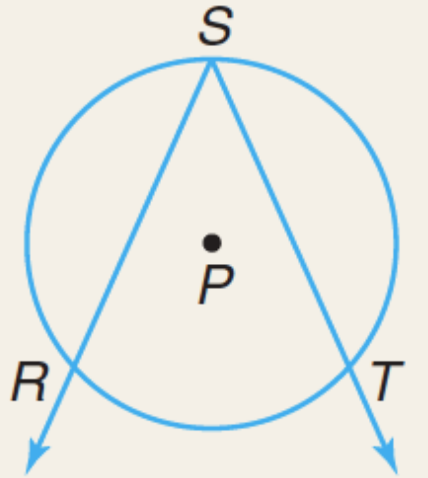
14.1- INSCRIBED ANGLES

Inscribed Angle

Definition of Inscribed Angle

Words: An angle is inscribed if and only if its vertex lies on the circle and its sides contain chords of the circle.

Model:

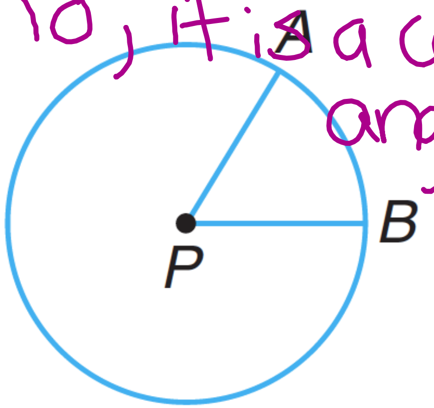


Symbols:
$\angle RST$ is inscribed in $\odot P$.

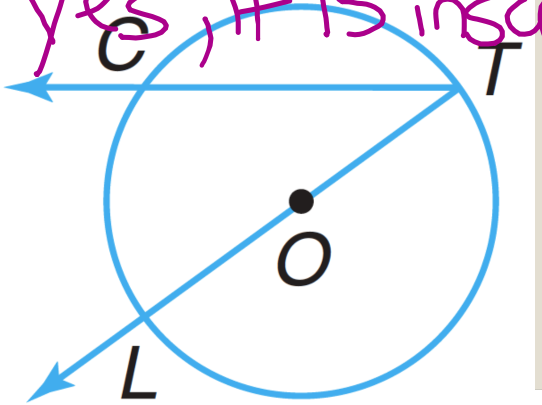
1) Vertex is on the circle

2) Sides are contained in the circle.

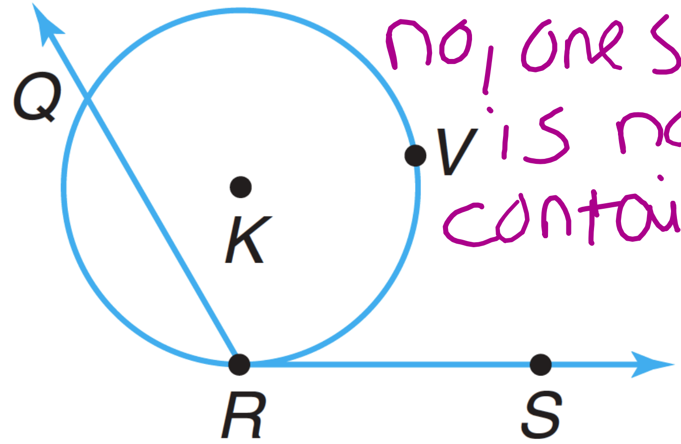
no, it is a central angle.



yes, it is inscribed



no, one side is not contained



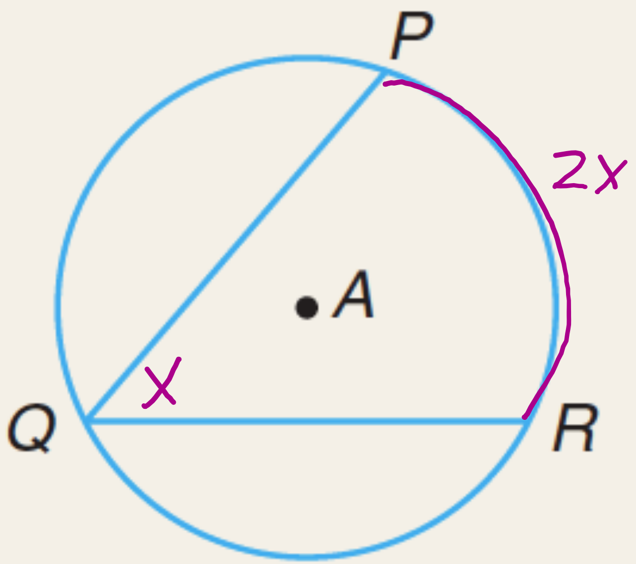
angle → arc: multiply by 2
arc → angle: divide by 2

Measure of inscribed angles

Theorem 14–1

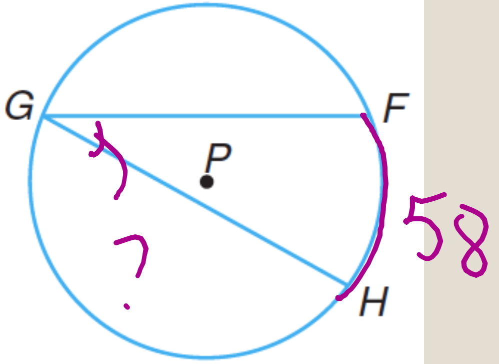
Words: The degree measure of an inscribed angle equals one-half the degree measure of its intercepted arc.

