



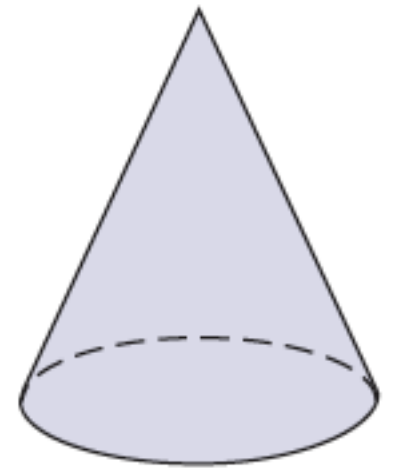
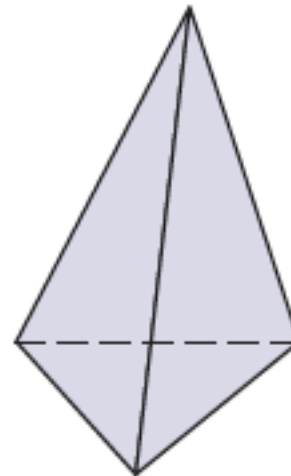
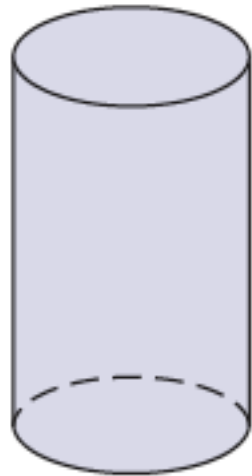
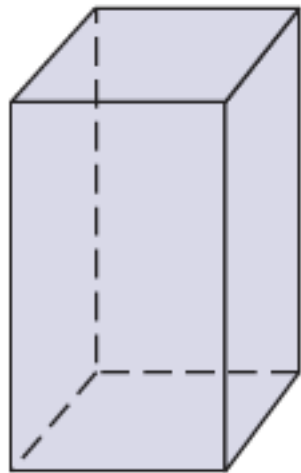
CHAPTER 12
SURFACE AREA AND VOLUME

12.1 – SOLID FIGURES

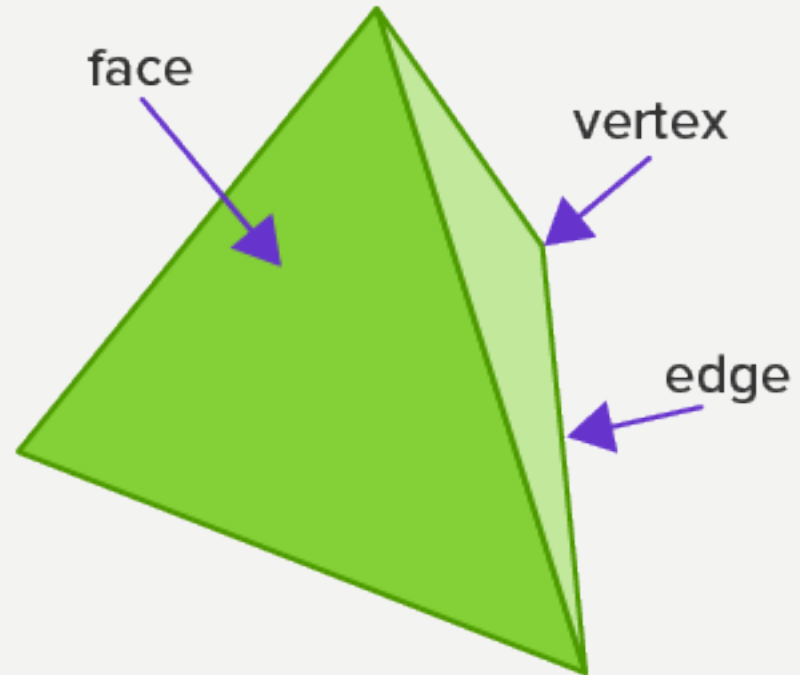
Solids

All figures above are examples of **solid figures** or **solids**.

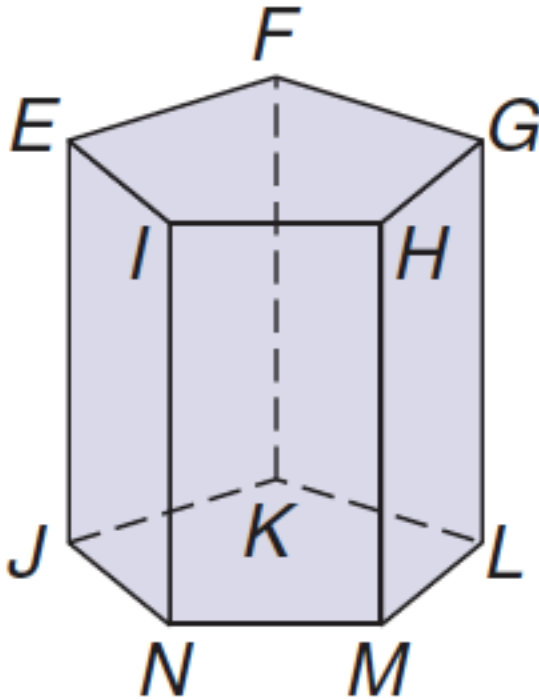
Solids with flat surface that are polygons are called **polyhedrons** or **polyhedra**.



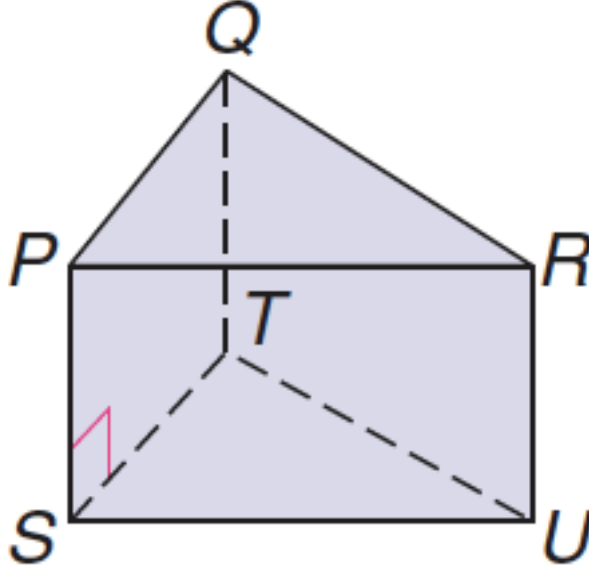
Parts of a polyhedron



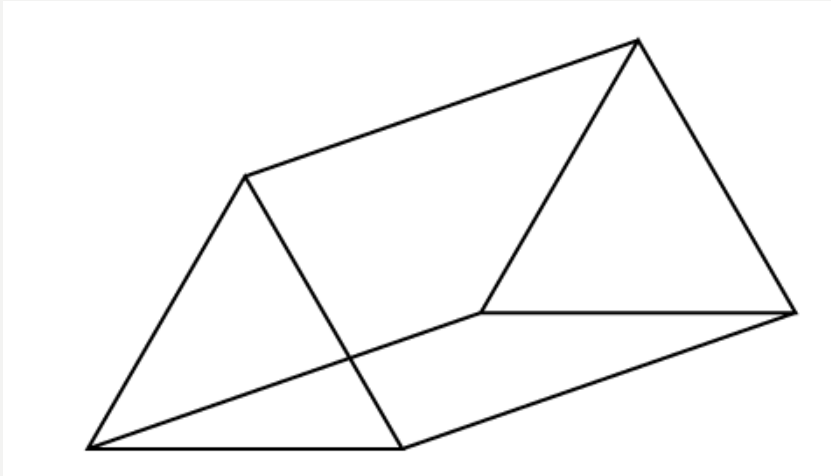
Name the faces, edges, and vertices of the polyhedron.



Name the faces, edges, and vertices of the polyhedron.

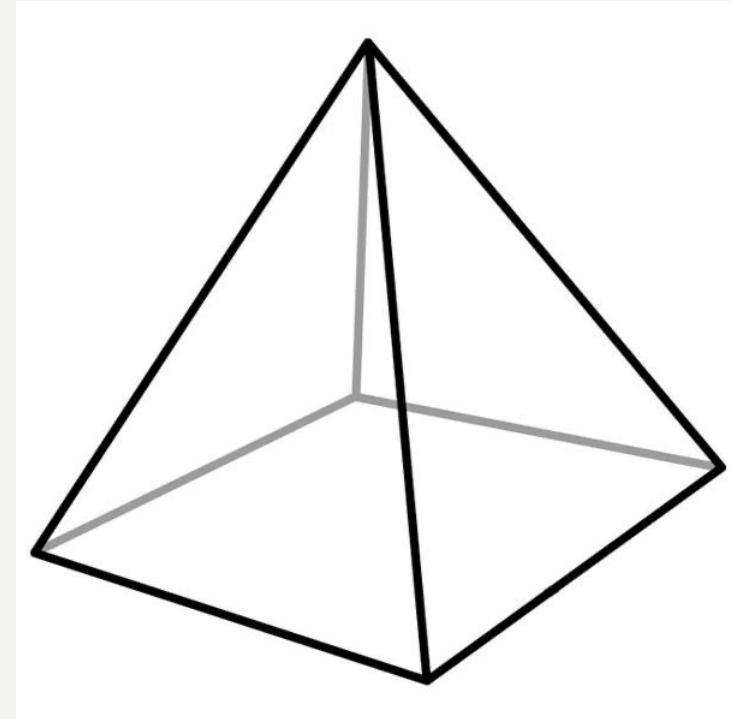


Prisms and Pyramids



Prism

- Lateral faces are rectangular.
- Two bases.

