CHAPTER 13 – RIGHT TRIANGLES AND TRIGONOMETRY

13.1 – SIMPLIFYING SQUARE ROOTS

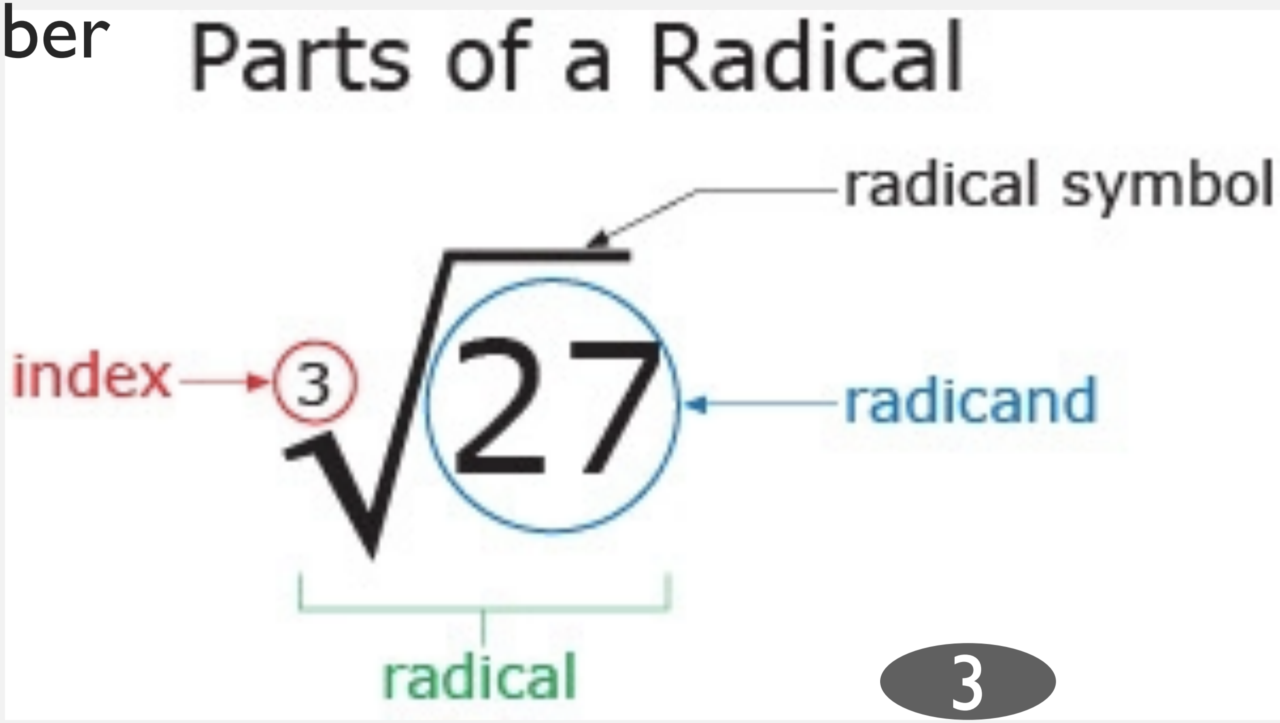
DEFINITION: SQUARE ROOTS

Square roots are the inverse of squaring.

They answer the question "what number squared gives ____?"

Ex: $\sqrt{4} = 2$ because $2^2 = 4$

what number squared gives 2?



Simplify each expression.

1 $\sqrt{49} = 7$

2 $\sqrt{64} = 8$

a. $\sqrt{25} = 5$

b. $\sqrt{144} = 12$

SIMPLIFYING SQUARE ROOTS

- 1. There are no perfect square factors other than 1 in the radicand.

  1

- 2. The radicand is not a fraction. $\sqrt{\frac{2}{3}}$ ← not simplified
- 3. The denominator does not contain a radical expression. $\frac{2}{\sqrt{3}}$ ↑

  not simplified

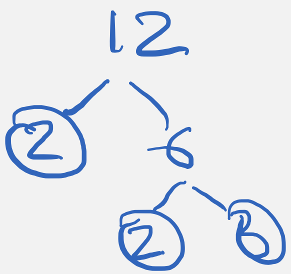
PULLING OUT PERFECT SQUARES

radicand

- 1) Break down the number into its prime factors.
- 2) Remove any numbers that appear twice, write them once in front of the radical.
- Ex: Simplify $\sqrt{12}$

$12 = 2^2 \cdot 3$

$2\sqrt{3}$

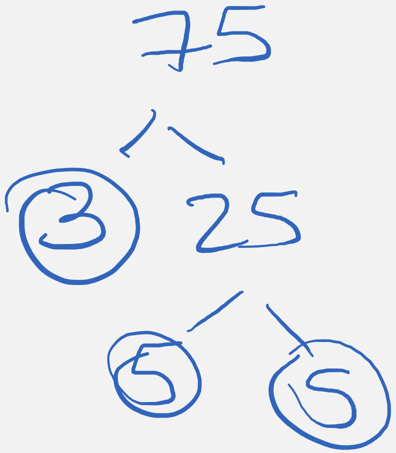


$\sqrt{12} = \sqrt{4 \cdot 3}$

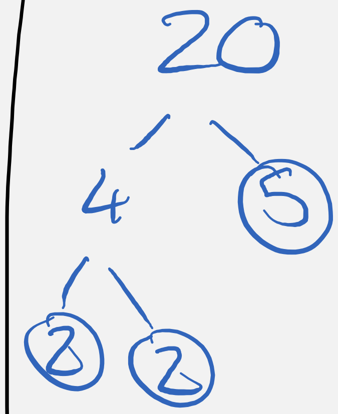
$2\sqrt{3}$

c. $\sqrt{8} = 2\sqrt{2}$

d. $\sqrt{75} = 5\sqrt{3}$



e. $\sqrt{20} = 2\sqrt{5}$



PROPERTIES OF SQUARE ROOTS

- Product property: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

  Ex: $\sqrt{6} = \sqrt{2} \cdot \sqrt{3}$

$\sqrt{2+3} \neq \sqrt{2} + \sqrt{3}$

common mistake

- Quotient property: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

  Ex: $\sqrt{\frac{5}{2}} = \frac{\sqrt{5}}{\sqrt{2}}$

Simplify $\sqrt{3} \cdot \sqrt{6}$. $= \sqrt{18} = \sqrt{2 \cdot 9} = \sqrt{2} \cdot \sqrt{9}$

3

$= 3\sqrt{2}$

f. $\sqrt{5} \cdot \sqrt{10}$

$= \sqrt{50} = \sqrt{2 \cdot 25}$

$= \sqrt{2}\sqrt{25}$

$= 5\sqrt{2}$

g. $\sqrt{3} \cdot \sqrt{15}$

$= \sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9}\ \sqrt{5}$

$= 3\sqrt{5}$

Simplify each expression.