Model:



Symbols:
$m\angle PQR = \frac{1}{2}m\widehat{PR}$

If $m\widehat{FH} = 58$, find $m\angle FGH$.

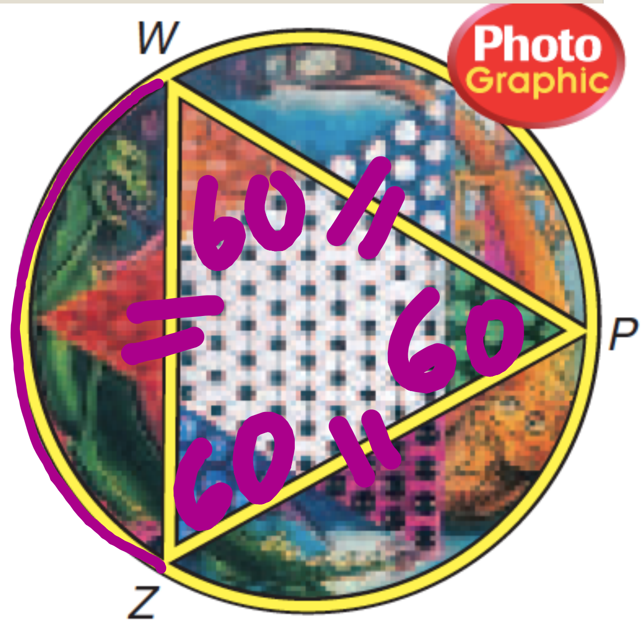


$$m\angle FGH = m\widehat{FH} \div 2$$
$$= 58 \div = 29^\circ$$

In the game shown at the right, ΔWPZ is equilateral. Find $m\widehat{WZ}$.

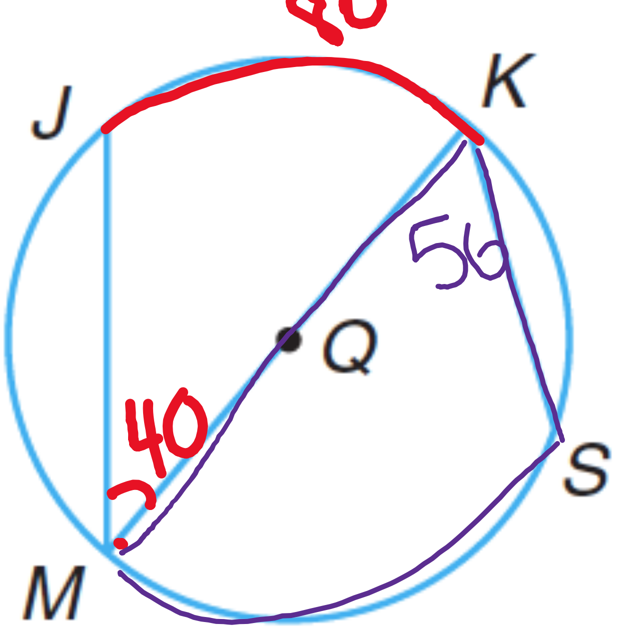
$$m\widehat{WZ} = 2 \cdot m\angle WPZ$$
$$= 2 \cdot 60 = 120^\circ$$

?



c. If $m\overset{\frown}{JK} = 80$, find $m\angle JMK$.

d. If $m\angle MKS = 56$, find $m\overset{\frown}{MS}$.



$80 \div 2 = 40$ $56 \times 2 = 112$

$m\overset{\frown}{JK} \div 2 = m\angle JMK$ $m\overset{\frown}{MS} = m\angle MKS \cdot 2$

$80 \div 2 = m\angle JMK$ $m\overset{\frown}{MS} = 56 \cdot 2$

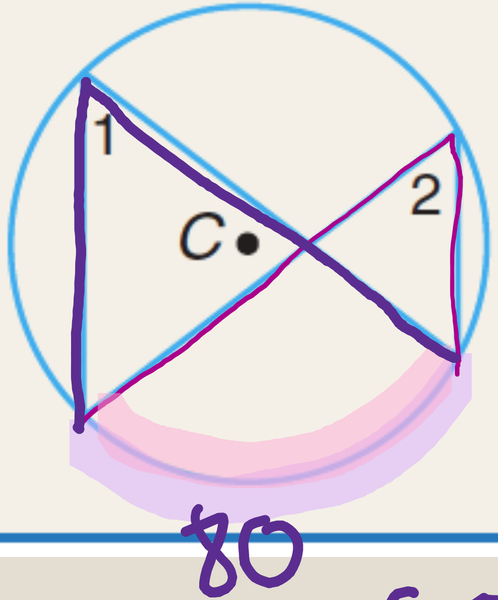
$40 = m\angle JMK$ $m\overset{\frown}{MS} = 112$

Congruent inscribed angles

Theorem 14–2

Words: If inscribed angles intercept the same arc or congruent arcs, then the angles are congruent.