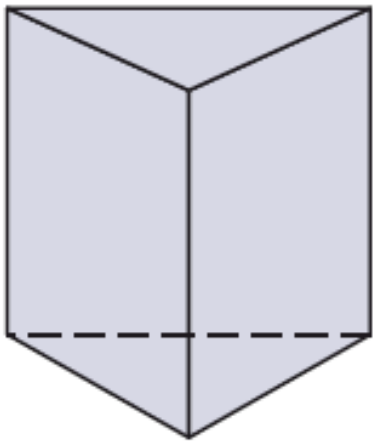


Pyramid

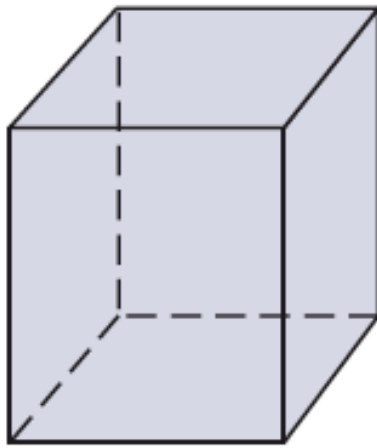
- Lateral faces are triangular.
- One base.

Classification

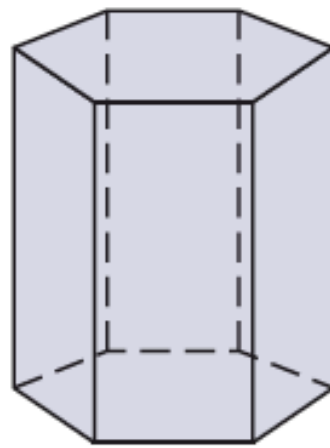
Prisms and pyramids are classified according to the shape of their base.



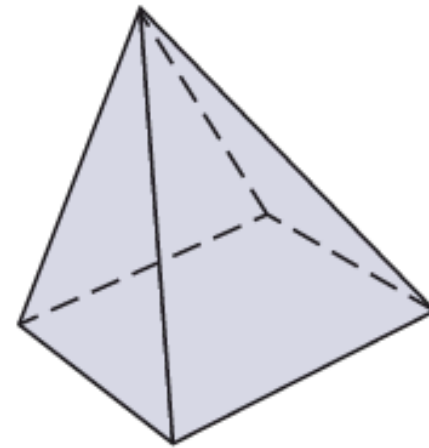
**triangular
prism**



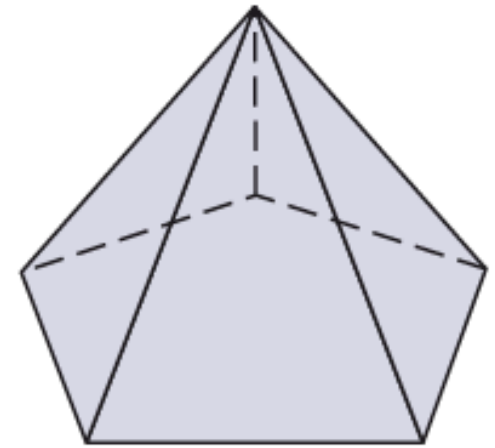
**rectangular
prism**



**hexagonal
prism**



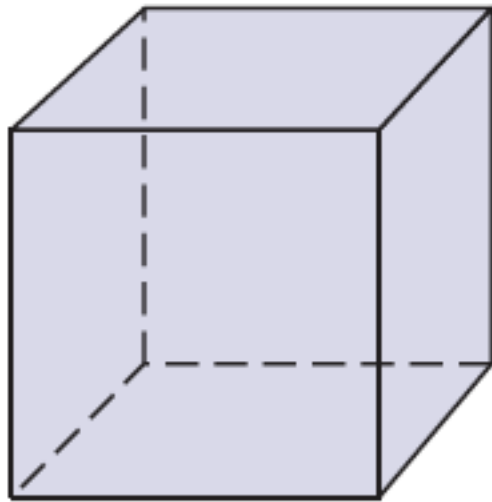
**rectangular
pyramid**



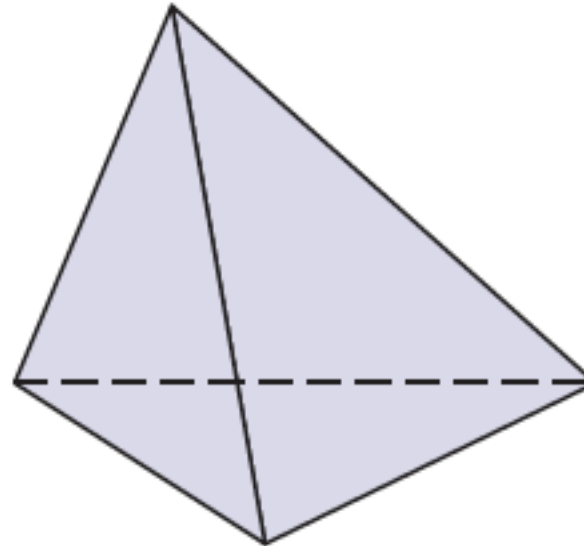
**pentagonal
pyramid**

Special solids

Prisms and pyramids are classified according to the shape of their base.



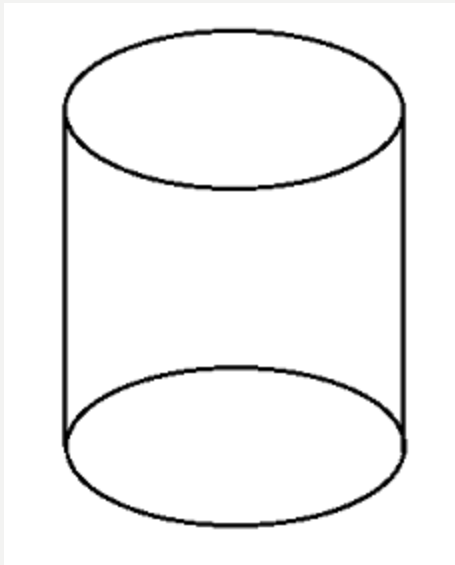
cube



tetrahedron

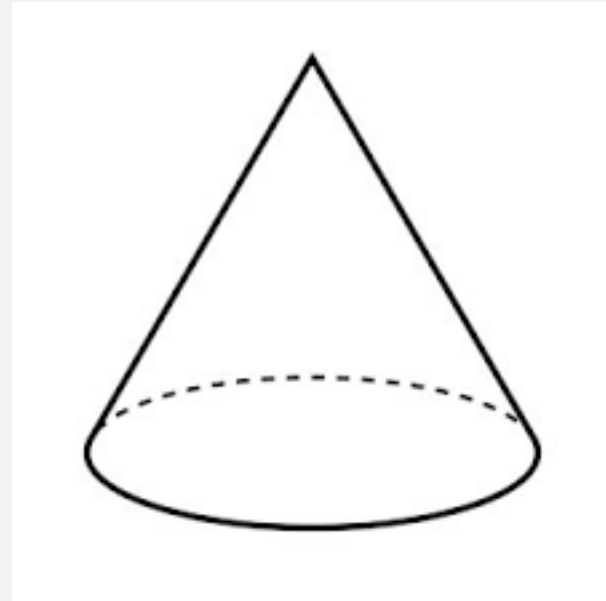
Cylinders and cones

Cylinders and cones are not polyhedral because they have curved lateral faces.



Cylinder

- Two bases.