5 $\frac{\sqrt{16}}{\sqrt{8}} = \sqrt{\frac{16}{8}} = \sqrt{2}$

6 $\sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$

i. $\frac{\sqrt{81}}{\sqrt{100}} = \frac{9}{10}$

j. $\sqrt{\frac{49}{64}} = \frac{\sqrt{49}}{\sqrt{64}} = \frac{7}{8}$

RADICALS IN THE DENOMINATOR

- When radicals are present in the denominator, rationalizing the denominator is necessary.

$$\sqrt{5}\cdot\sqrt{5} = 5$$

$$(\sqrt{5})^2$$

Ex: **Simplify** $\dfrac{\sqrt{3}}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\sqrt{15}}{5}$

$$= 1$$

Simplify $\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

k. $\frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{14}}{2}$

l. $\frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

13.2 – 45-45-90 TRIANGLES

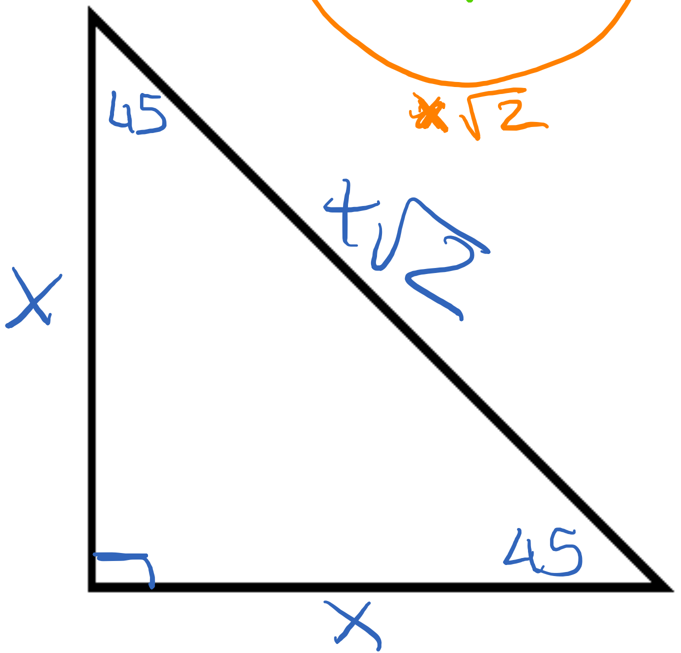
45-45-90

Special Right Triangle

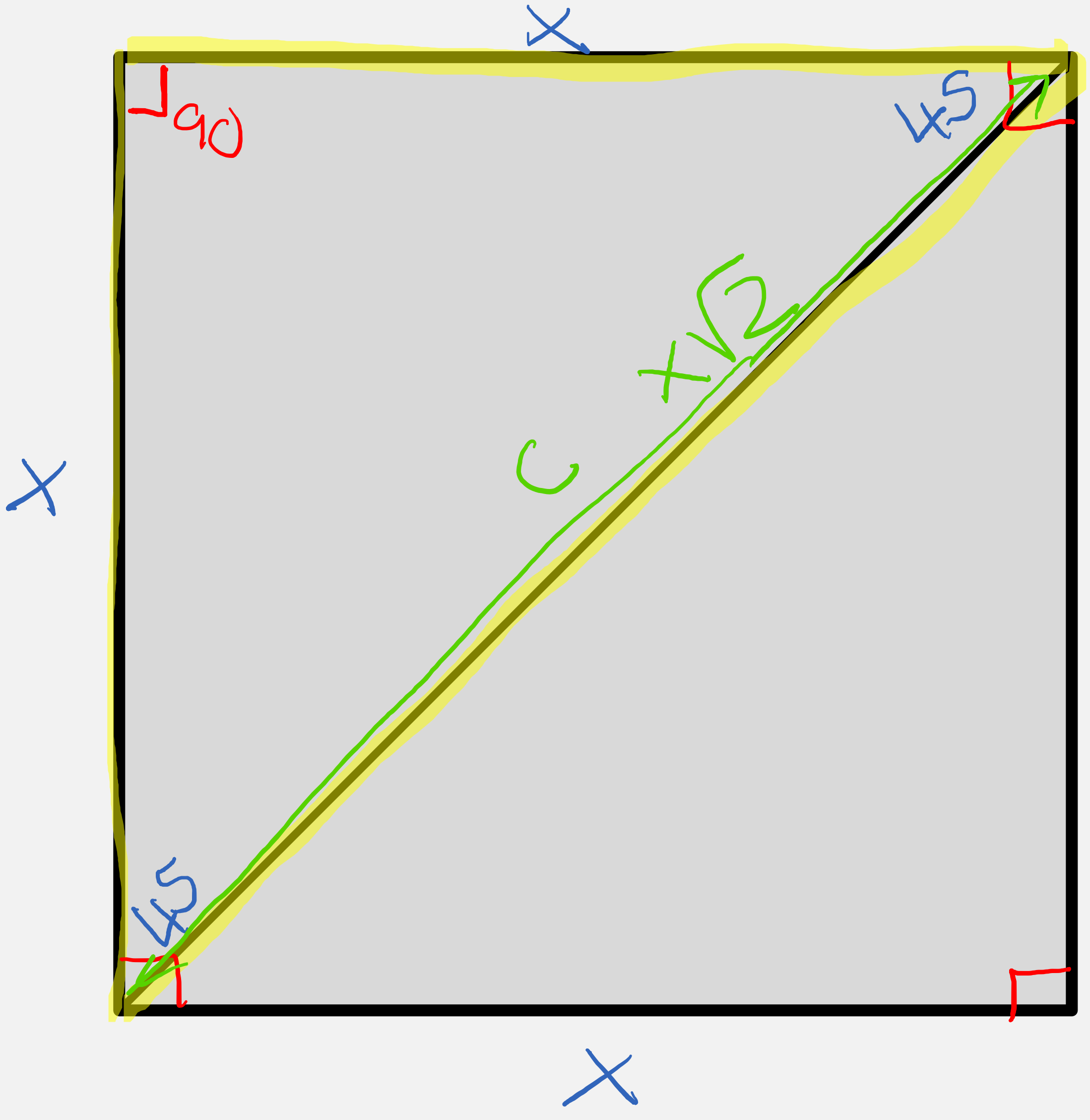
Angle Measures		
45	45	90
Side Measures		
x	x	$x\sqrt{2}$