Model:



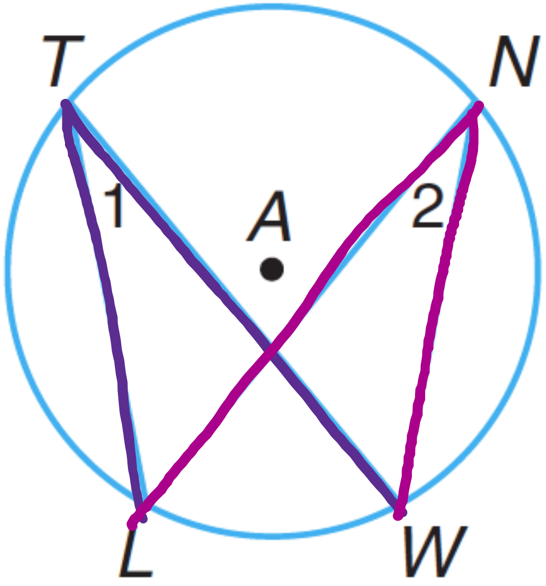
Symbols: $\angle 1 \cong \angle 2$

If the arc measures 80°, then $m\angle 1 = 40$

$m\angle 2 = 40$

so $\angle 1 \cong \angle 2$

In $\odot A$, $m\angle 1 = 2x$ and $m\angle 2 = x + 14$. Find the value of x.

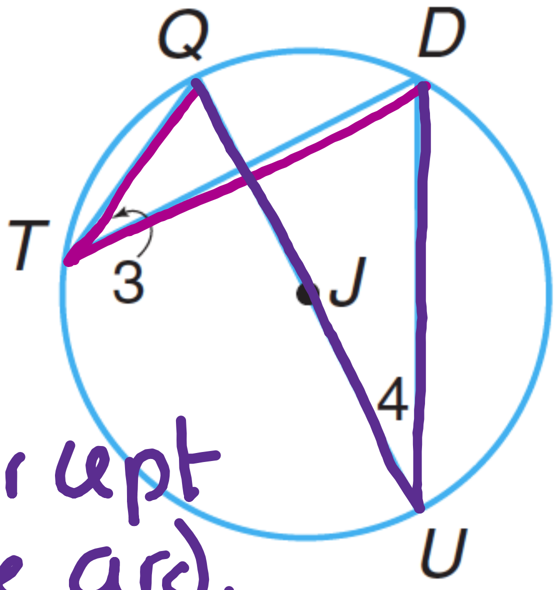


$\angle 1 \cong \angle 2$

$2x = x + 14$

$x = 14$

In $\odot J$, $m\angle 3 = 3x$ and $m\angle 4 = 2x + 9$. Find the value of x.



$\angle 3 \cong \angle 4$

(they intercept the same arc).

$3x = 2x + 9$

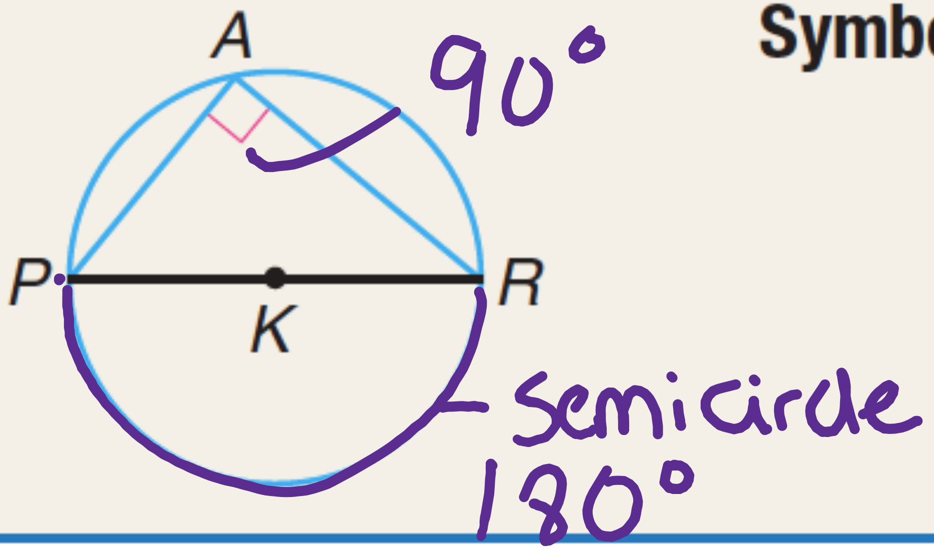
$x = 9$

Inscribed right triangles

Theorem 14–3

Words: If an inscribed angle of a circle intercepts a semicircle, then the angle is a right angle.

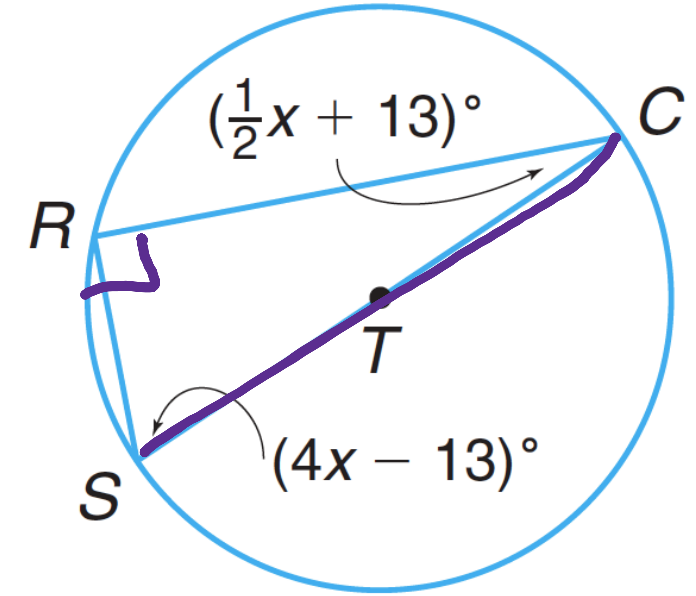
Model:



Symbols: $m\angle PAR = 90$

If an inscribed angle intercepts a semicircle, then it is a right angle.