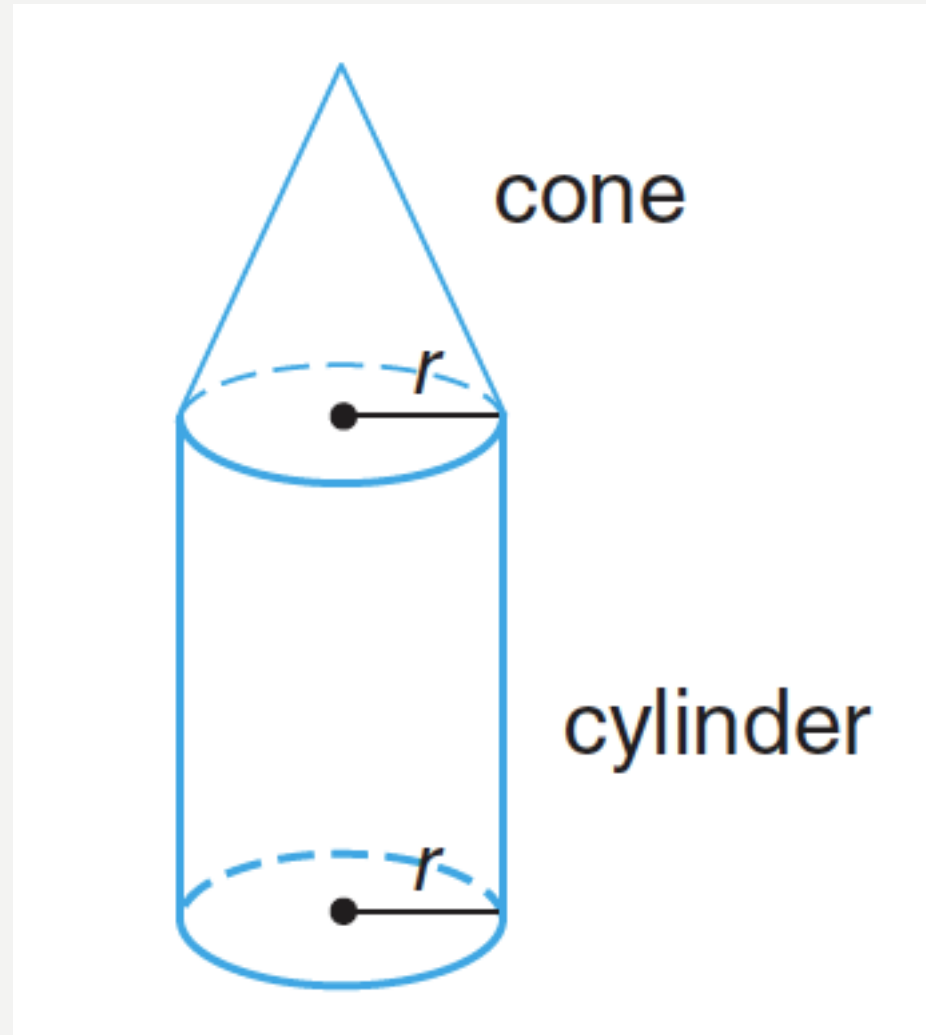


Cone

- One base.

Composite solids

Composite solids are formed when several solids are combined to form a new solid.

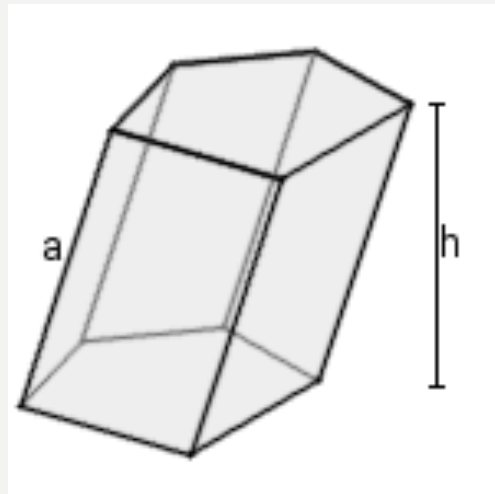


12.2 – SURFACE AREA OF PRISMS AND CYLINDERS

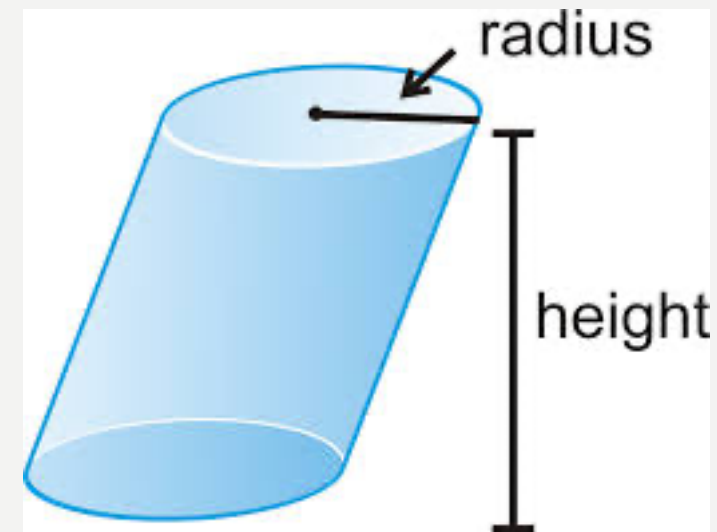
Oblique prisms and cylinders.

Solids are **oblique** when they are slanted. In this case, the height of the prisms and cylinder does not correspond to the edges.

Oblique prism:



Oblique cylinder:



Area definitions

- **Lateral area** includes the area of all the lateral faces.
- **Surface area** includes the area of lateral faces and bases.

For prisms and cylinders:

$$SA = LA + 2 \cdot A_{base}$$

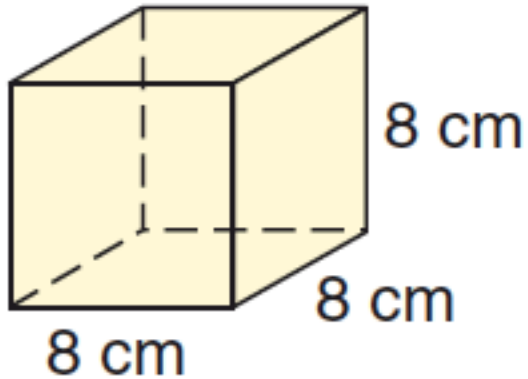
For pyramids and cones:

$$SA = LA + A_{base}$$

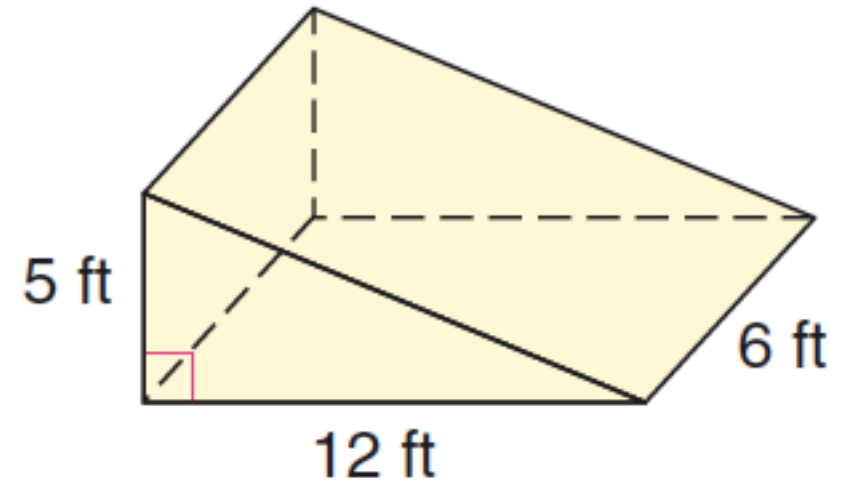
Area using nets

Find the lateral area and the surface area of each prism.

a.



b.



Prisms and cylinders as layering of shapes

It can be more convenient to think of prisms and cylinders as a stack of shapes.

Using this method, we can use the formula:

$$LA = P_{base} \cdot h$$

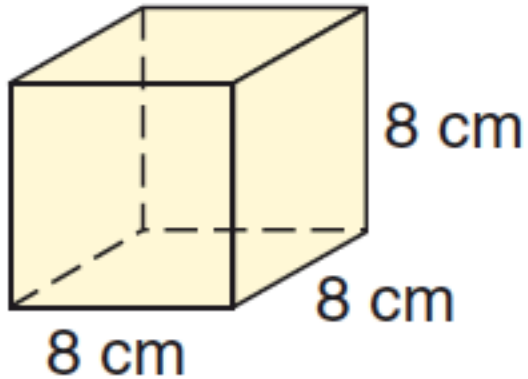
$$SA = P_{base} \cdot h + 2A_{base}$$



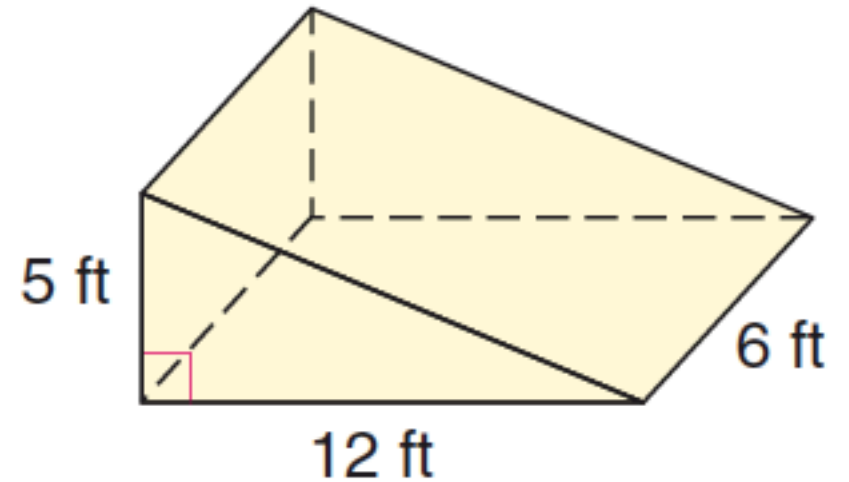
Area using stacking

Find the lateral area and the surface area of each prism.