$\div\sqrt{2}$

$\times\sqrt{2}$

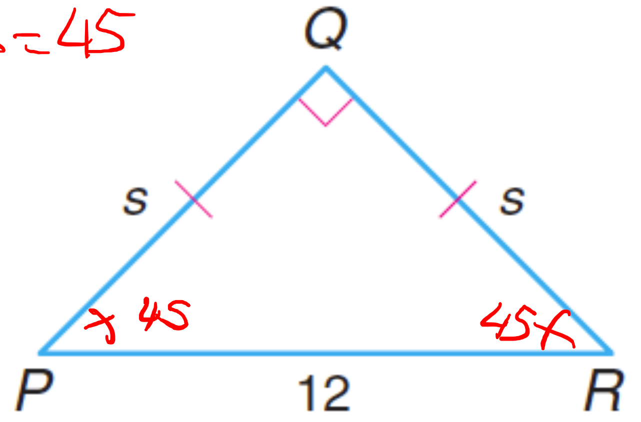


$$c^2 = x^2 + x^2$$
$$\sqrt{c^2} = \sqrt{2x^2}$$
$$c = x\sqrt{2}$$



2 If ΔPQR is an isosceles right triangle and the measure of the hypotenuse is 12, find s. Write the answer in simplest form.

$m\angle P = m\angle R = 90 \div 2 = 45$



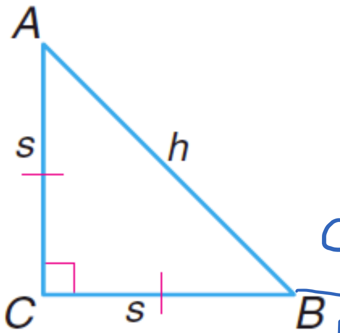
45	45	90
x	x	$x\sqrt{2}$
s	s	12
$6\sqrt{2}$	$6\sqrt{2}$	

$$s = \frac{12}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$$

ΔABC is an isosceles right triangle. Find s for each value of h.

a. 4 $\quad$ b. 5 $\quad$ c. $3\sqrt{2}$ $\quad$ 6

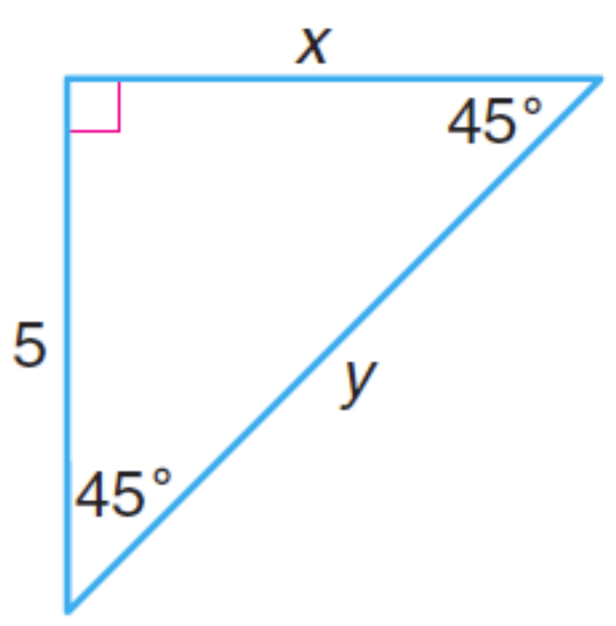
$4\sqrt{2}$ $\quad$ $5\sqrt{2}$



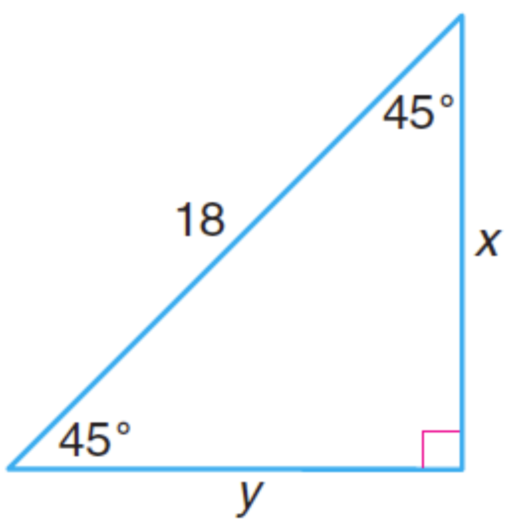
$$\underbrace{3\sqrt{2} \cdot \sqrt{2}} = 3 \cdot 2$$
$$= 6$$

	45	45	90
	x	x	$x\sqrt{2}$
a	4	4	$4\sqrt{2}$
b	5	5	$5\sqrt{2}$
c	$3\sqrt{2}$	$3\sqrt{2}$	6

Find the missing measures. Write all radicals in simplest form.