In $\odot T$, $\overline{CS}$ is a diameter. Find the value of x.



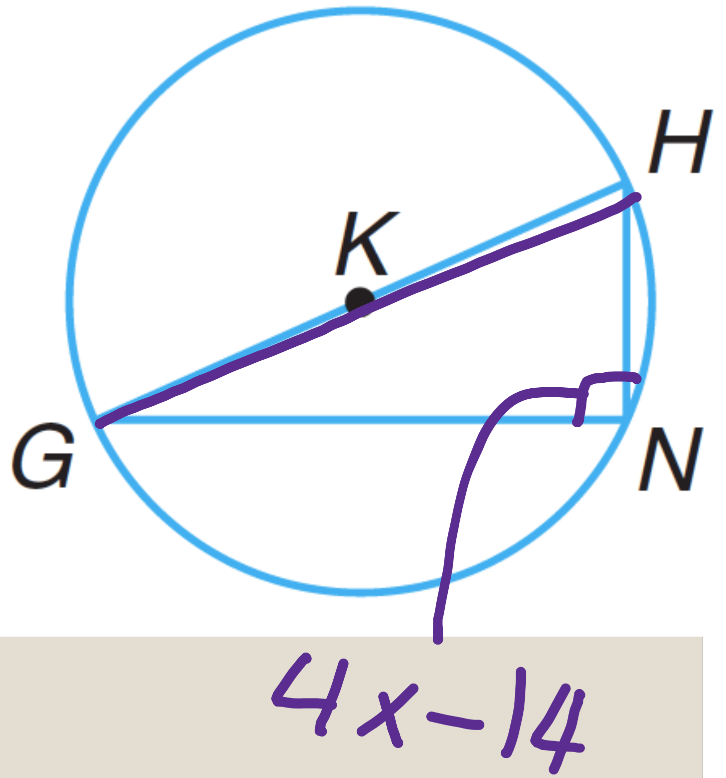
$$90+\left(\frac{1}{2}x+13\right)+(4x-13)=180$$

$$4.5x+90=180$$

$$4.5x=90$$

$$x=20°$$

In $\odot K$, $\overline{GH}$ is a diameter and $m\angle GNH = 4x - 14$. Find the value of x.



$$4x - 14 = 90$$
$$4x = 104$$
$$x = 26$$

14.1 Recap

- Inscribed angles have their vertex on the circle and sides contained in the circle.
- The inscribed angle measure is half the arc it intercepts.
- Inscribed angles are congruent if they intercept the same arc.
- Inscribed right triangles intercept semicircles.

14.2- TANGENTS TO A CIRCLE

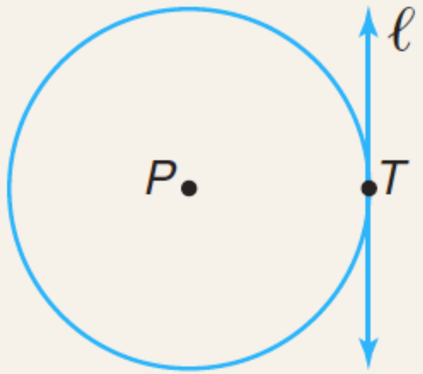
Tangents

A tangent touches a circle at exactly one point.

Definition of a Tangent

Words: In a plane, a line is a tangent if and only if it intersects a circle in exactly one point.

Model:

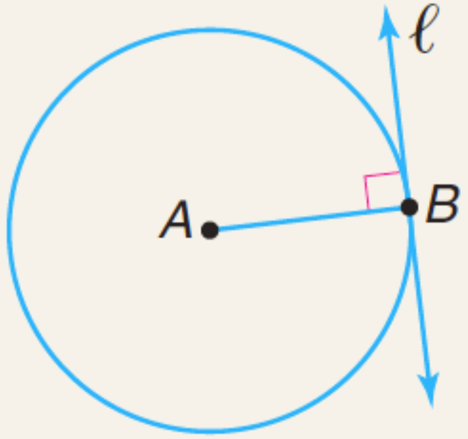


Symbols: Line ℓ is tangent to $\odot P$. T is called the **point of tangency**.

Theorem 14–4

Words: In a plane, if a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

Model:



Tangents are $\perp$ to the radius at the point of tangency.

Symbols: If line ℓ is tangent to $\odot A$ at point B, then $\overline{AB} \perp \ell$.

Theorem 14–5

Words: In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is a tangent.

Symbols: If $\overline{AB} \perp \ell$, then ℓ is tangent to $\odot A$ at point B.

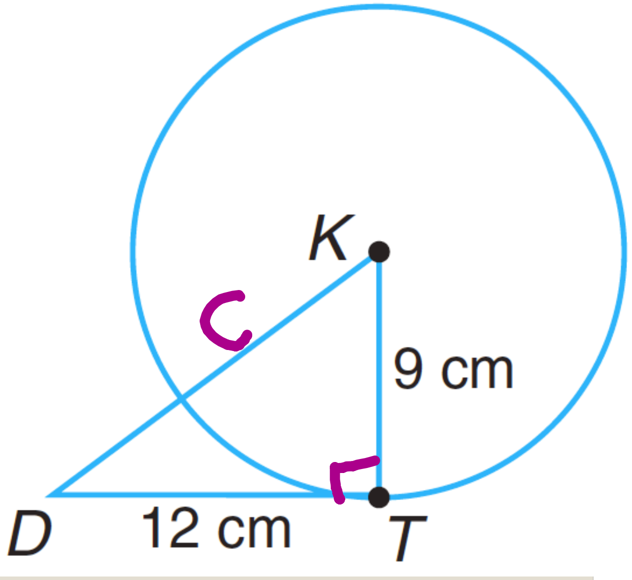
$\overline{TD}$ is tangent to $\odot K$ at T. Find KD.