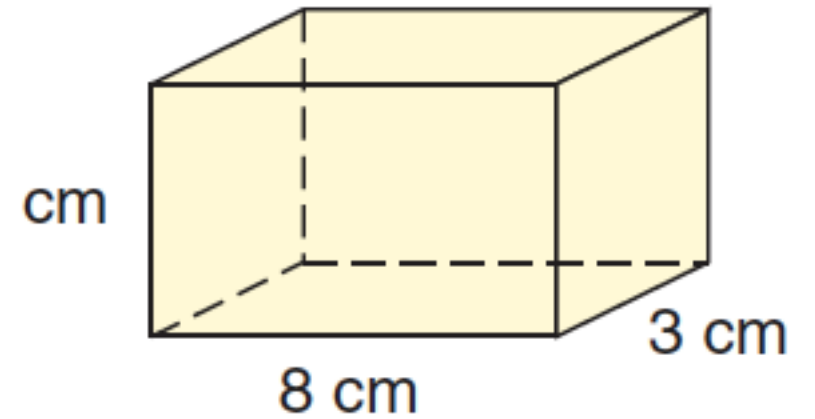
a.



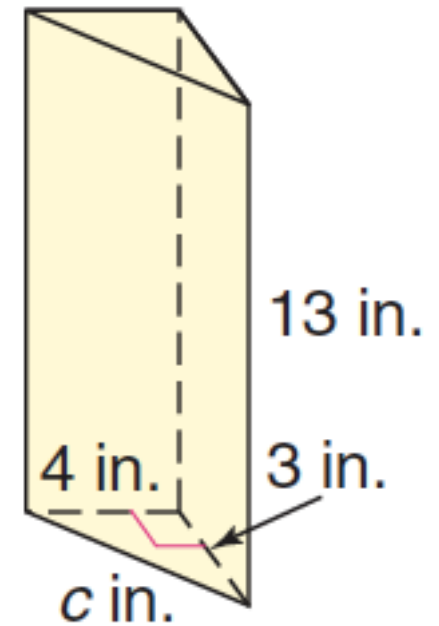
b.



Find the lateral area and the surface area of the rectangular prism.



Find the lateral area and the surface area of the triangular prism.



Prisms and cylinders as layering of shapes

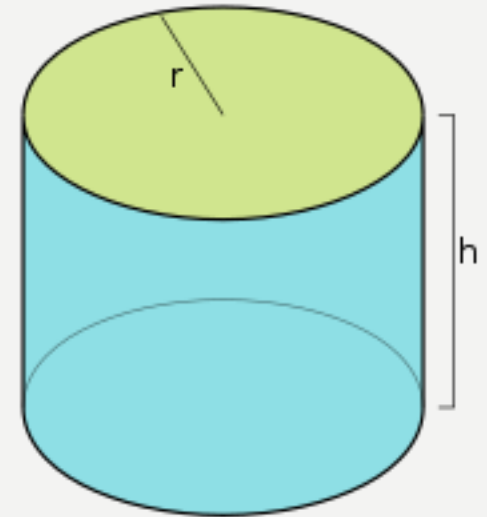
$$LA = P_{base} \cdot h$$

$$SA = P_{base} \cdot h + 2A_{base}$$

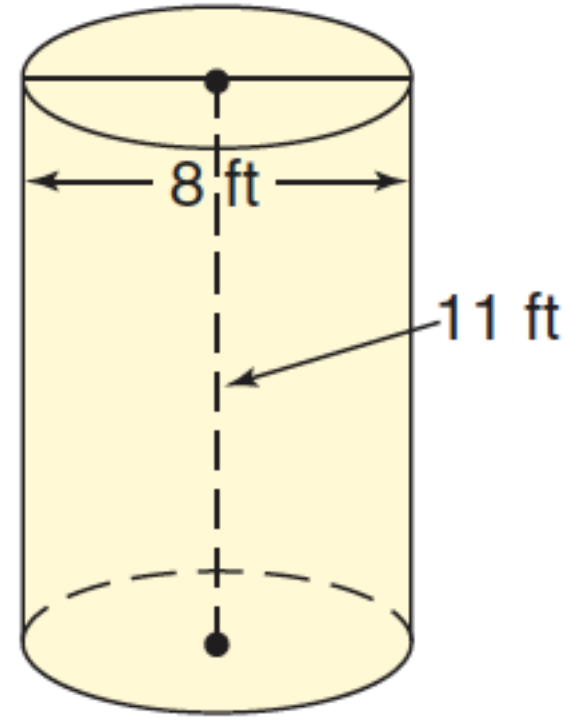
For cylinders, we can replace the perimeter and area by their formulas, which give us:

$$LA = 2\pi r \cdot h$$

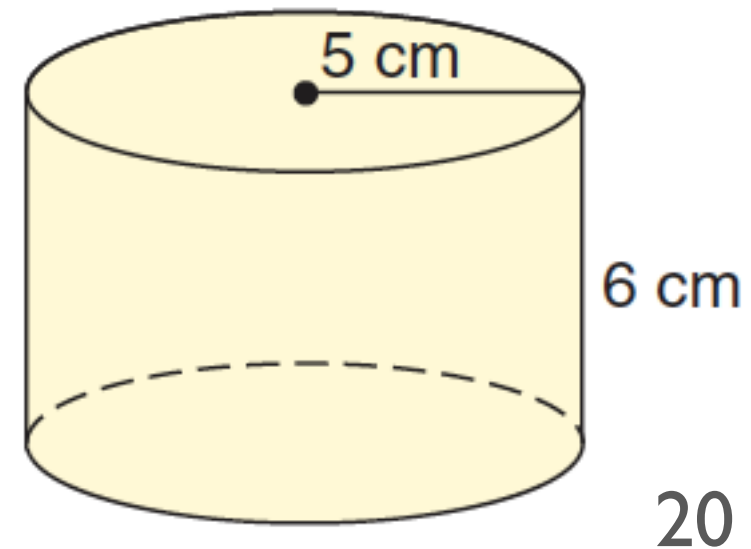
$$SA = 2\pi r h + 2\pi r^2$$



Find the lateral area and surface area of the cylinder to the nearest hundredth.



Find the lateral area and the surface area of the cylinder to the nearest hundredth.



12.3 – VOLUME OF PRISMS AND CYLINDERS

Definition: Volume

Volume is the amount of space contained in a solid. It is measured in cubic units.

Prisms and cylinders as layering of shapes

Formula:

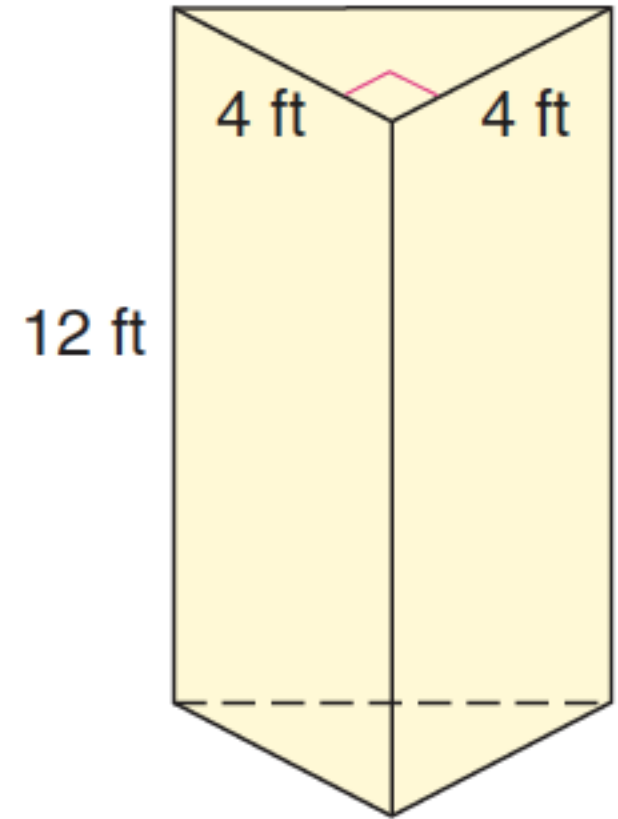
$$V = A_{base} \cdot h$$

For cylinders, you can replace the area by the formula for the area of a circle.