45	45	90
x	x	$x\sqrt{2}$
5	5	$5\sqrt{2}$
$9\sqrt{2}$	$9\sqrt{2}$	18



$x = 5$

$y = 5\sqrt{2}$

$\frac{18}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{18\sqrt{2}}{2} = 9\sqrt{2}$

$x = 9\sqrt{2}$

$y = 9\sqrt{2}$

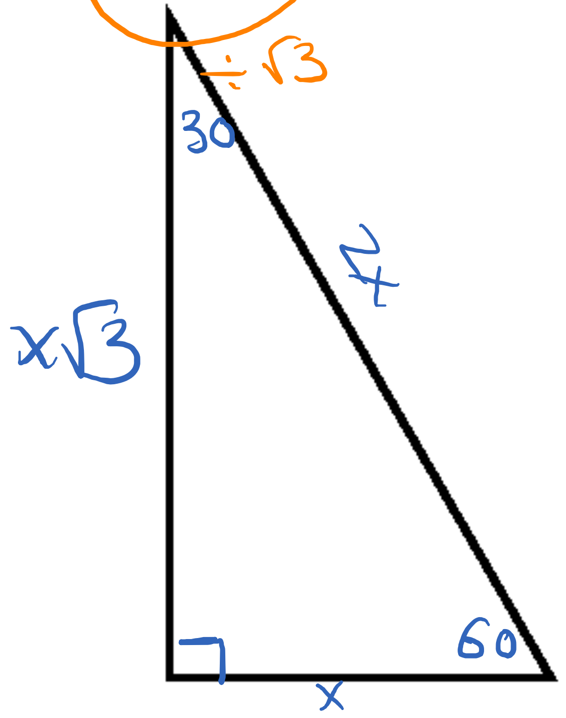
13.3 – 30-60-90 TRIANGLES

Special Right Triangle

Angle Measures		
30	60	90
Side Measures		
x	$x\sqrt{3}$	$2x$

$\times\sqrt{3}$

$\div\sqrt{3}$

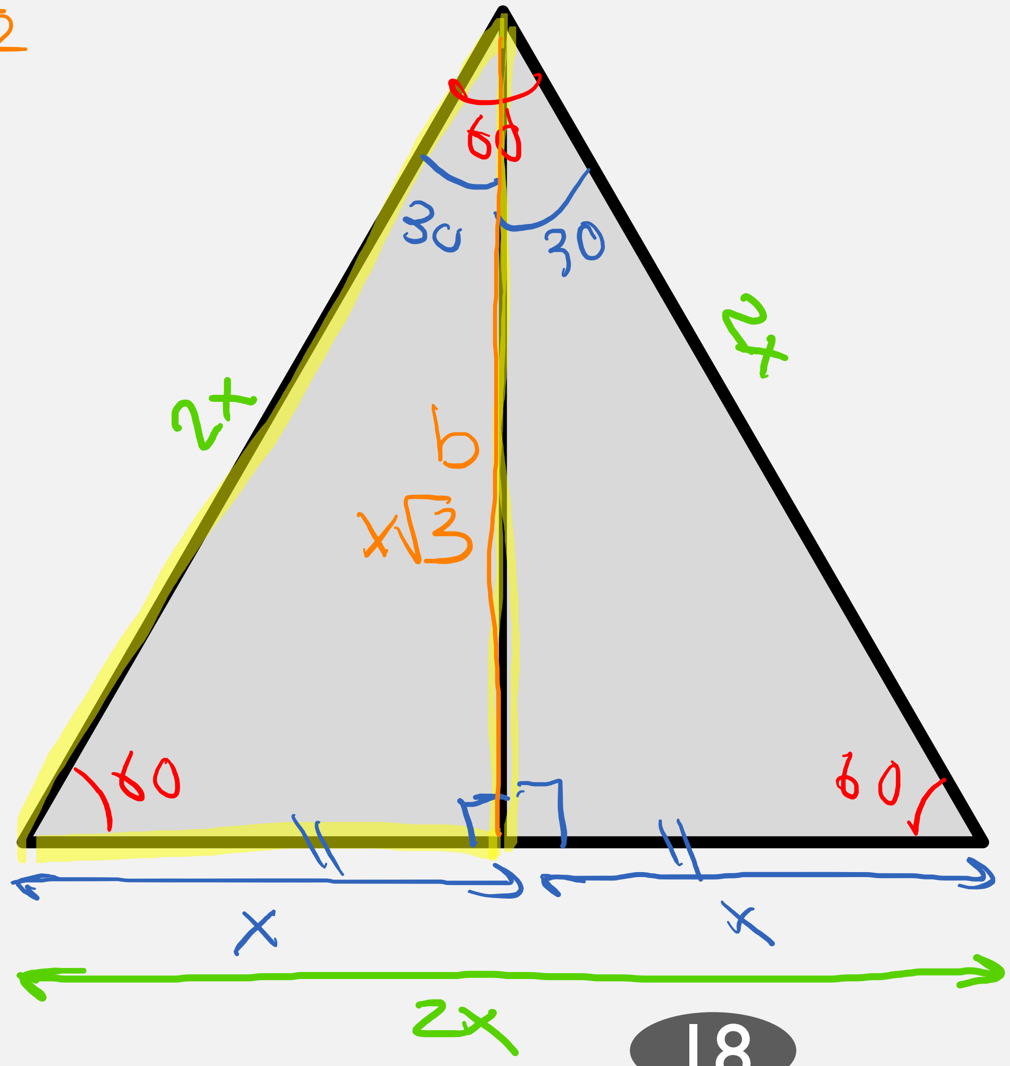


$x^2+b^2=(2x)^2$

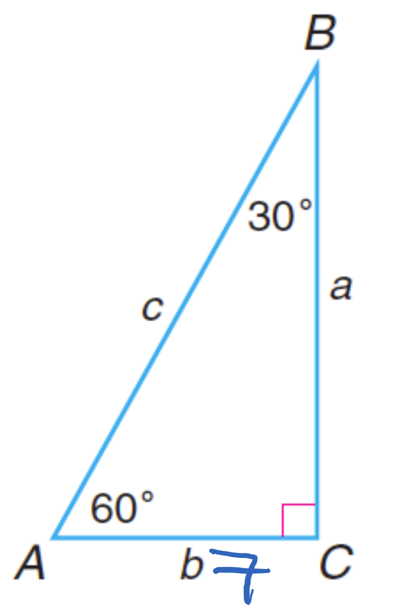
$x^2+b^2=4x^2$

$\sqrt{b^2}=\sqrt{3x^2}$

$b=x\sqrt{3}$



1 In $\triangle ABC$, $b = 7$. Find a and c. Write in simplest form.

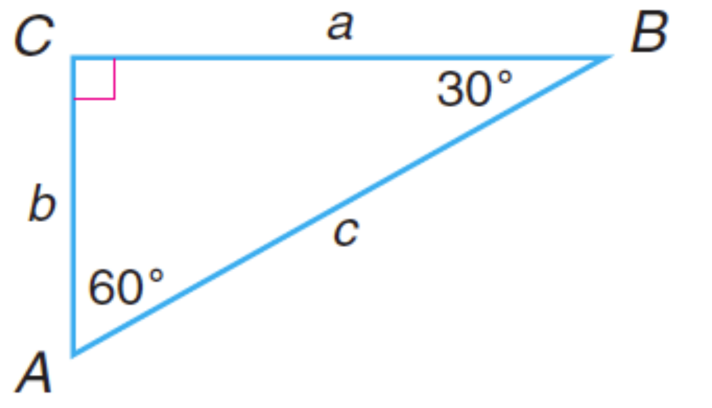


$a = 7\sqrt{3}$

$c = 14$

30	60	90
x	$x\sqrt{3}$	$2x$
7	$7\sqrt{3}$	14
9	$9\sqrt{3}$	18

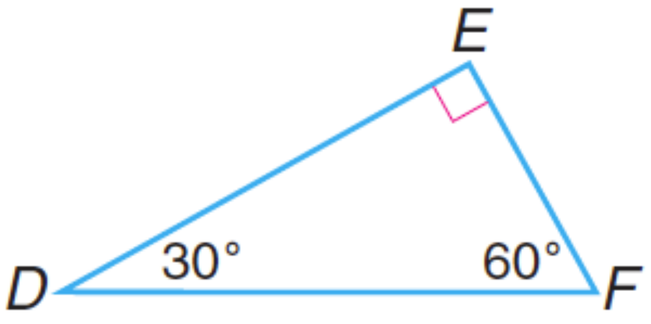
