CHAPTER 12
SURFACE AREA AND VOLUME

12.1 – SOLID FIGURES

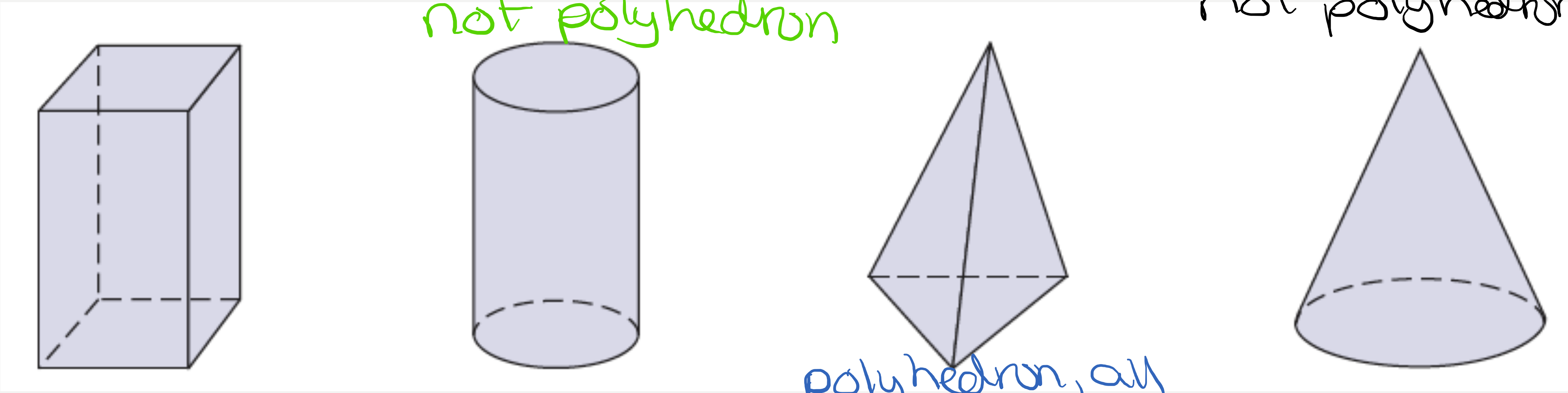
Solids

All figures above are examples of **solid figures** or **solids**.

Solids with flat surface that are polygons are called **polyhedrons** or **polyhedra.**

not polyhedron

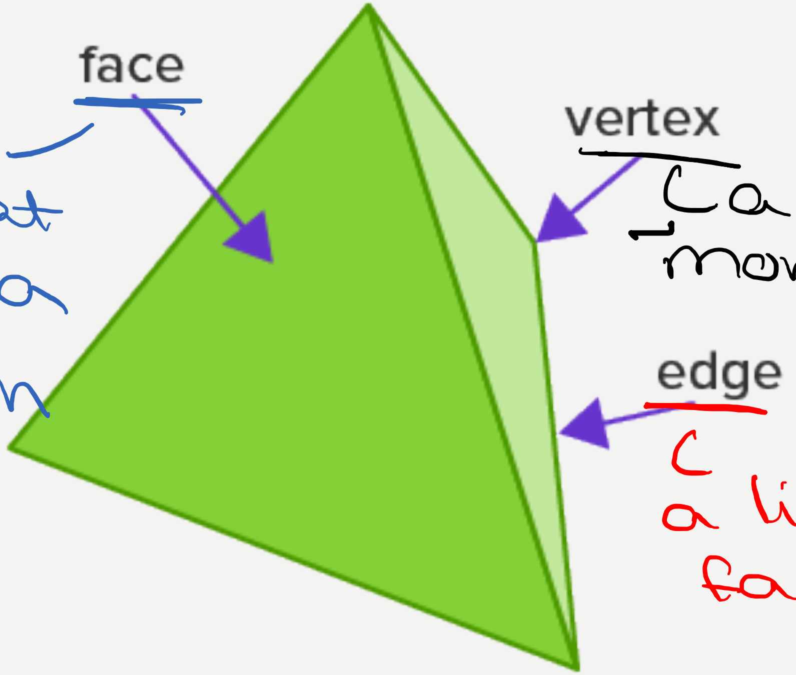
not polyhedron



polyhedron, all faces are Δs

polyhedron
all faces are squares or rectangles

Parts of a polyhedron



planes that make up a polyhedron

a point where 3 or more edges meet

a line where two faces meet.

Name the faces, edges, and vertices of the polyhedron.

Faces

EFG
MNJ
EFK
FGL
GHM
IHM
NJE

Edges

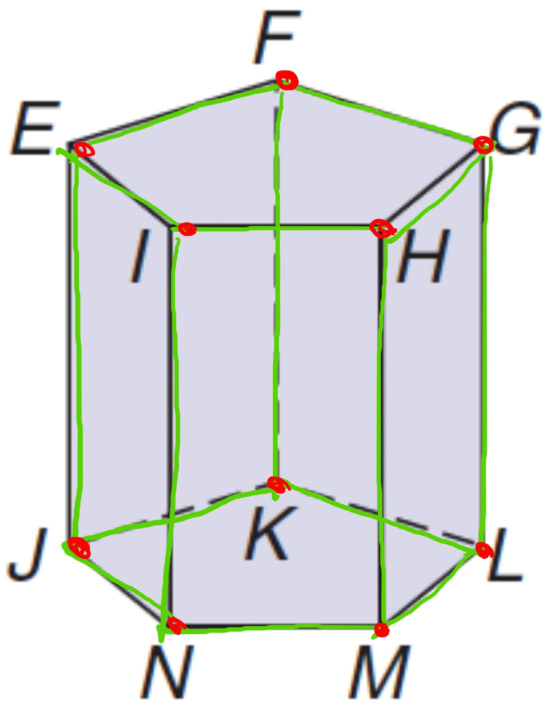
$\overline{EF}$, $\overline{FG}$, $\overline{GH}$, $\overline{HI}$, $\overline{IE}$, $\overline{KL}$, $\overline{LM}$, $\overline{MN}$, $\overline{NJ}$, $\overline{JK}$