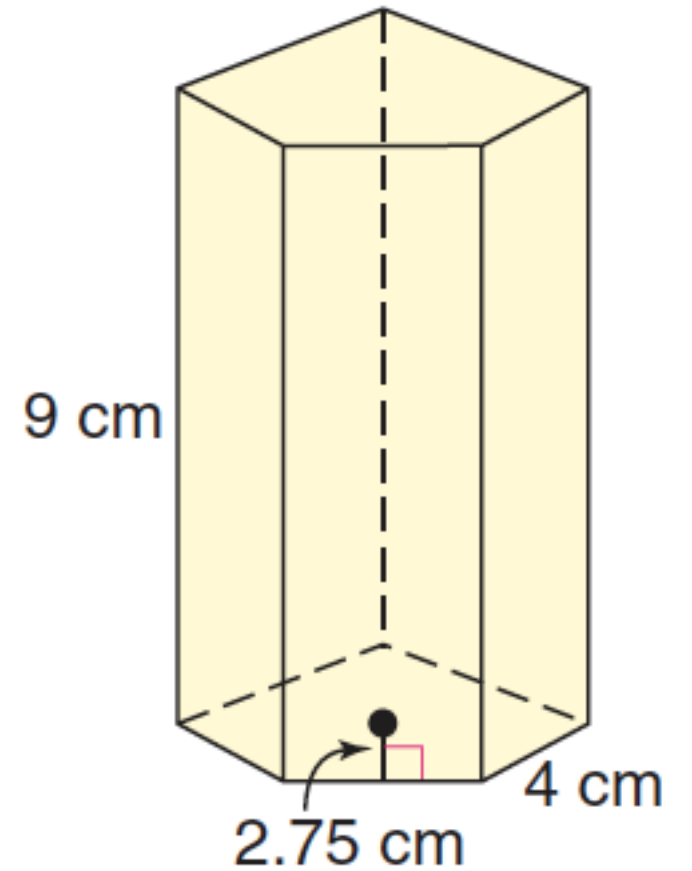
$$V = \pi r^2 \cdot h$$



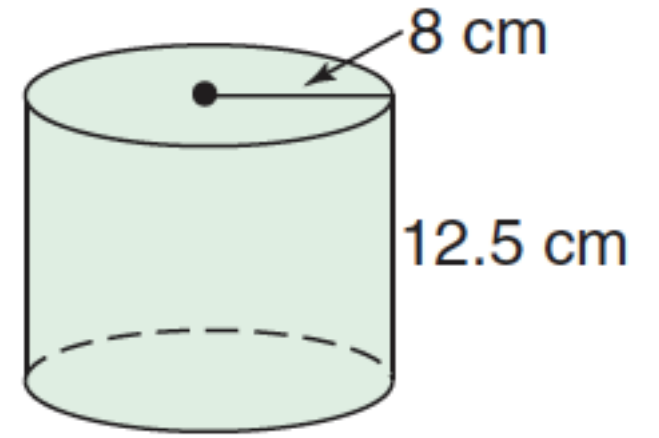
Find the volume of the triangular prism.



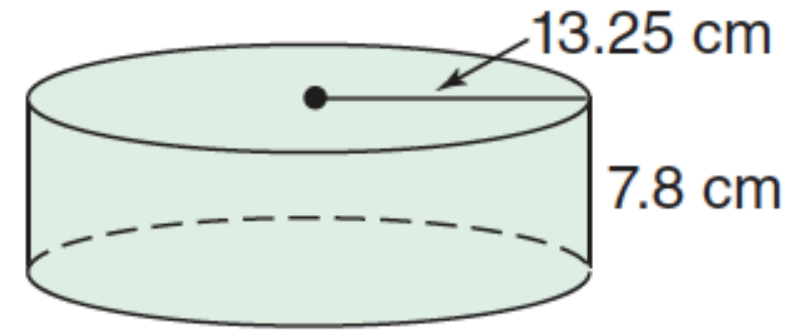
The base of the prism is a regular pentagon with sides of 4 centimeters and an apothem of 2.75 centimeters. Find the volume of the prism.



Find the volume of the cylinder to the nearest hundredth.



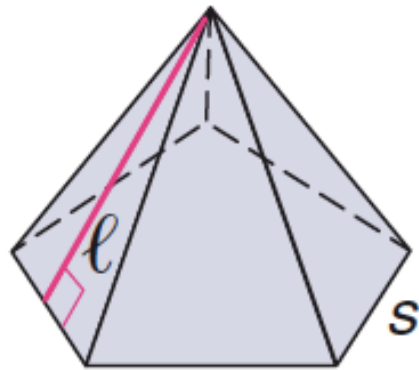
Find the volume of the cylinder to the nearest hundredth.



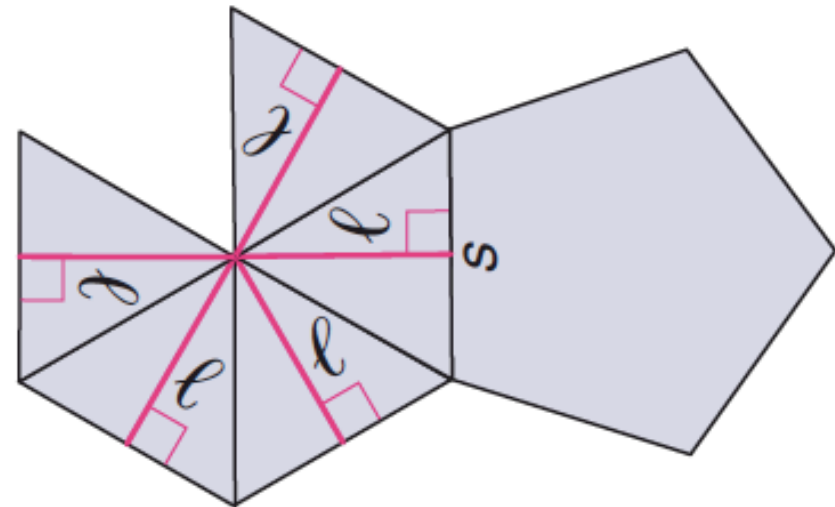
12.4 – SURFACE AREA OF PYRAMIDS AND CONES

Area using nets

Solid



Net



Area using perimeter

$$LA = \frac{1}{2} P_{base} \cdot l$$

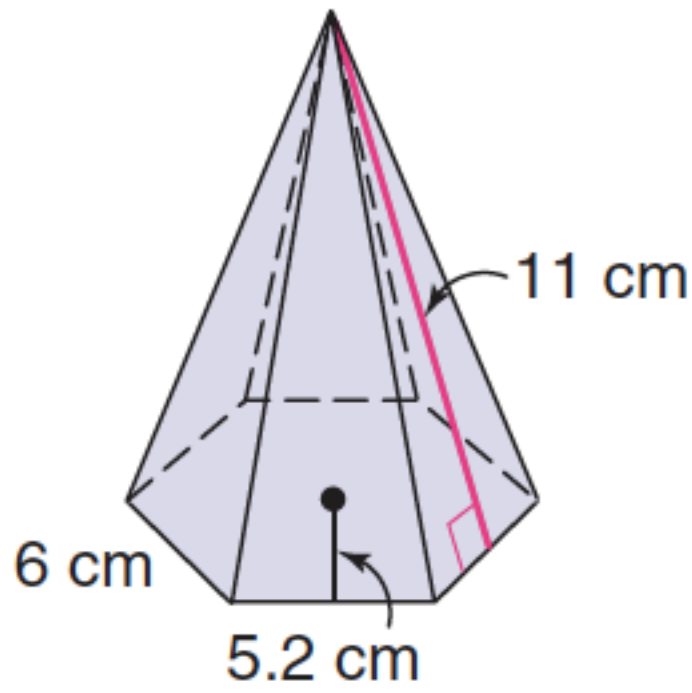
$$SA = \frac{1}{2} P_{base} l + A_{base}$$

For a cone, you can replace perimeter and area of circle by their formula.

$$LA = \pi r l$$

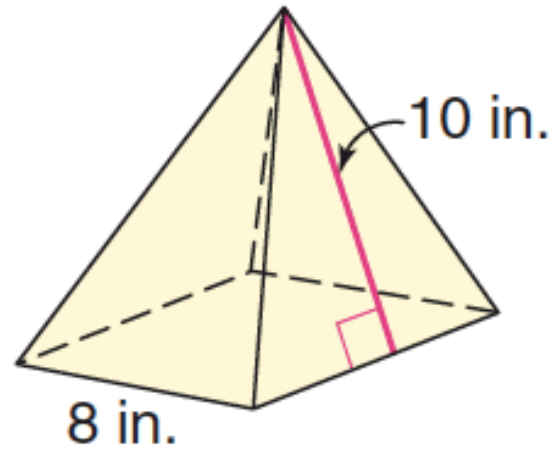
$$SA = \pi r l + \pi r^2$$

Find the lateral area and the surface area of the regular hexagonal pyramid.

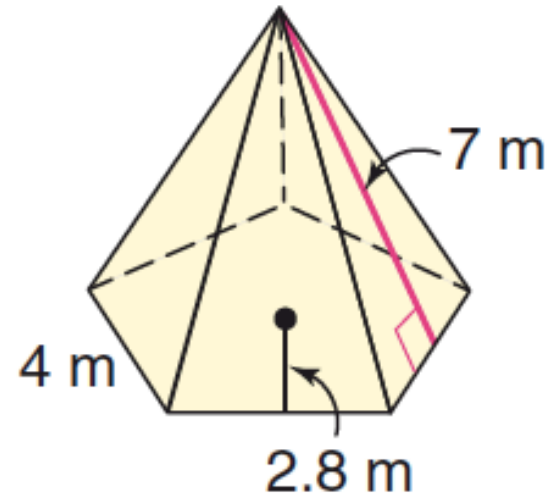


Find the lateral area and the surface area of each regular pyramid.