2 In $\triangle ABC$, $c = 18$. Find a and b. Write in simplest form.



$b = 9$

$a = 9\sqrt{3}$

3 In ΔDEF, $DE = 12$. Find EF and DF. Write in simplest form.



$$\frac{12}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

$EF = 4\sqrt{3}$

$DF = 8\sqrt{3}$

30	60	90
x	$x\sqrt{3}$	$2x$
$4\sqrt{3}$	12	$8\sqrt{3}$
$\frac{8\sqrt{3}}{3}$	8	$\frac{16\sqrt{3}}{3}$

c. Refer to ΔDEF above. If $DE = 8$, find EF and DF.

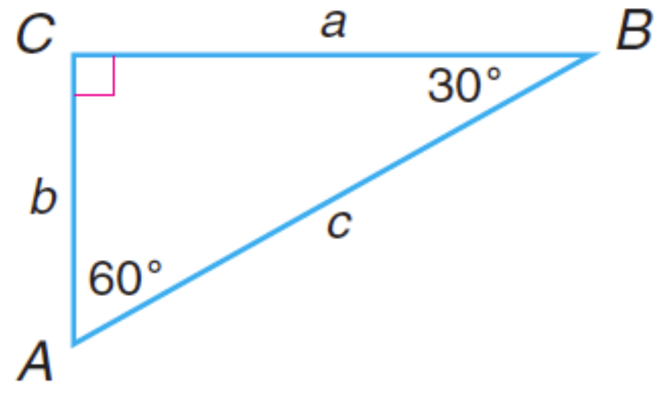
$$\frac{8}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$$

$EF = \frac{8\sqrt{3}}{3}$

$DF = \frac{16\sqrt{3}}{3}$

a. Refer to $\triangle ABC$ above. If $b = 8$, find a and c.

b. Refer to $\triangle ABC$ above. If $c = 10$, find a and b.



a) $a = 8\sqrt{3}$

$c = 16$

b) $b = 5$

$a = 5\sqrt{3}$

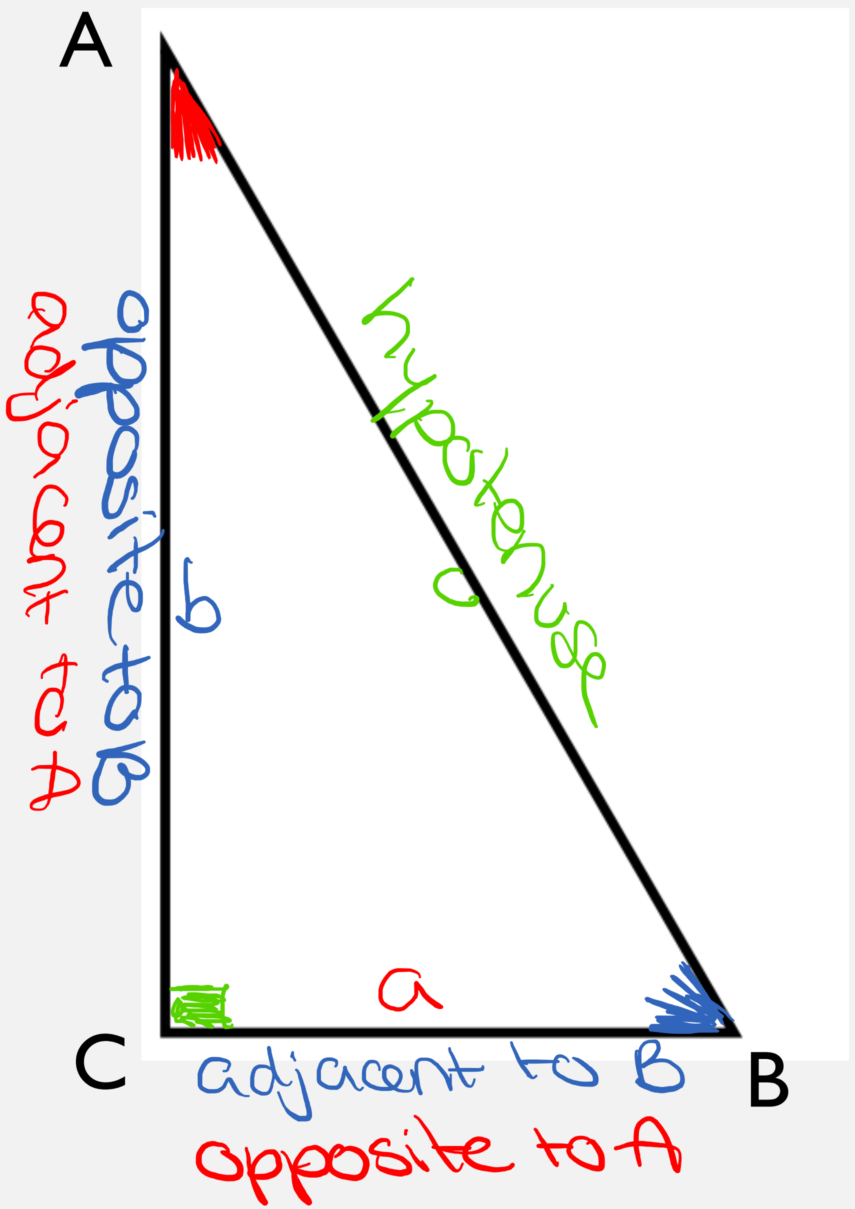
30	60	90
x	$x\sqrt{3}$	$2x$
8	$8\sqrt{3}$	16
5	$5\sqrt{3}$	10