$\overline{FK}$, $\overline{GL}$, $\overline{HM}$, $\overline{IN}$, $\overline{EJ}$

Vertices

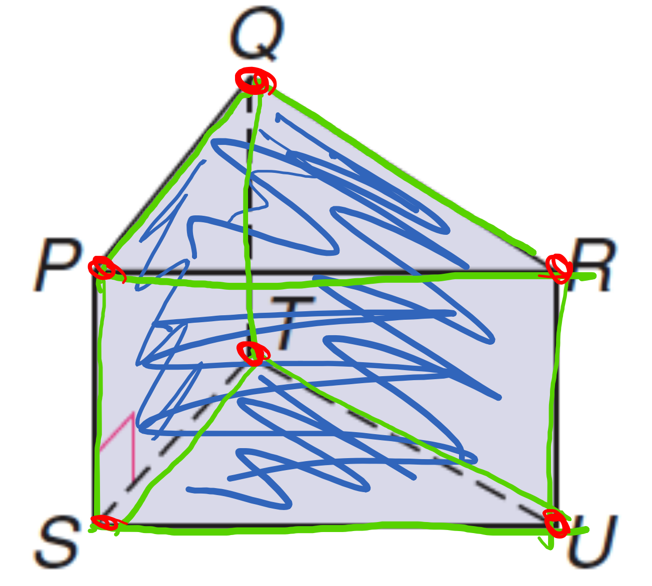
F, G, H, I, E,

J, K, L, M, N

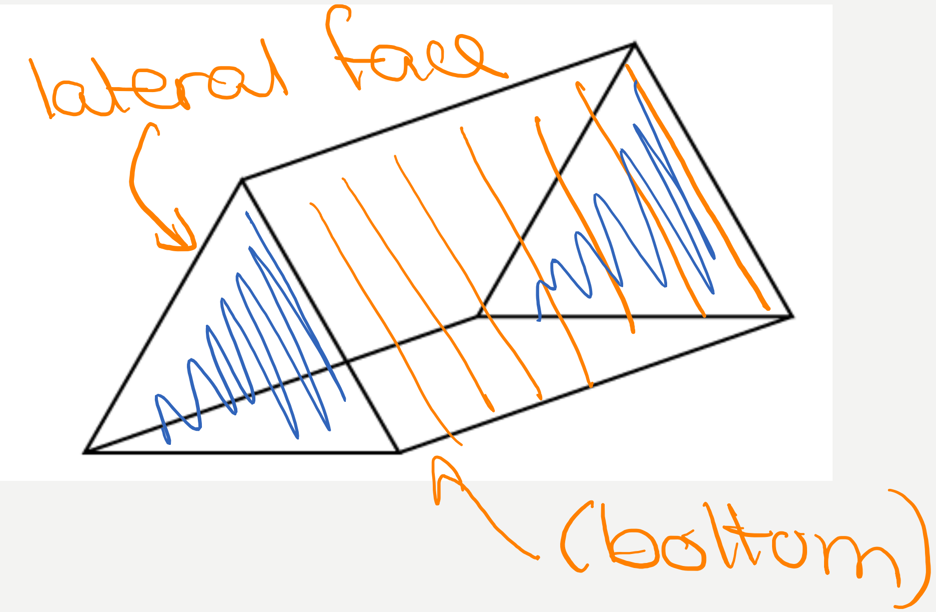


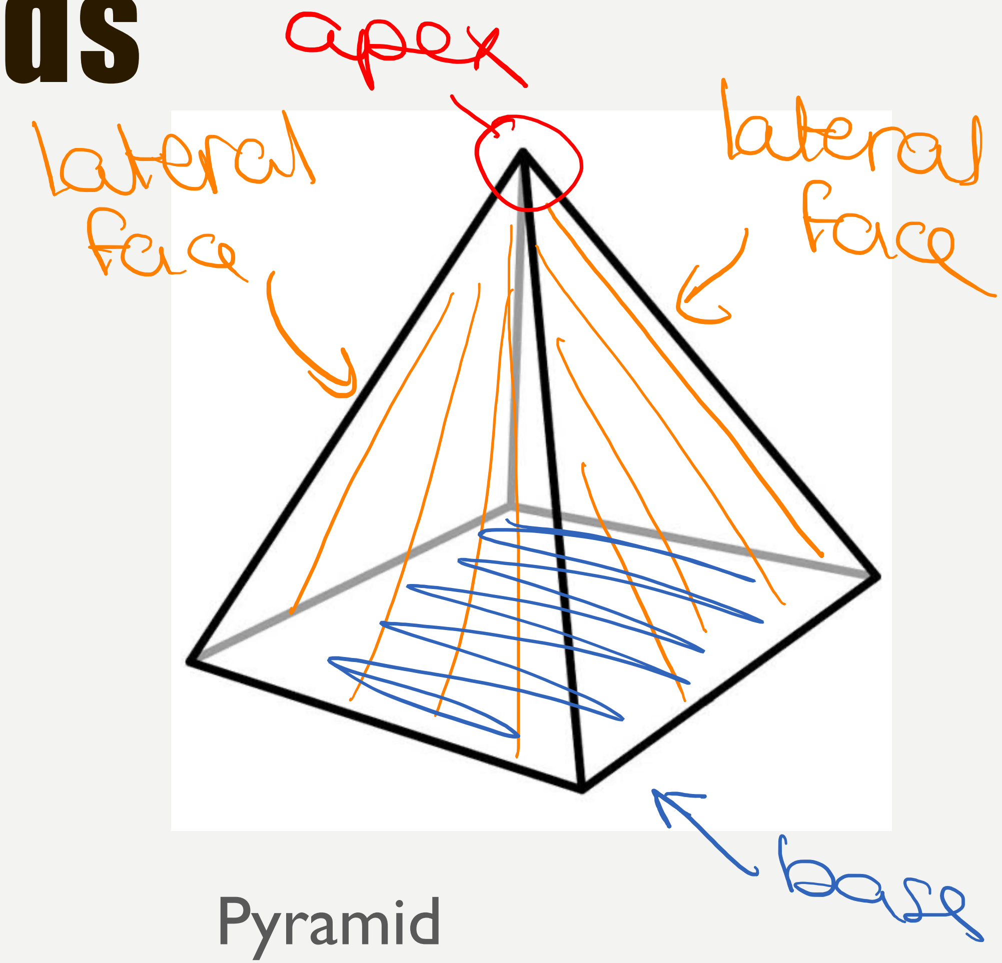
Name the faces, edges, and vertices of the polyhedron.