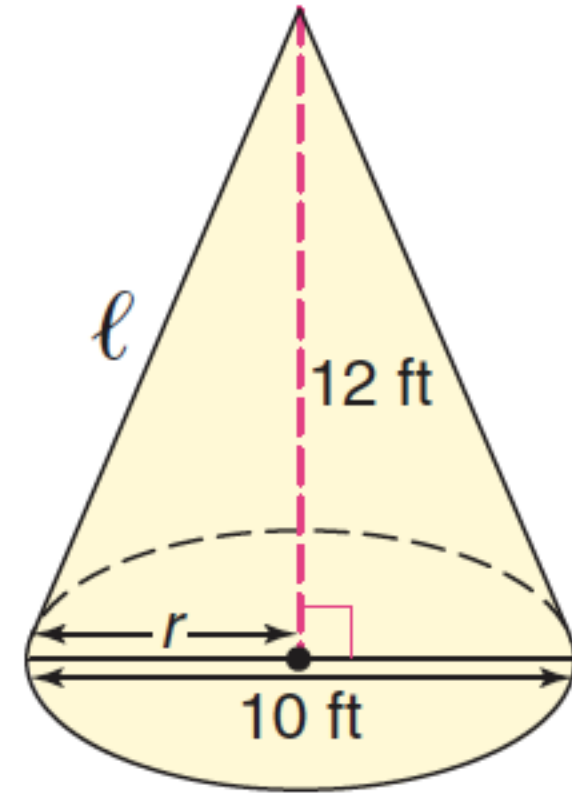
a.



b.

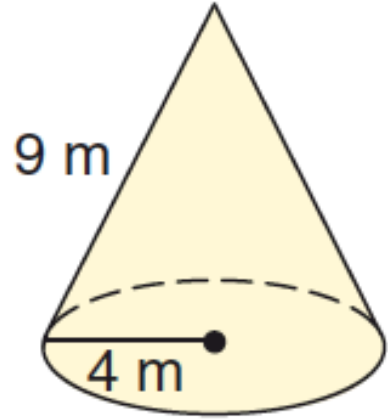


Find the lateral area and the surface area of the cone to the nearest hundredth.

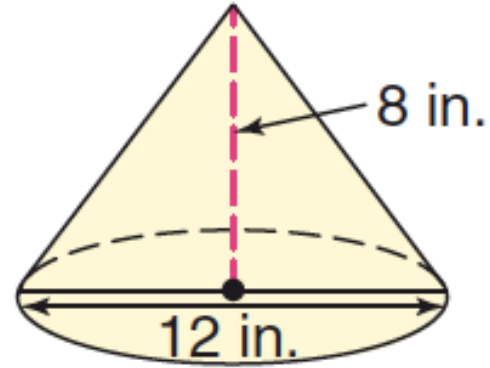


Find the lateral area and the surface area of each cone. Round to the nearest hundredth.

c.



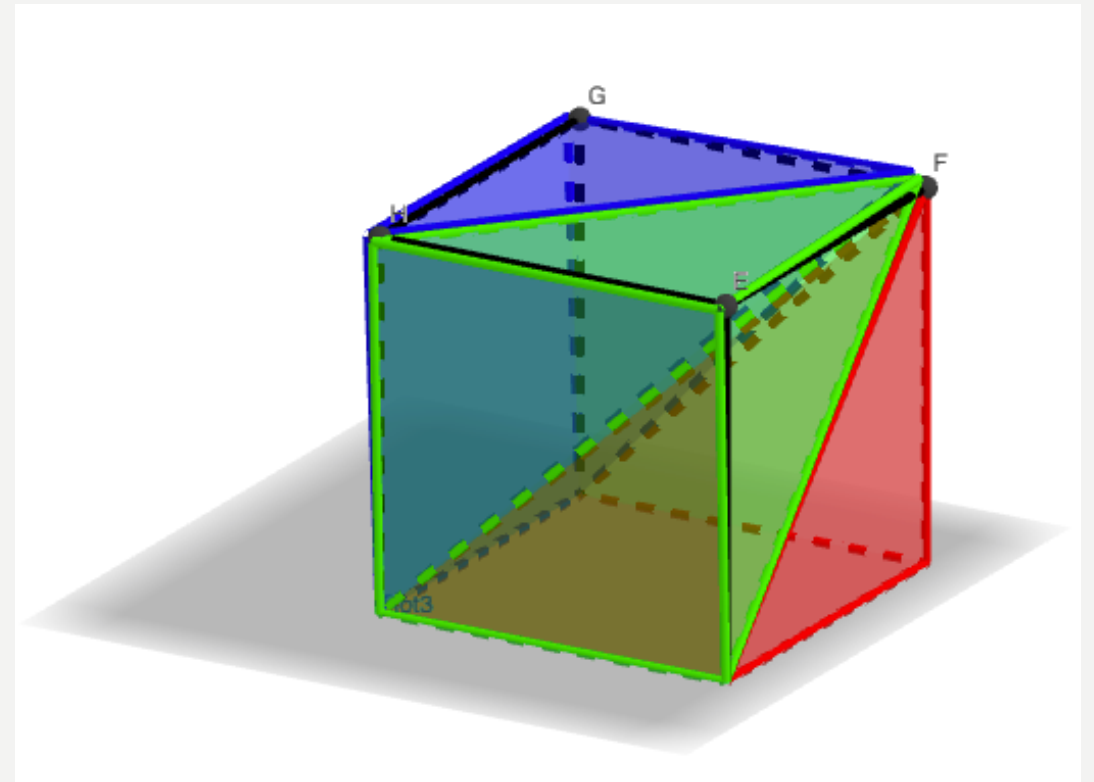
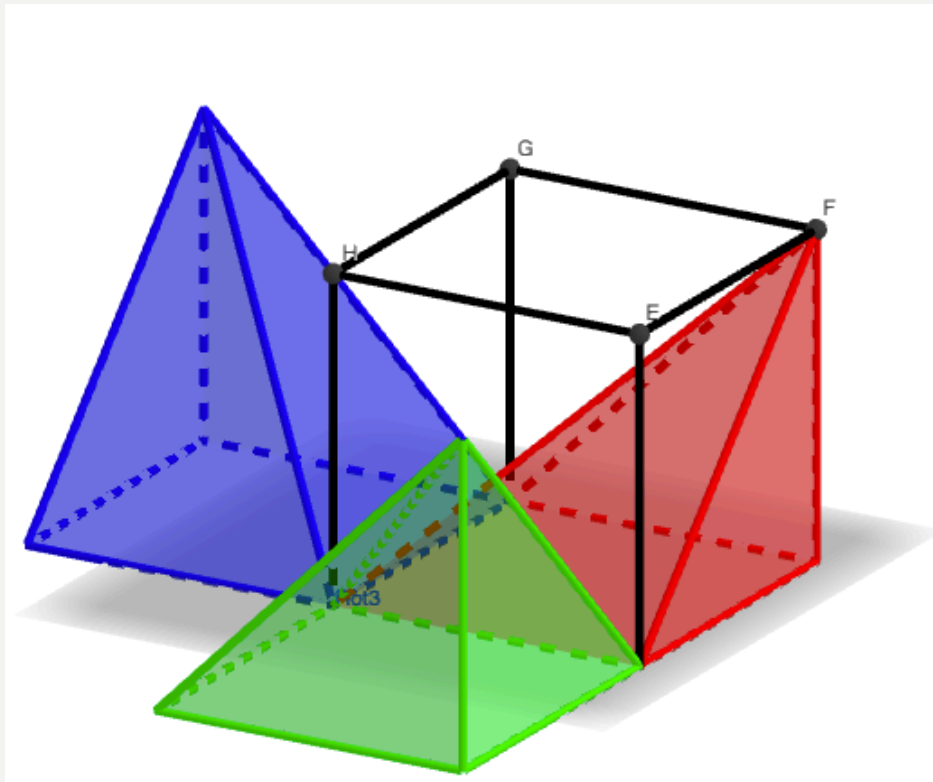
d.