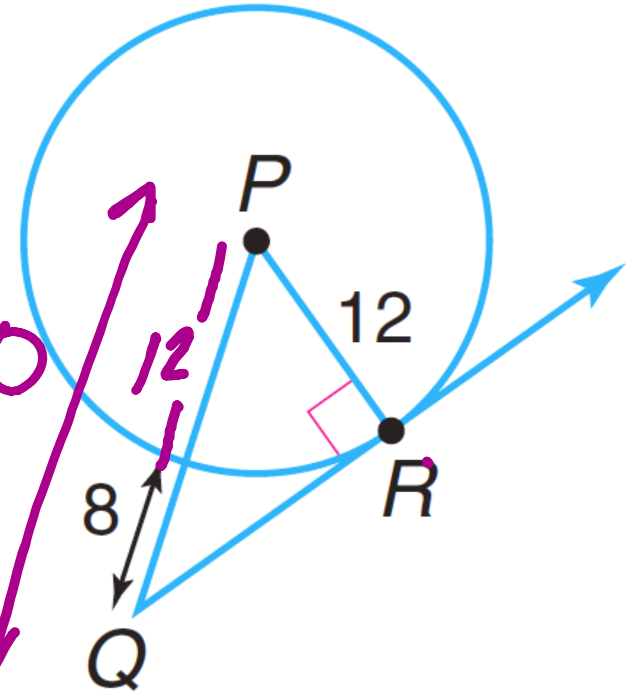
$KD^2 = 9^2 + 12^2$

$KD^2 = 81 + 144$

$KD^2 = 225$

$KD = \sqrt{225} = 15$





$\overrightarrow{QR}$ is tangent to $\odot P$ at R. Find RQ.

$20^2 = 12^2 + RQ^2$

$400 - 144 = RQ^2$

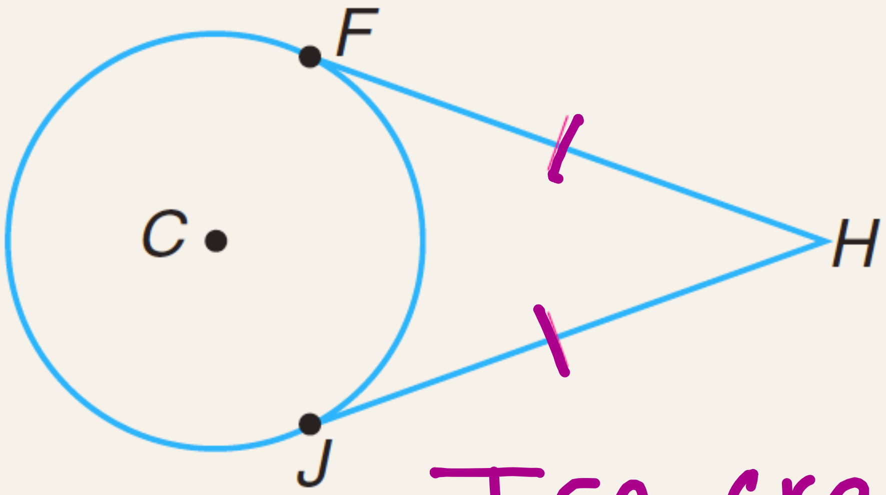
$256 = RQ^2$

$RQ = \sqrt{256} = 16$

Theorem 14–6

Words: If two segments from the same exterior point are tangent to a circle, then they are congruent.

Model:

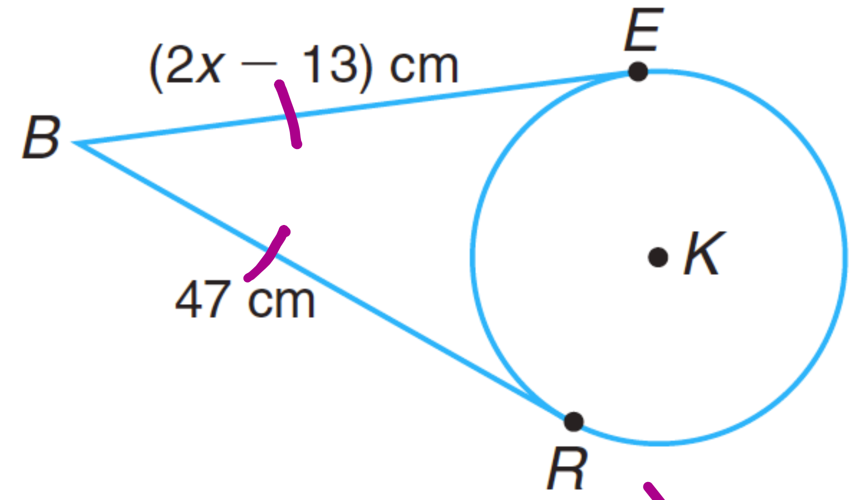


Ice cream cone

Symbols: If $\overline{HF}$ and $\overline{HJ}$ are tangent to $\odot C$, then $\overline{HF} \cong \overline{HJ}$.