Faces	Edges	Vertices
QRP	$\overline{PQ}$	Q, R, P, T,
TUS	$\overline{QR}$	U, S
SPQ	$\overline{PR}$	
QRU	$\overline{TU}$	
PRU	$\overline{SU}$	
	$\overline{TS}$	
	$\overline{QT}$	
	$\overline{RU}$	
	$\overline{PS}$	



Prisms and Pyramids





Prism

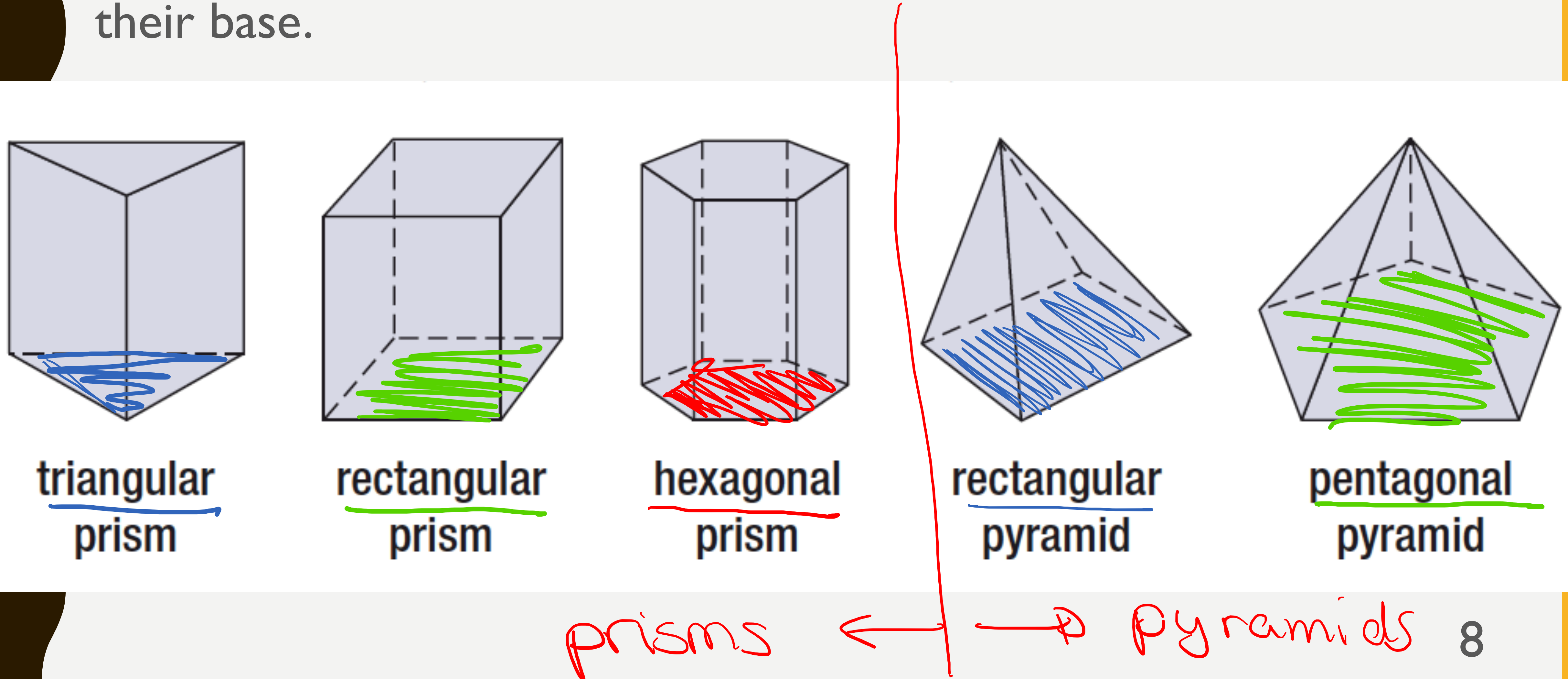
- Lateral faces are rectangular.
- Two bases.

Pyramid

- Lateral faces are triangular.
- One base.