13.4/5 TRIGONOMETRIC RATIOS

DEFINITIONS

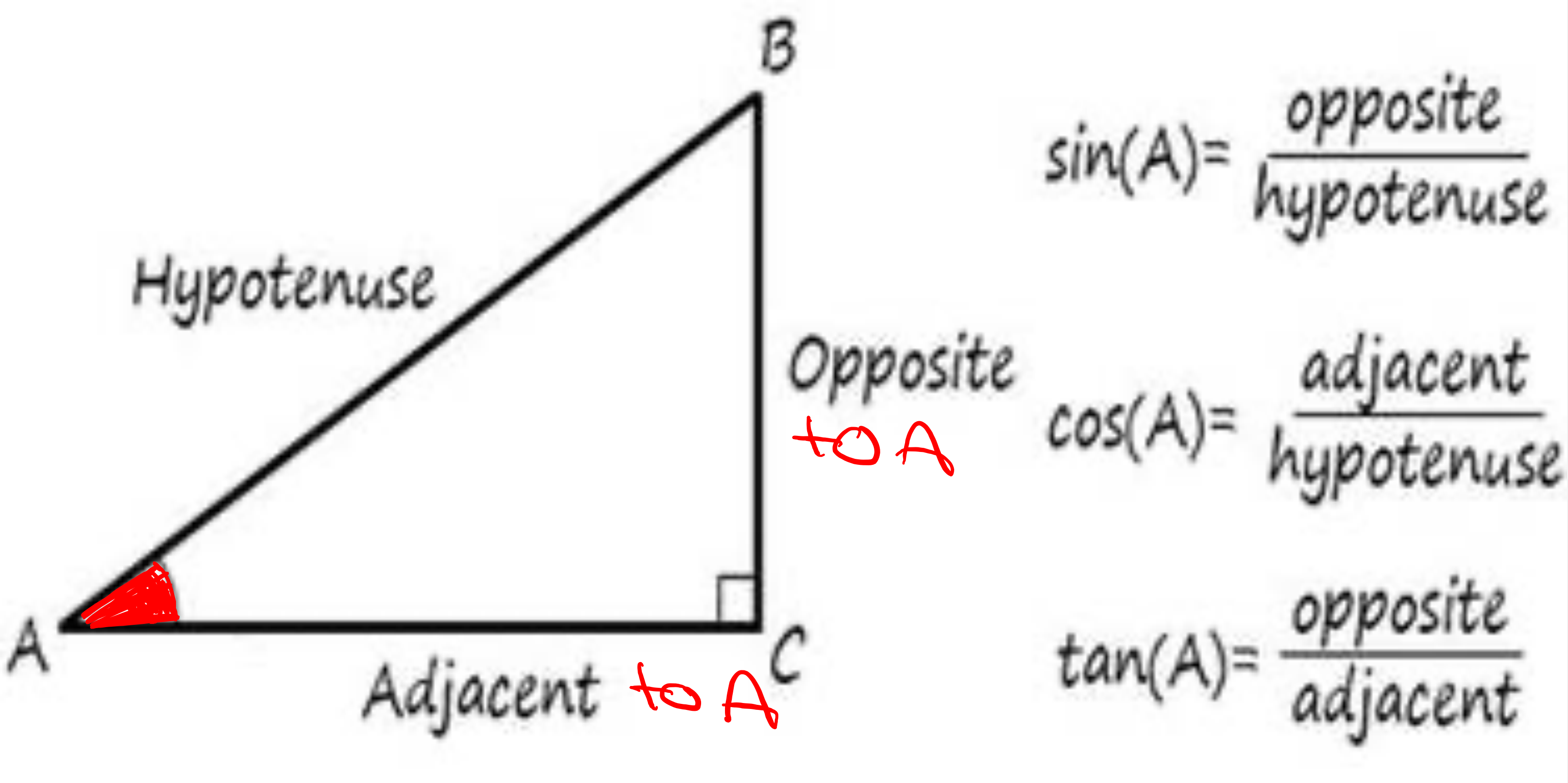
- **Trigonometry** comes from Greek: *trigon* means triangles and *metron* means measure. Trigonometry involves the measure of triangles.
- A **trigonometric ratio** is a ratio of the lengths of two sides of a triangle. Trig ratios are constant for any given angle measure (due to similarity properties).