The sides of the cone are congruent.

$\overline{BE}$ and $\overline{BR}$ are tangent to $\odot K$. Find the value of x.



(Ice cream cone)

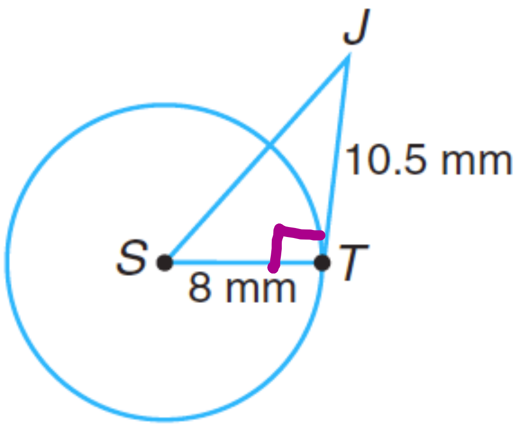
$\overline{BE} \cong \overline{BR}$

$2x - 13 = 47$

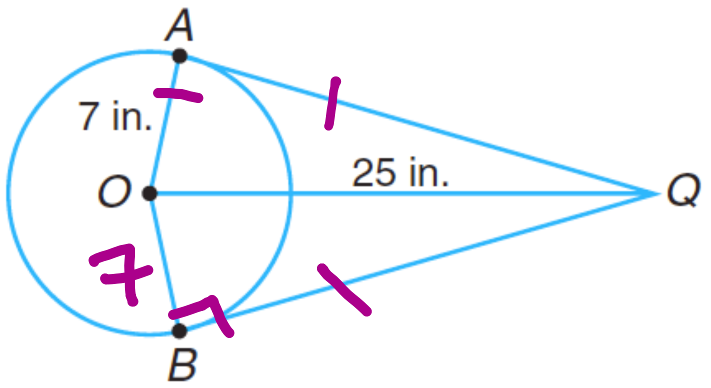
$2x = 60$

$x = 30$

6. $\overline{JT}$ is tangent to $\odot S$ at T. Find SJ to the nearest tenth.



7. $\overline{QA}$ and $\overline{QB}$ are tangent to $\odot O$. Find QB.



$SJ^2 = 10.5^2 + 8^2$

$SJ^2 = 110.25 + 64$

$SJ^2 = 174.25$

$SJ = 13.2$

$25^2 = 7^2 + QB^2$

$625 = 49 + QB^2$

$576 = QB^2$

$24 = QB$

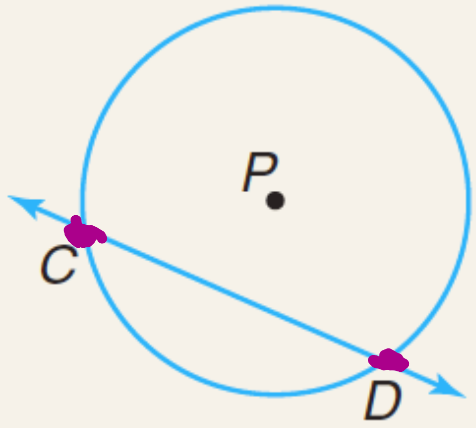
Recap

- Tangent lines to a circle touch it in exactly one place.
- Tangents are perpendicular to the radius / diameter they intersect. Lines perpendicular to radii are tangents to a circle.
- Two segments that are tangent to a circle and passing through the same point are congruent.

14.3- SECANT ANGLES

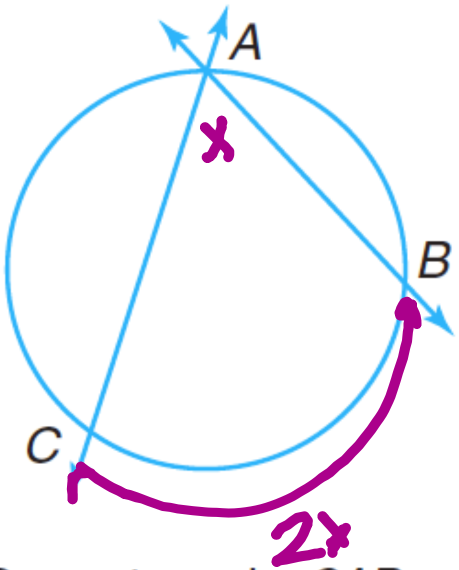
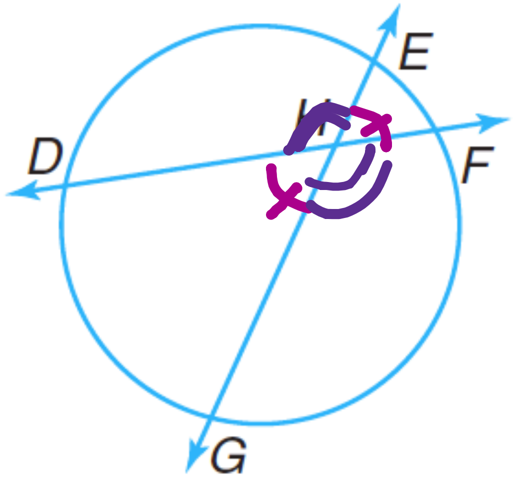
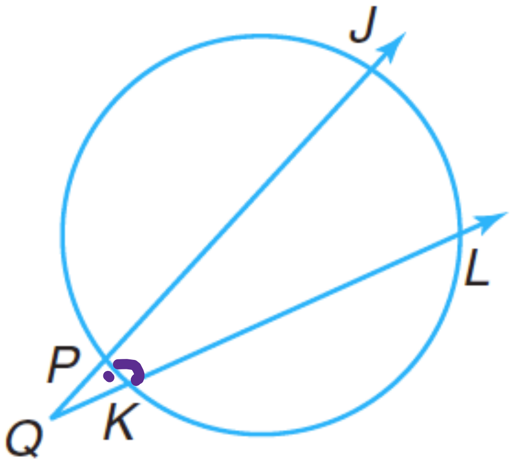
Secant segments

Secants intersect a circle in two places.