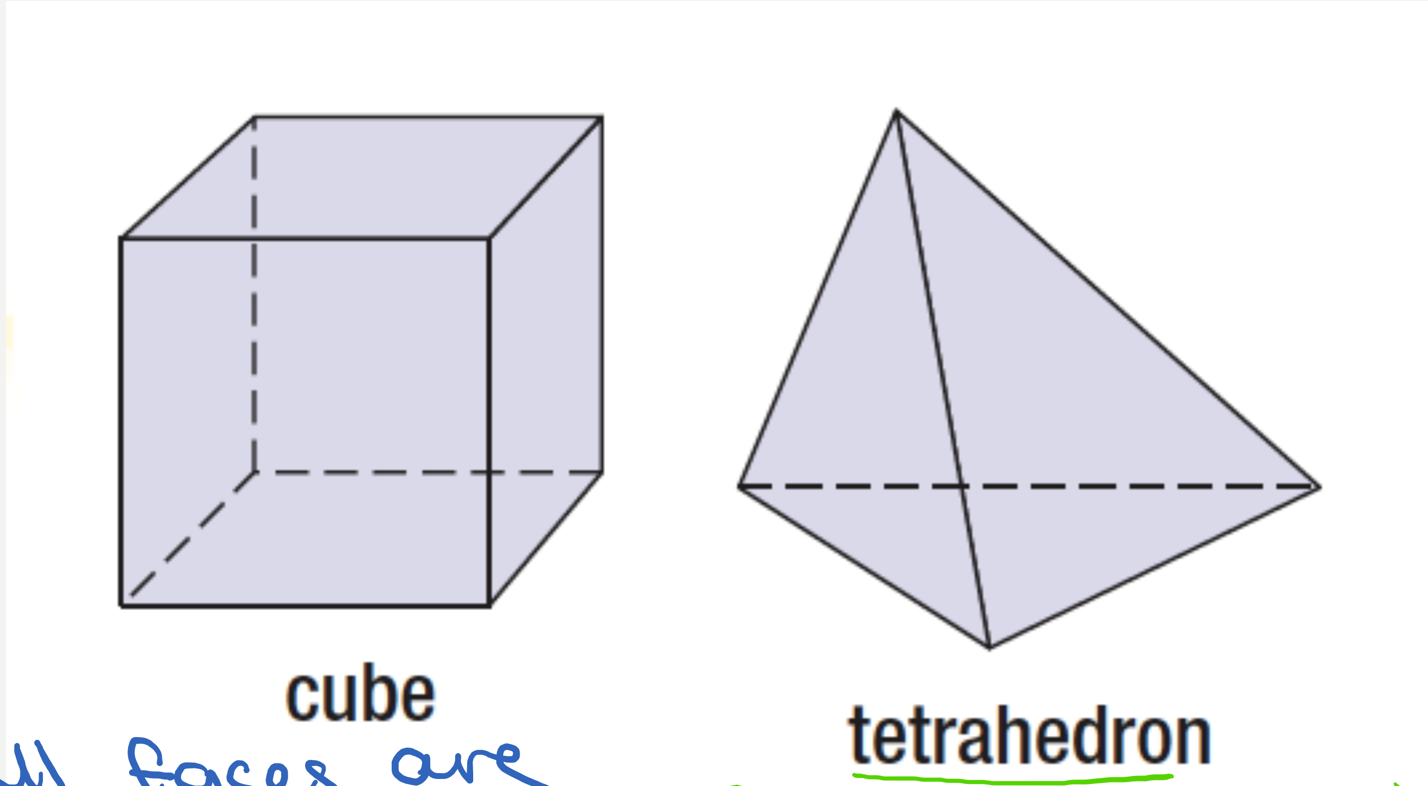
Classification

Prisms and pyramids are classified according to the shape of their base.



Special solids

Prisms and pyramids are classified according to the shape of their base.

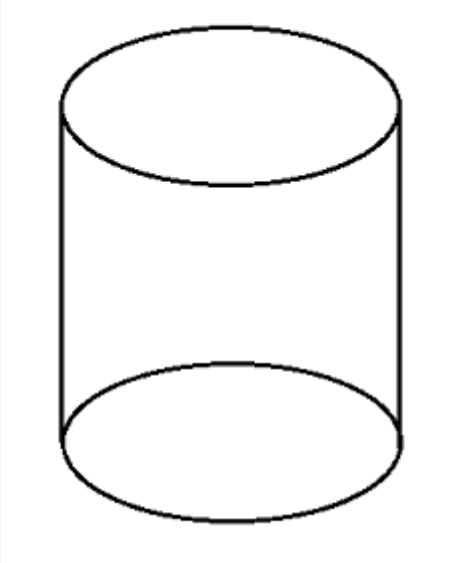


Cube: all faces are squares.
Type of prism.

Type of pyramid
All faces are triangles.

Cylinders and cones → non-polyhedra

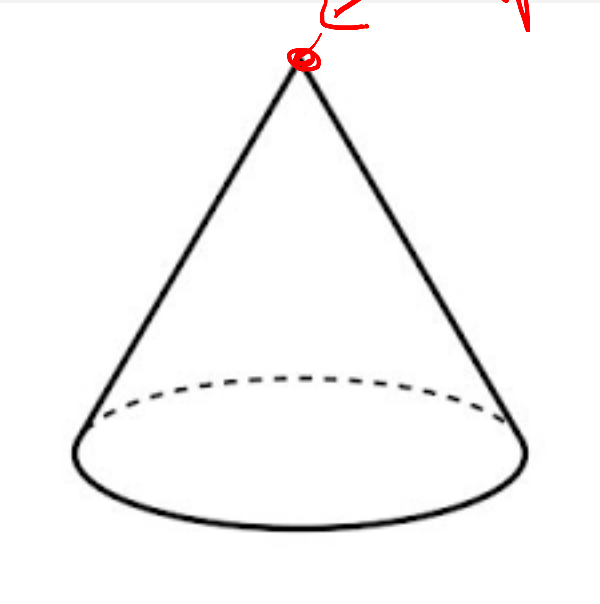
Cylinders and cones are not polyhedral because they have curved lateral faces.