Labelling Triangles



TRIGONOMETRIC RATIOS

$S\frac{O}{H}$ $C\frac{A}{H}$ $T\frac{O}{A}$



FINDING A RATIO

- 1) Identify the angle you are working with.
- 2) Identify the angles opposite and adjacent sides. to that angle.
- 3) Write the ratio.

Express each ratio as a fraction and as a decimal to the nearest hundredth.

$S\frac{O}{H}$ $C\frac{A}{H}$ $T\frac{O}{A}$

a. $\sin P = \frac{15}{17}$

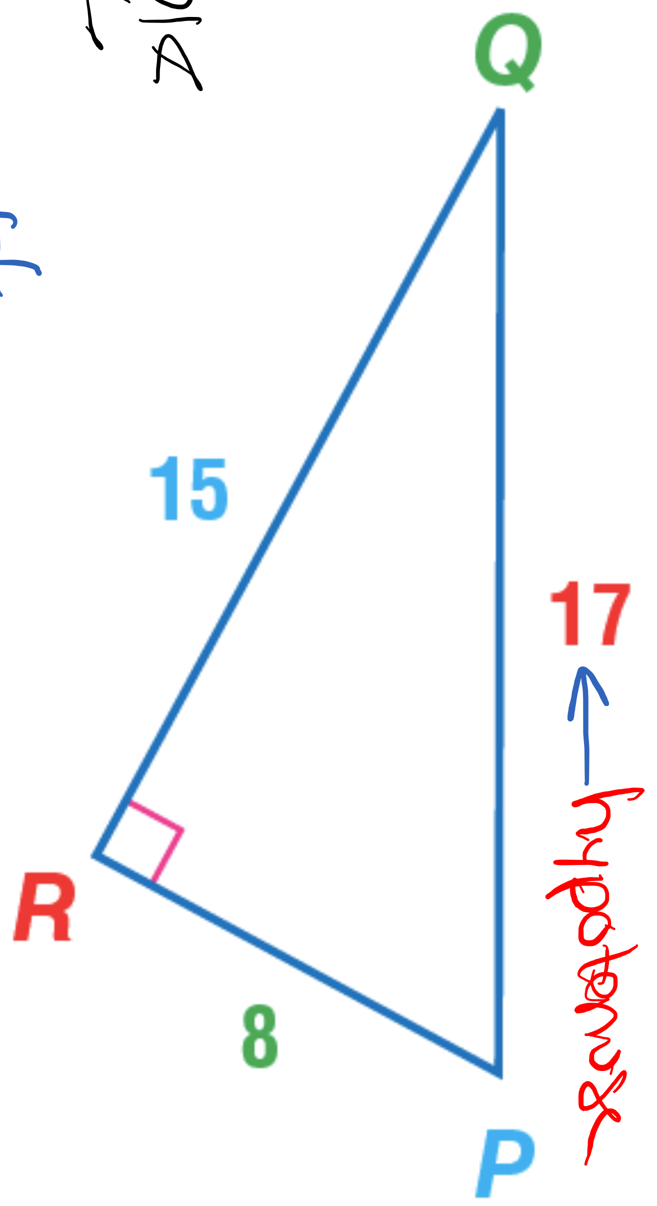
b. $\cos P = \frac{8}{17}$

c. $\tan P = \frac{15}{8}$

d. $\sin Q = \frac{8}{17}$

e. $\cos Q = \frac{15}{17}$

f. $\tan Q = \frac{8}{15}$



FINDING MISSING MEASURES

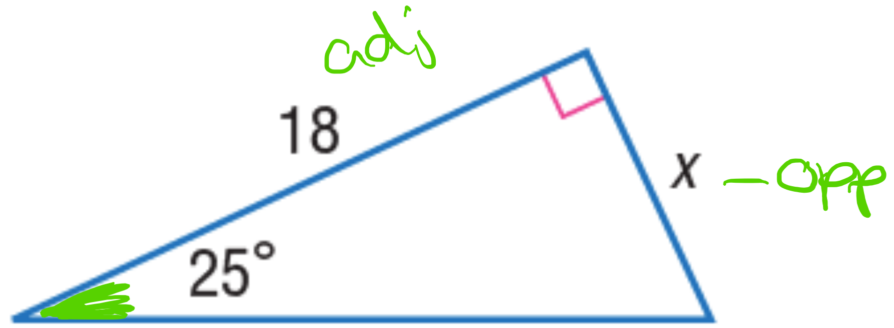
- 1) Identify the angle you have or are looking for.
- 2) Identify one side you have, and one you have or are looking for.
- 3) Determine the trigonometric ratio that relates all 3 items.
- 4) Write the ratio and solve it.

- Note: when looking for an angle, you must use the inverse to find the angle. In the calculator $\sin^{-1}$ $\cos^{-1}$ $\tan^{-1}$
- Solving a right triangle means finding all angle and side measures.

Find x to the nearest hundredth.

$S\frac{O}{H}$ $C\frac{A}{H}$ $T\frac{O}{A}$

3A.

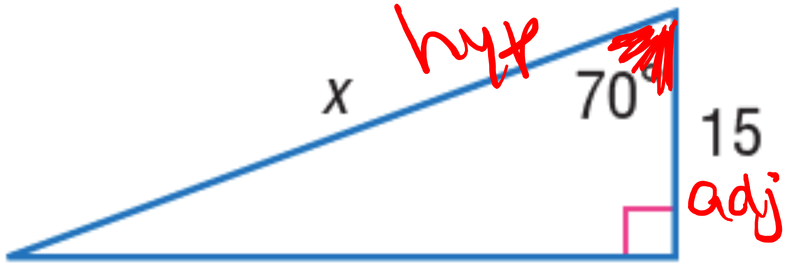


$$\frac{\tan(25)}{1} = \frac{x}{18}$$

$$18\tan(25) = x$$

$$x = 8.39$$

3B.