12.5 – VOLUME OF PYRAMIDS AND CONES

Volume of a pyramid demo

<https://www.geogebra.org/m/jwf5y73q>



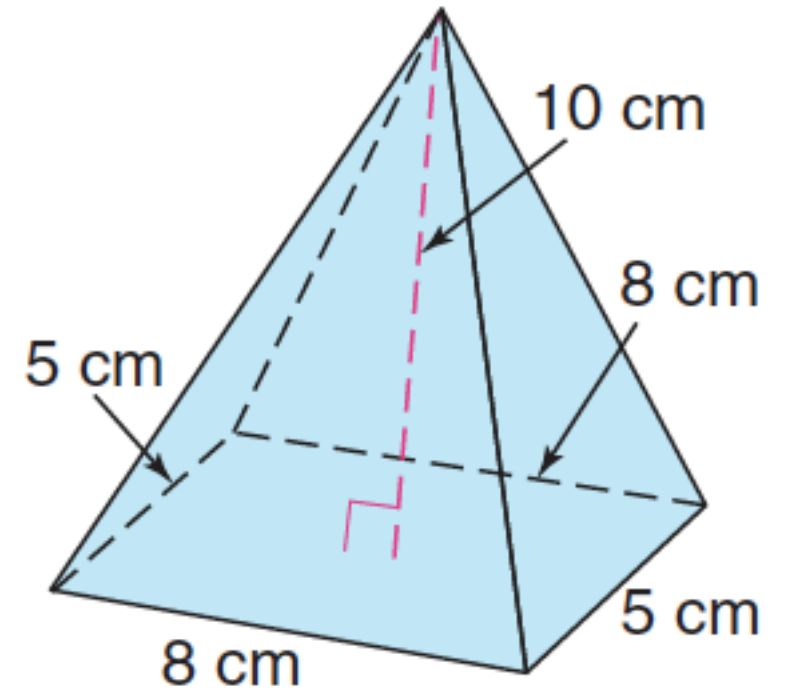
Volume of pyramids and cones

$$V = \frac{1}{3} A_{base} \cdot h$$

For a cone, replace with formula for area of a circle:

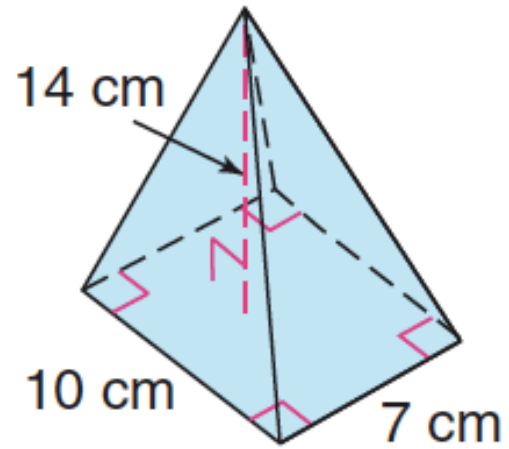
$$V = \frac{1}{3} \pi r^2 h$$

Find the volume of the rectangular pyramid to the nearest hundredth.

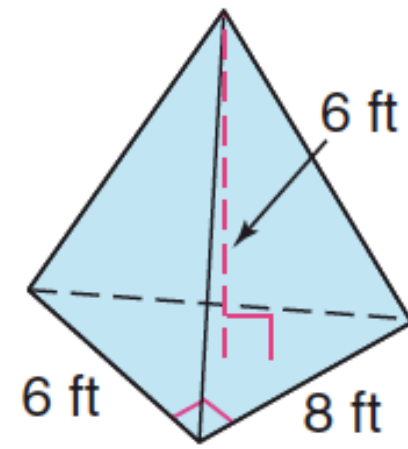


Find the volume of each pyramid. Round to the nearest hundredth.

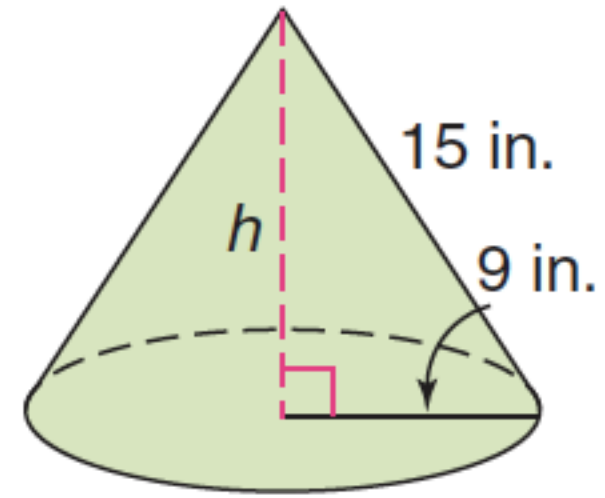
a.



b.

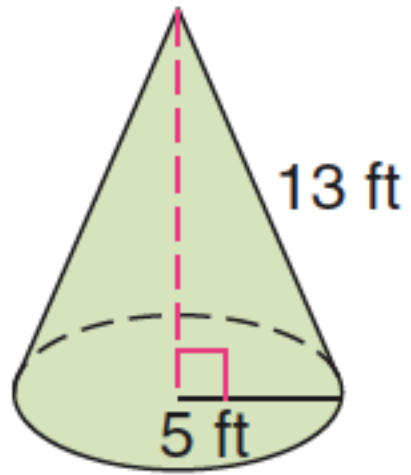


Find the volume of the cone to the nearest hundredth.

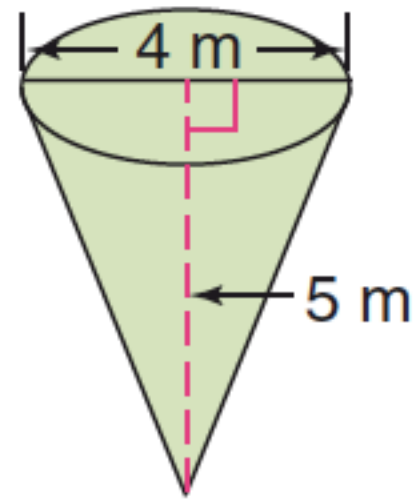


Find the volume of each cone to the nearest hundredth.

c.



d.



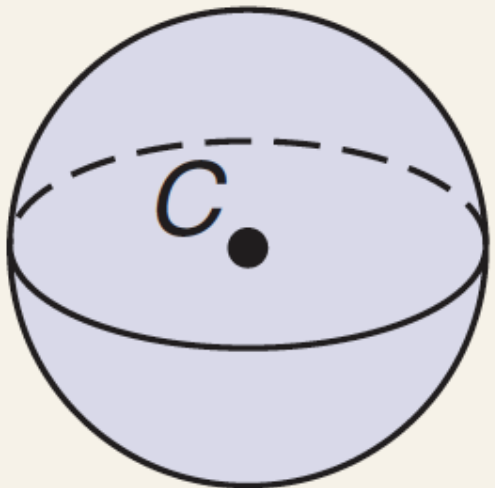
12.6 – SPHERES

Area and Volume Formulas

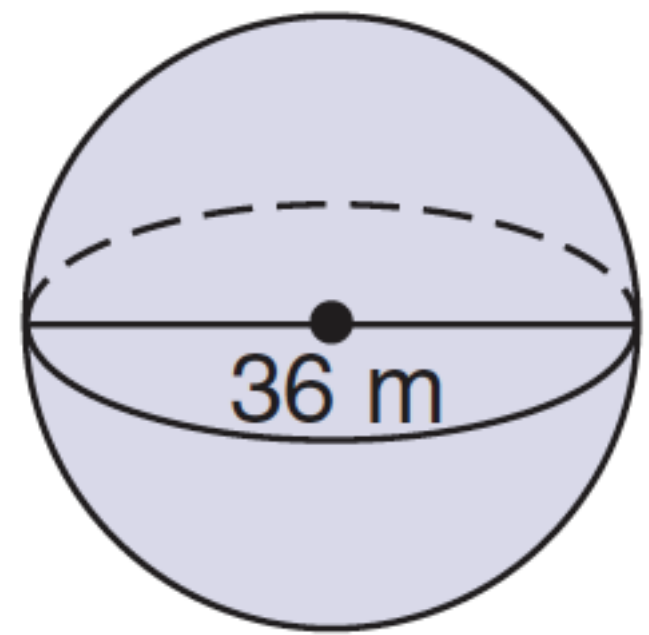
Spheres have no base, so there is only one area (no distinction between lateral and surface areas.)

$$\text{Surface Area: } S = 4\pi r^2$$

$$\text{Volume: } V = \frac{4}{3}\pi r^3$$

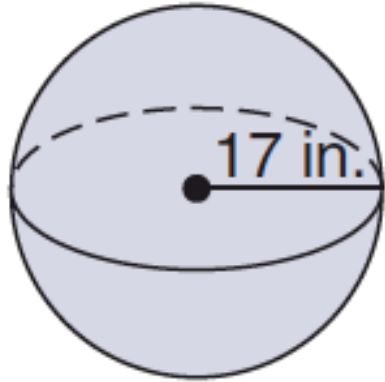


Find the surface area and volume of the sphere.

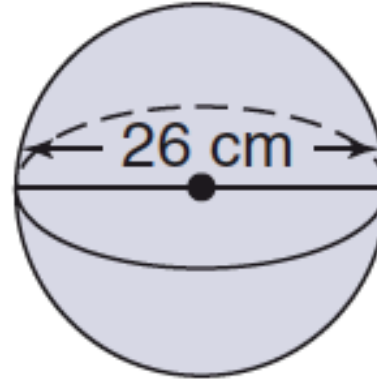


Find the surface area and volume of each sphere. Round to the nearest hundredth.

a.