Cylinder

- Two bases.

apex

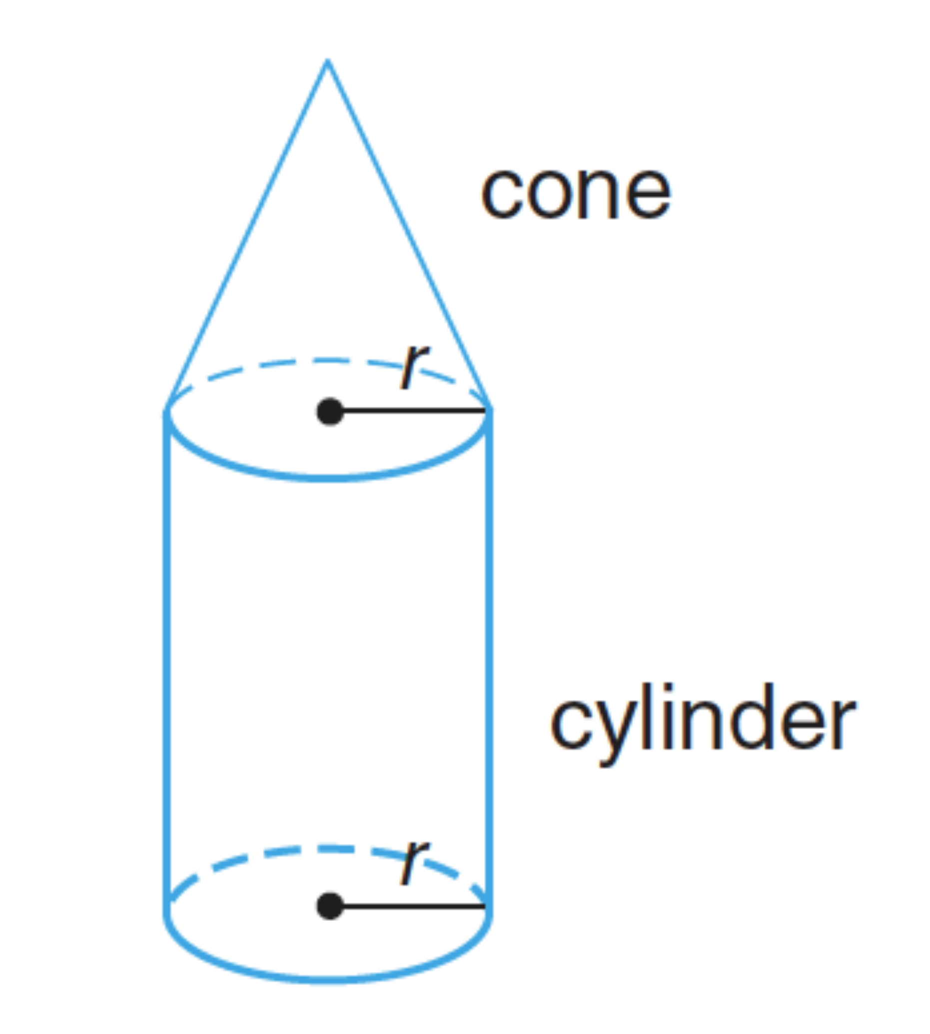


Cone

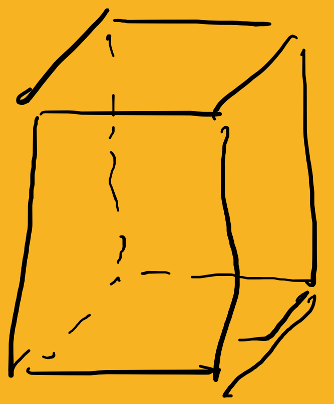
- One base.

Composite solids

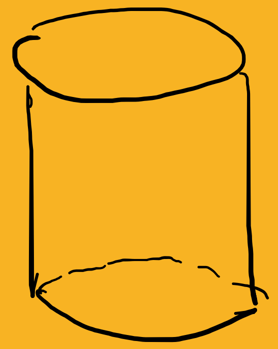
Composite solids are formed when several solids are combined to for a new solid.



12.2 – SURFACE AREA OF PRISMS AND CYLINDERS



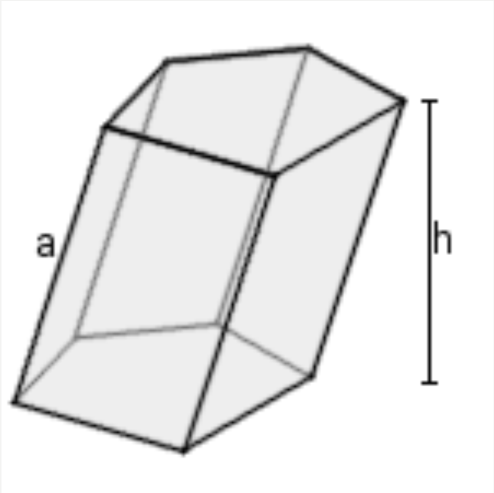
2 bases

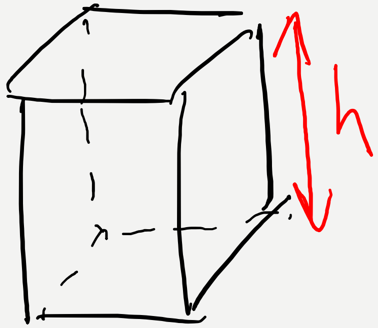


Oblique prisms and cylinders.