$$\frac{\cos(70)}{1} = \frac{15}{x}$$

$$\frac{x\,\cancel{\cos(70)}}{\cancel{\cos(70)}} = \frac{15}{\cos(70)}$$

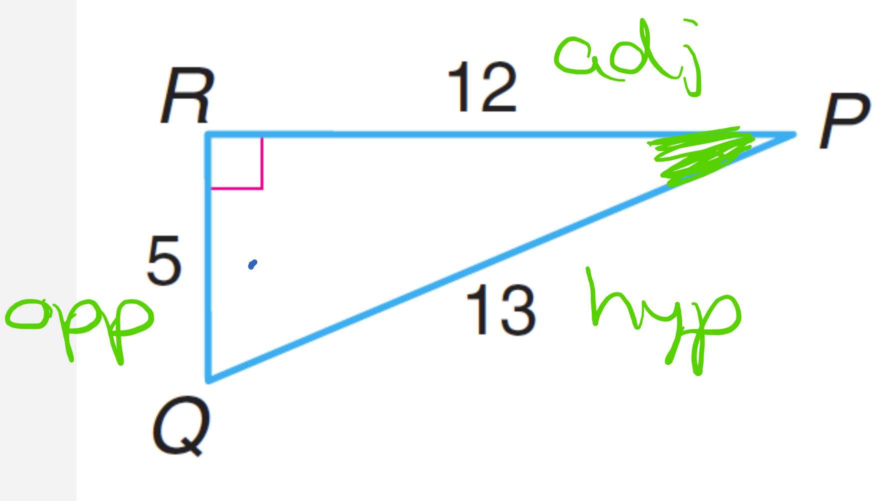
$$x = 43.86$$

Find the measures of each angle.

$\sin P = \frac{5}{13}$

$P = \sin^{-1}\left(\frac{5}{13}\right)$

$P = 22.62^\circ$

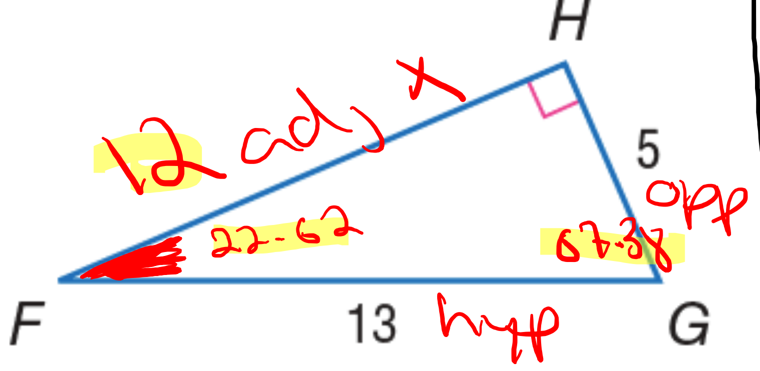


$Q = 180 - 90 - 22.62 = 67.38^\circ$

Solve each right triangle. Round side measures to the nearest tenth and angle measures to the nearest degree.

S $\frac{O}{H}$ C $\frac{A}{H}$ T $\frac{O}{A}$

5A.



$\sin F = \frac{5}{13}$

$F = \sin^{-1}\left(\frac{5}{13}\right) = 22.62$

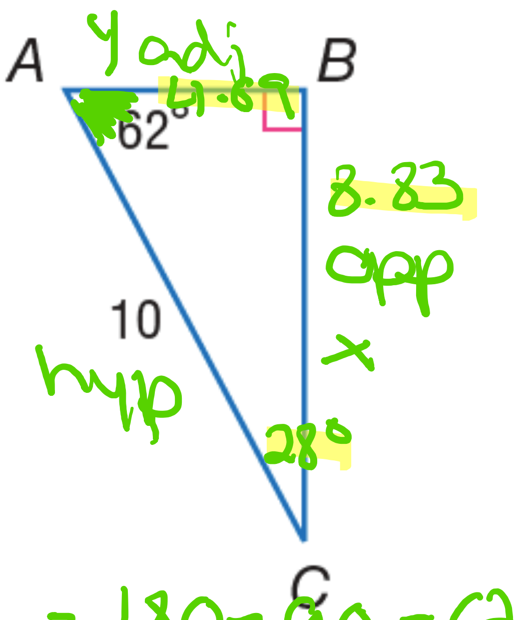
$G = 180 - 90 - 22.62 = 67.38$

$\cos(22.62) = \frac{x}{13}$

$x = 13\cos(22.62)$

$x = 12$

5B.



$C = 180 - 90 - 62 = 28°$

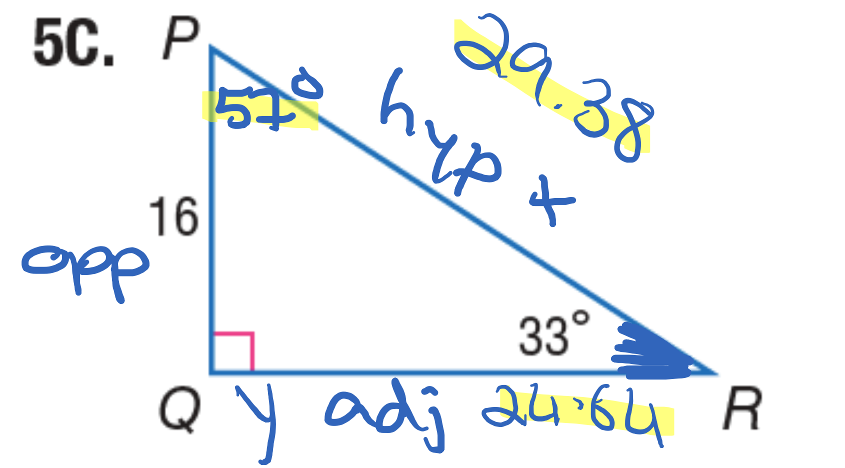
$\sin(62) = \frac{x}{10}$

$x = 10\sin(62) = 8.83$

$\cos(62) = \frac{y}{10}$

$y = 10\cos(62) = 4.69$

5C.



$P = 180 - 90 - 33 = 57°$

$\frac{\sin(33)}{1} = \frac{16}{x}$

$x\sin(33) = 16$

$x = \frac{16}{\sin(33)} = 29.38$

$\tan(33) = \frac{16}{y}$

$y = \frac{16}{\tan(33)} = 24.64$