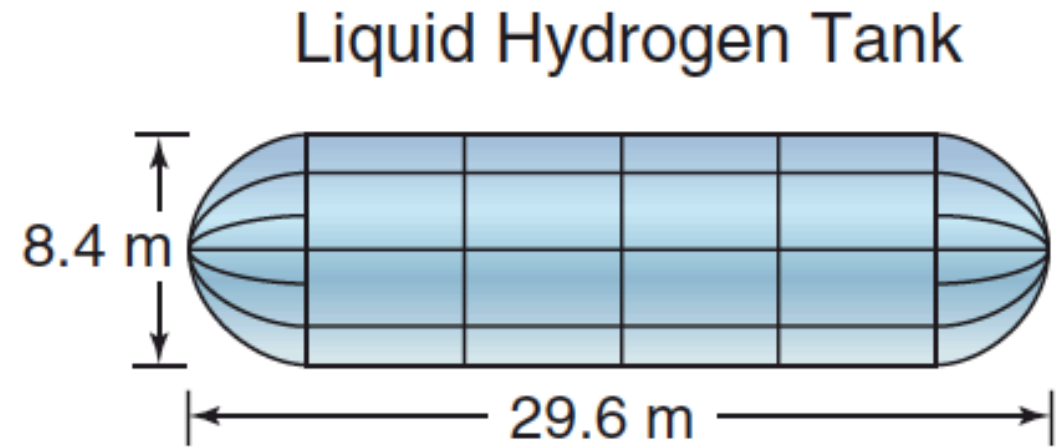
b.



Area and volume of composite solids

- To find the area or volume of composite solids, calculate the area or volume of the individual solids they are made up of and add them together.

The large external tank attached to the space shuttle at the time of launch contains the propellants for takeoff. It holds three tanks, including the liquid hydrogen tank. If the ends of the liquid hydrogen tank are hemispheres, find the volume of this tank to the nearest hundredth.



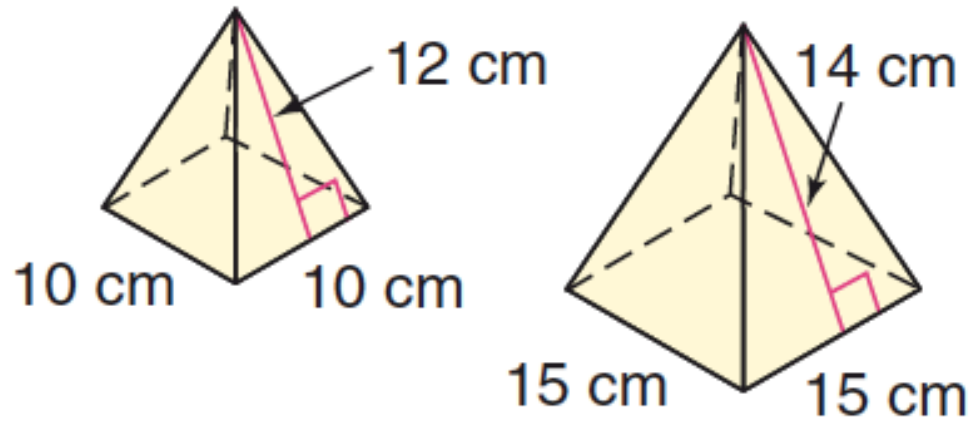
12.7 – SIMILARITY OF SOLID FIGURES

Similar solids

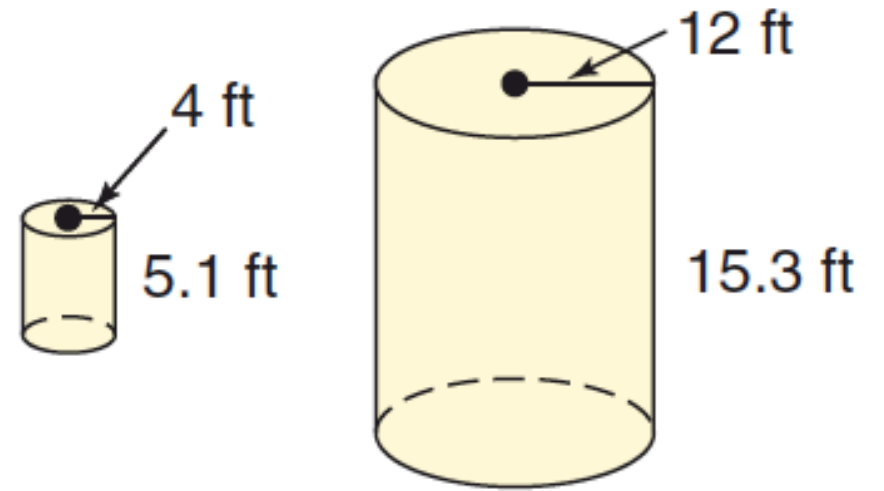
Just like similar figures, **similar solids** have the same shape, but not the same size. All their measures are proportional.

Determine whether each pair of solids is similar.

1



2



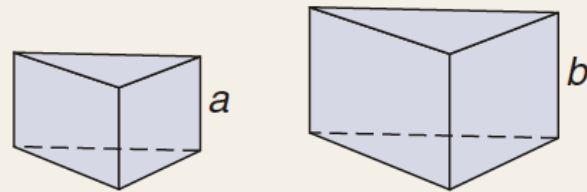
Scale factor relationships

In similar solids, the areas and volumes are also proportional, but their scale factors are squares for area and cubed for volume.

Theorem 12–15

Words: If two solids are similar with a scale factor of $a:b$, then the surface areas have a ratio of $a^2:b^2$ and the volumes have a ratio of $a^3:b^3$.

Model:

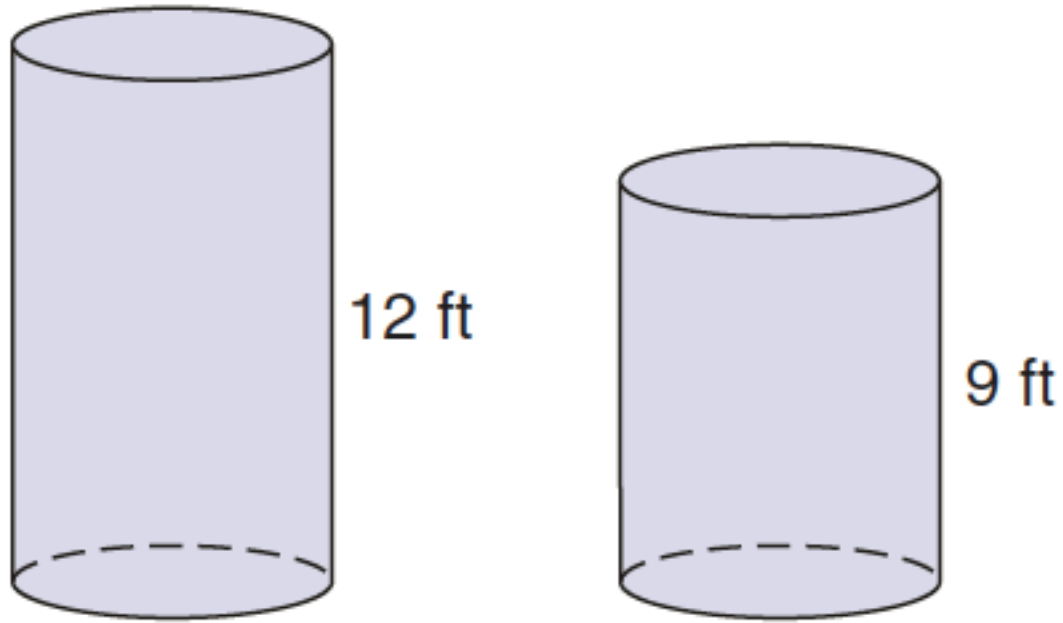


Solid A

Solid B

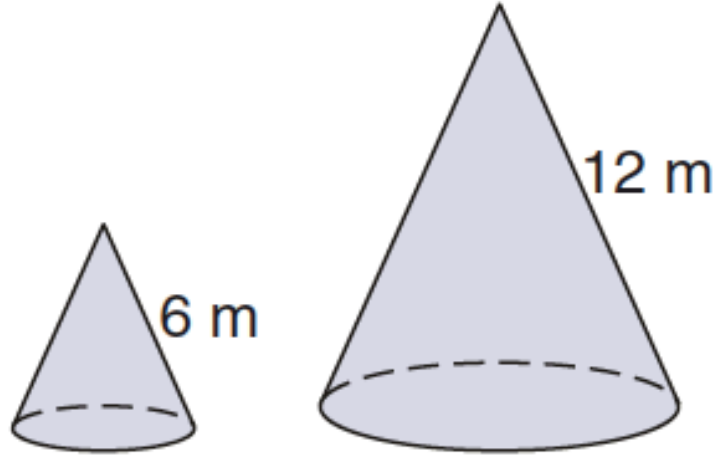
Symbols: scale factor of solid A to solid B = $\frac{a}{b}$
 $\frac{\text{surface area of solid A}}{\text{surface area of solid B}} = \frac{a^2}{b^2}$
 $\frac{\text{volume of solid A}}{\text{volume of solid B}} = \frac{a^3}{b^3}$

For the similar cylinders, find the scale factor of the cylinder on the left to the cylinder on the right. Then find the ratios of the surface areas and the volumes.



For each pair of similar solids, find the scale factor of the solid on the left to the solid on the right. Then find the ratios of the surface areas and the volumes.

c.



d.