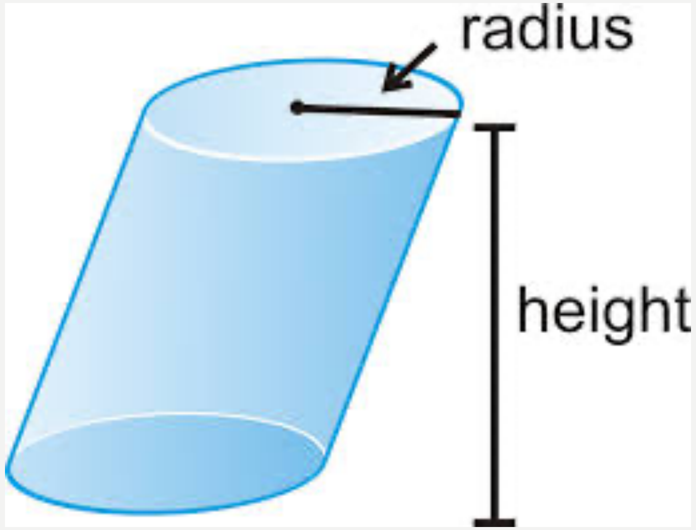
Solids are **oblique** when they are slanted. In this case, the height of the prisms and cylinder does not correspond to the edges.

Oblique prism:





Oblique cylinder:



Area definitions

lateral faces do not include the base.

- **Lateral area** includes the area of all the lateral faces.
- **Surface area** includes the area of lateral faces and bases.

Total area

For prisms and cylinders: - 2 bases

$$SA = LA + 2 \cdot A_{base}$$

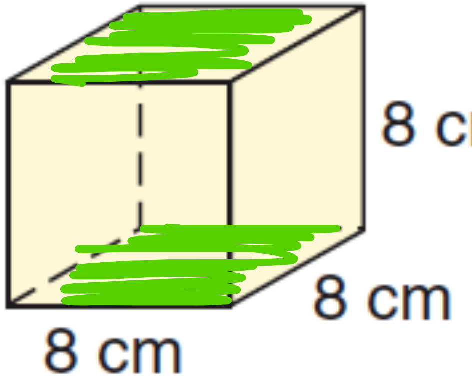
For pyramids and cones: - 1 base

$$SA = LA + A_{base}$$

Area using nets

Find the lateral area and the surface area of each prism.

a.



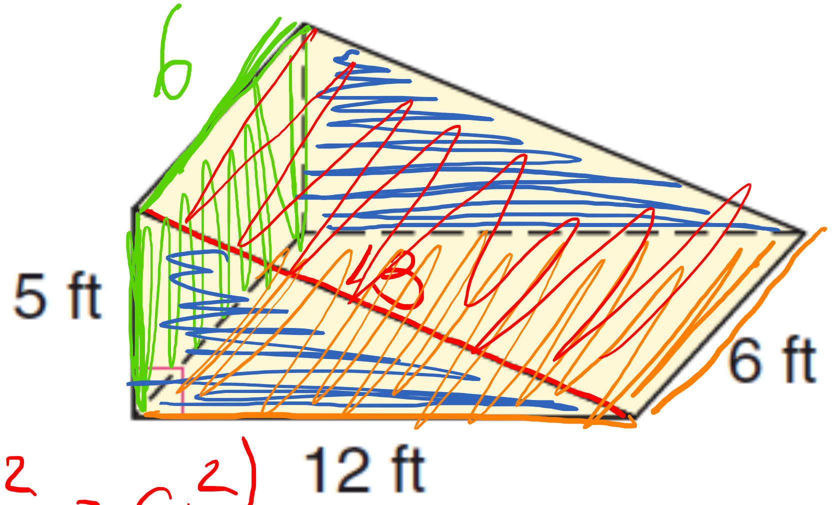
2 bases
4 lateral faces.

Each face : $8 \times 8 = 64 cm^2$

LA: $4 \times 64 = 256 cm^2$

SA: $256 + 2 \times 64 = 384 cm^2$

b.



$5^2 + 12^2 = c^2$
$25 + 144 = c^2$
$169 = c^2$
$13 = c$

$13 \times 6 = 78$

$5 \times 6 = 30$

$\frac{12 \times 5}{2} = 30$ (2)

$12 \times 6 = 72$

$LA = 30 + 72 + 78$
$LA = 180 ft^2$
$SA = 180 + 2 \times 30$
$= 240 ft^2$

Prisms and cylinders as layering of shapes

It can be more convenient to think of prisms and cylinders as a stack of shapes.

Using this method, we can use the formula:

$$LA = P_{base} \cdot h$$
$$SA = P_{base} \cdot h + 2A_{base}$$

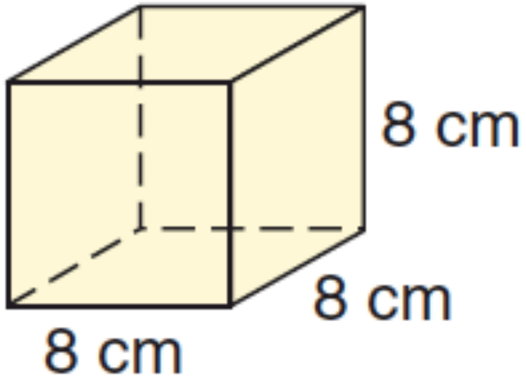
Area using stacking

$LA: P_{base} \cdot h$

$SA: LA + 2A_{base}$

Find the lateral area and the surface area of each prism.

a.



$P_{base}: 8 \times 4 = 32cm$

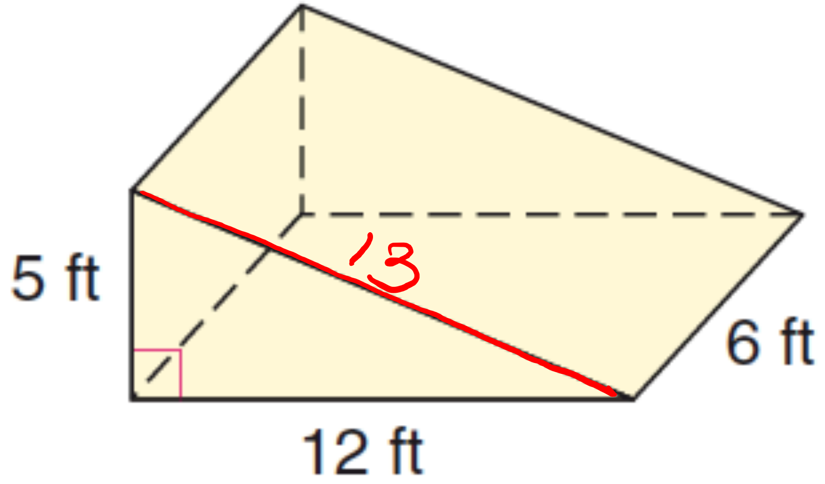
$h = 8cm$

$LA: 32 \cdot 8 = 256cm^2$

$SA: A_{base} = 8 \times 8 = 64cm^2$

$SA = 256 + 2 \times 64 = 384cm^2$

b.