Theorem 14–7	**Words:** A line or line segment is a secant to a circle if and only if it intersects the circle in two points. **Model:**  **Symbols:** $\overleftrightarrow{CD}$ is a secant of $\odot P$. Chord CD is a secant segment.

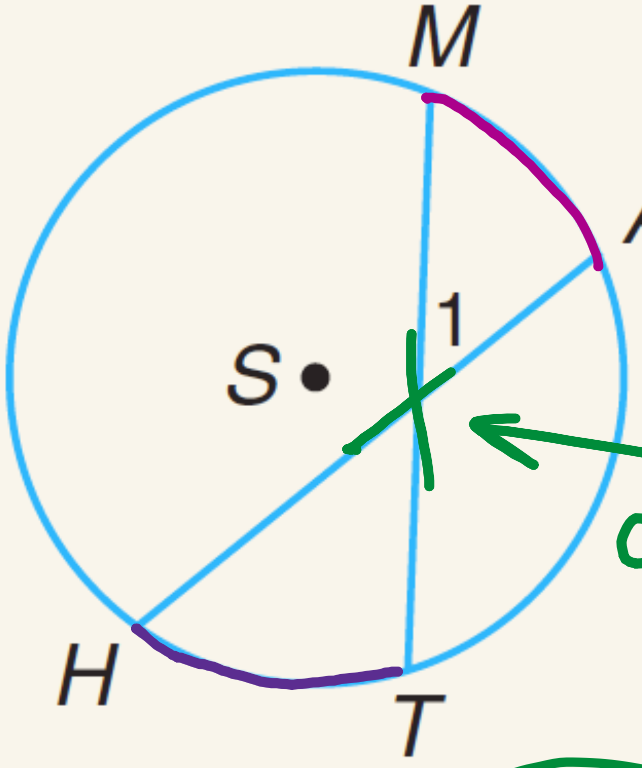
Secant Angles

- Secant Angles are formed when two or more secants segments intersect.

Case 1 Vertex On the Circle	Case 2 Vertex Inside the Circle	Case 3 Vertex Outside the Circle
		
Secant angle *CAB* intercepts $\overset{\frown}{BC}$ and is an inscribed angle.	Secant angle *DHG* intercepts $\overset{\frown}{DG}$, and its vertical angle intercepts $\overset{\frown}{EF}$.	Secant angle *JQL* intercepts $\overset{\frown}{JL}$ and $\overset{\frown}{PK}$.