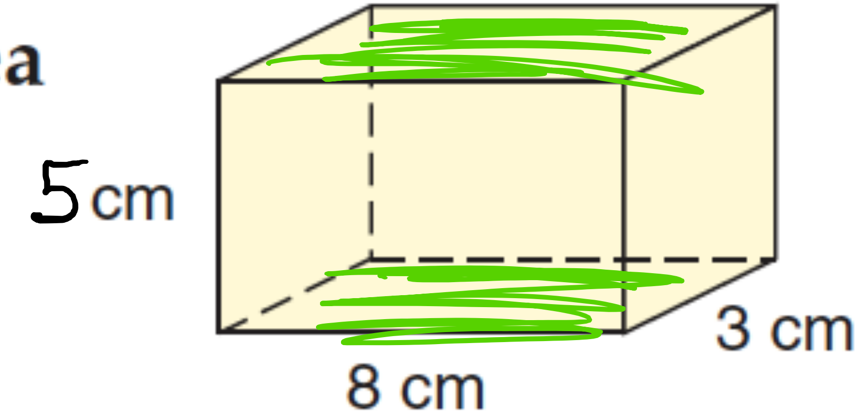
$P_{base}: 5 + 12 + 13 = 30ft$

$LA = 30 \times 6 = 180ft^2$

$A_{base} = \frac{12 \times 5}{2} = 30$

$SA = 180 + 2 \times 30 = 240ft^2$

Find the lateral area and the surface area of the rectangular prism.



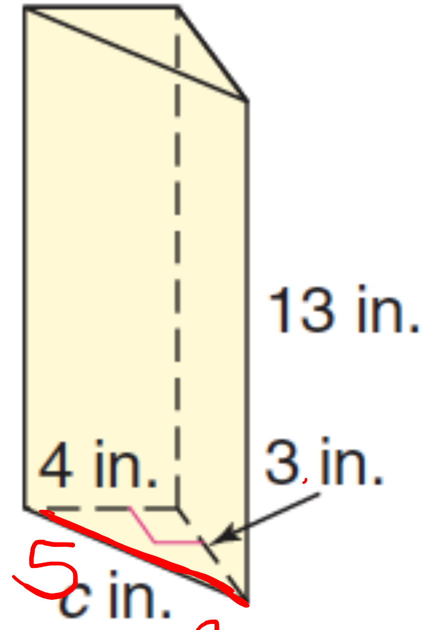
$P_{base} = 2(8+3) = 22\text{cm}$

$LA = 22 \cdot 5 = 110\text{ cm}^2$

$A_{base} = 8 \times 3 = 24$

$SA = 110 + 2 \cdot 24 = 158\text{ cm}^2$

Find the lateral area and the surface area of the triangular prism.



$P_{base} = 4+3+5 = 12\text{cm}$

$LA = 12 \cdot 13 = 156\text{ cm}^2$

$A_{base} = \frac{4 \cdot 3}{2} = 6$

$SA = 156 + 2 \cdot 6 = 168\text{cm}^2$

$4^2 + 3^2 = c^2$

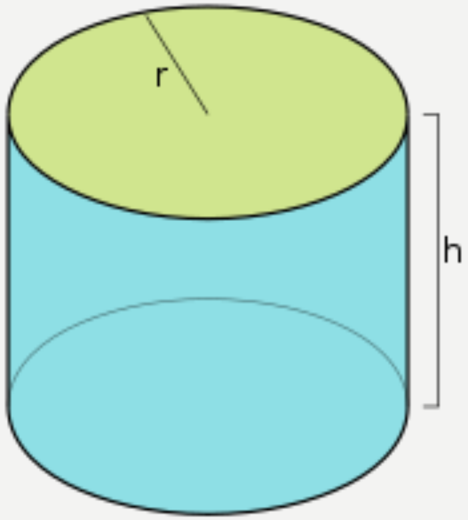
$25 = c^2$

$5 = c$

Prisms and cylinders as layering of shapes

$$LA = \overbrace{P_{base}}^{2\pi r} \cdot h = 2\pi r h$$

$$SA = \underbrace{P_{base}}_{\substack{\text{circle} \\ C = 2\pi r}} \cdot h + 2\underbrace{A_{base}}_{\pi r^2}$$



For cylinders, we can replace the perimeter and area by their formulas, which give us:

$$\underline{LA = 2\pi r \cdot} h$$

$$SA = 2\pi r h + 2\pi r^2$$

Find the lateral area and surface area of the cylinder to the nearest hundredth.