Secant Angle-Arc Relationships

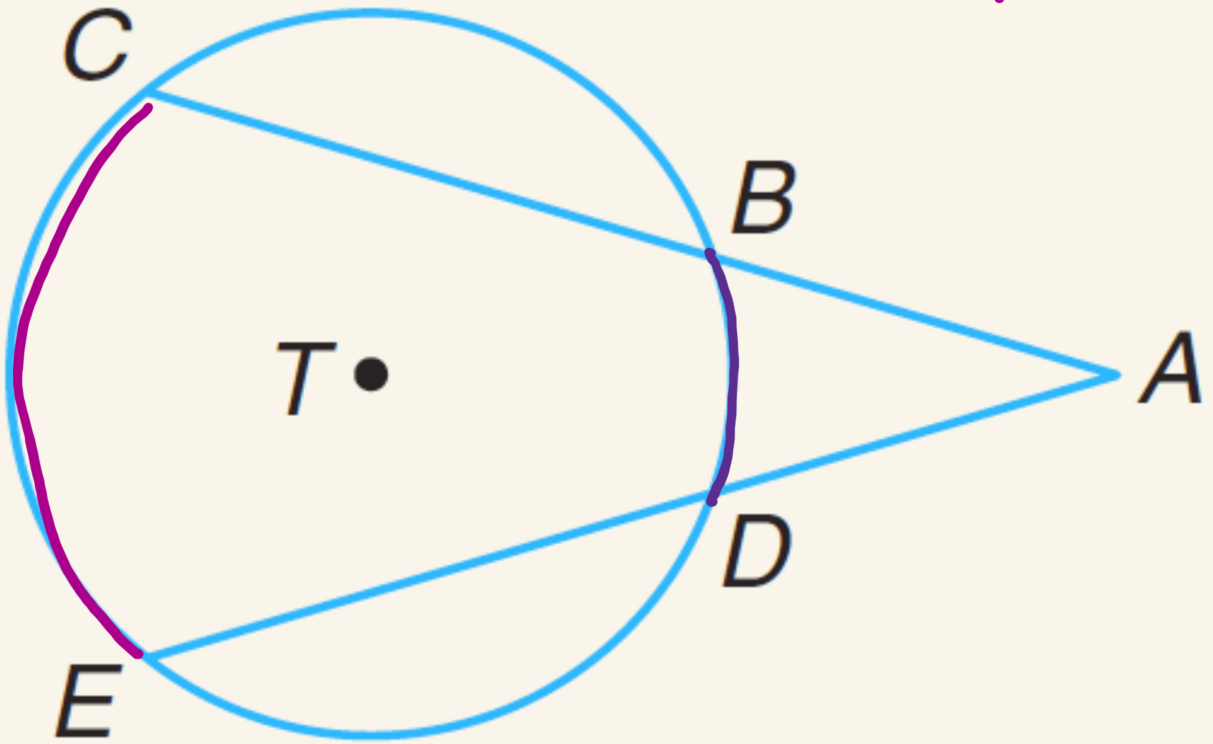
Intersection is inside



looks like a "plus" sign

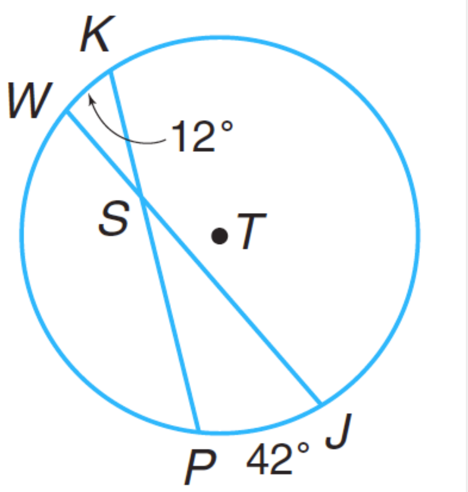
$$m\angle 1 = \frac{m\widehat{AM} + m\widehat{HT}}{2}$$

Intersection is outside



$$m\angle A = \frac{m\widehat{CE} - m\widehat{BD}}{2}$$

Find $m\angle WSK$.

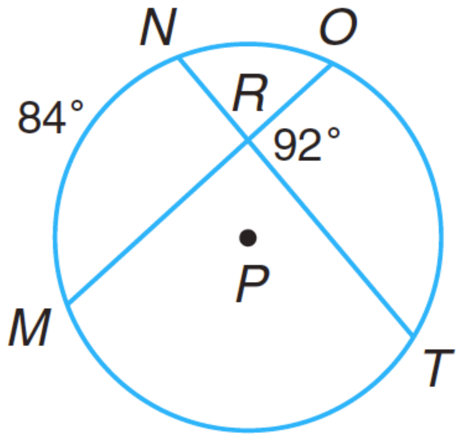


$m\angle WSK = \dfrac{m\overset{\frown}{WK} + m\overset{\frown}{PJ}}{2}$

$m\angle WSK = \dfrac{12+42}{2}$

$m\angle WSK = 27°$

Find $m\overset{\frown}{OT}$.



$m\angle ORT = \dfrac{m\overset{\frown}{OT} + m\overset{\frown}{NM}}{2}$

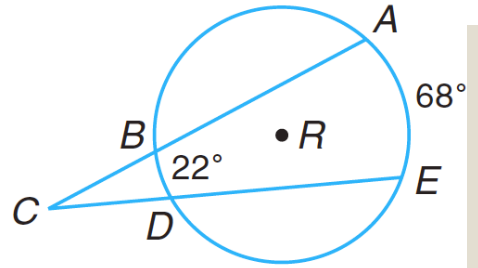
$2 \cdot 92 = \dfrac{m\overset{\frown}{OT} + 84}{2} \cdot 2$

$184 = m\overset{\frown}{OT} + 84$

$-84 \qquad -84$

$100 = m\overset{\frown}{OT}$

Find $m\angle C$.

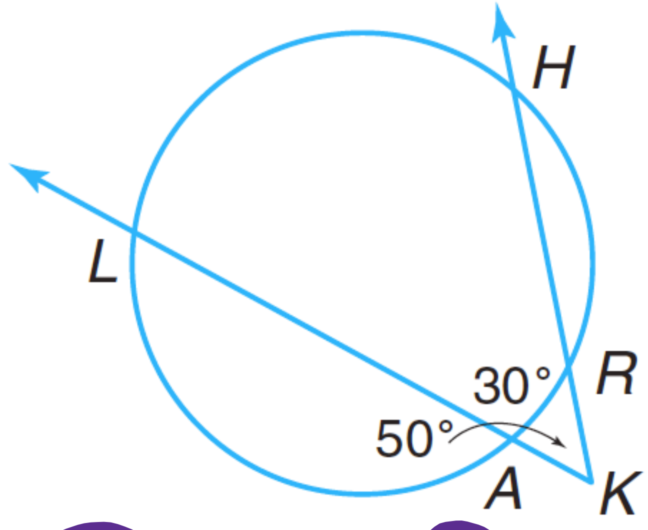


$$m\angle C = \frac{m\overset{\frown}{AE} - m\overset{\frown}{BD}}{2}$$

$$m\angle C = \frac{68-22}{2}$$

$$m\angle C = \frac{46}{2} = 23°$$

$m\overset{\frown}{LH}$



$$m\angle K = \frac{m\overset{\frown}{HL} - m\overset{\frown}{AR}}{2}$$

$$2 \cdot 50 = \frac{m\overset{\frown}{HL} - 30}{2} \cdot 2$$

$$100 = m\overset{\frown}{HL} - 30$$

$$+30 \qquad +30$$

$$130° = m\overset{\frown}{HL}$$

Recap

- Secant angles are formed when secants intersect in a circle.
- There is a relationship between the angle measures and the measures of the intercepted arcs (see previous slide for equations).