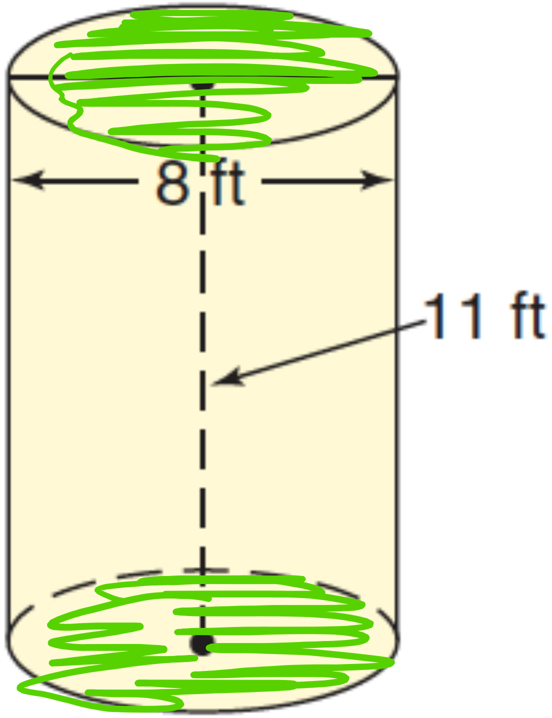
$d = 8$
$r = 4$
$h = 11$

$LA = 2\pi rh = 2\pi(4)\cdot 11 = 276.46\ ft^2$

$A_{base} = \pi r^2 = \pi(4)^2 = 50.27\ ft^2$

$SA = 276.46 + 2\cdot 50.27$

$SA = 376.99\ ft^2$



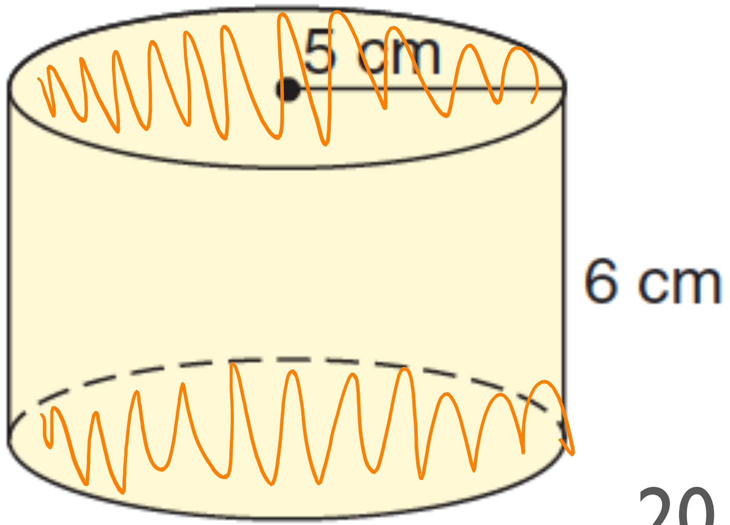
Find the lateral area and the surface area of the cylinder to the nearest hundredth.

$r = 5$
$h = 6$

$LA = 2\pi rh = 2\pi(5)(6) = 188.50\ cm^2$

$A_{bases} = \pi(5)^2 = 78.54\ cm^2$

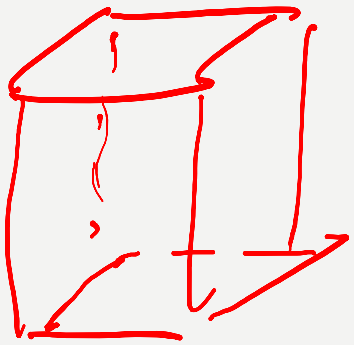
$SA = 188.50 + 2\cdot 78.54 = 345.58\ cm^2$



12.3 – VOLUME OF PRISMS AND CYLINDERS

Definition: Volume

Volume is the amount of space contained in a solid. It is measured in cubic units.



Prisms and cylinders as layering of shapes

will depend on the shape of the base

Formula:

$$V = A_{base} \cdot h$$

For cylinders, you can replace the area by the formula for the area of a circle.

$$V = \pi r^2 \cdot h$$

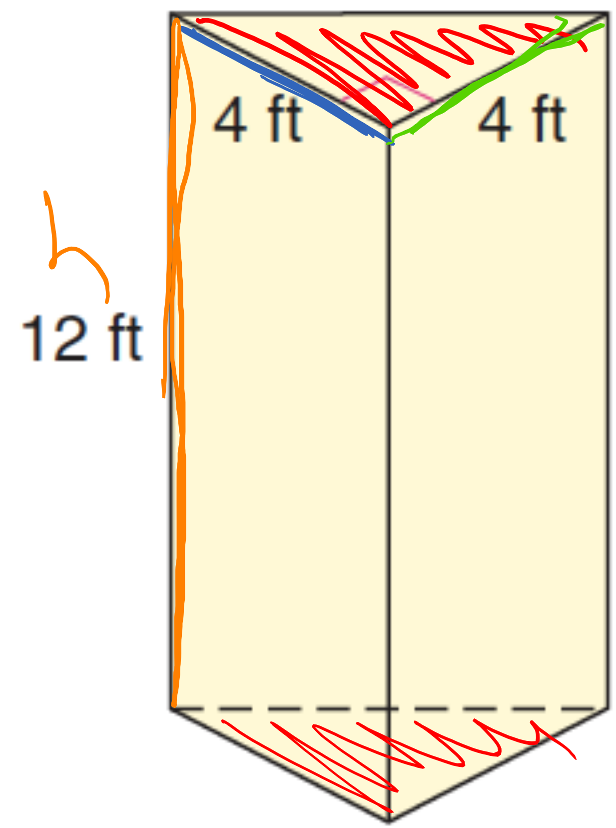
$A = \pi r^2$

Find the volume of the triangular prism.

$$A_{\triangle} = \frac{b \times h}{2}$$

$$V = A_{base} \times h$$

$h = 12ft$



$$A_{base} = \frac{4 \times 4}{2} = 8 ft^2$$

$$V = 8 \times 12 = 96 ft^3$$

The base of the prism is a regular pentagon with sides of 4 centimeters and an apothem of 2.75 centimeters. Find the volume of the prism.

Area of polygon $= \dfrac{n \times s \times a}{2}$

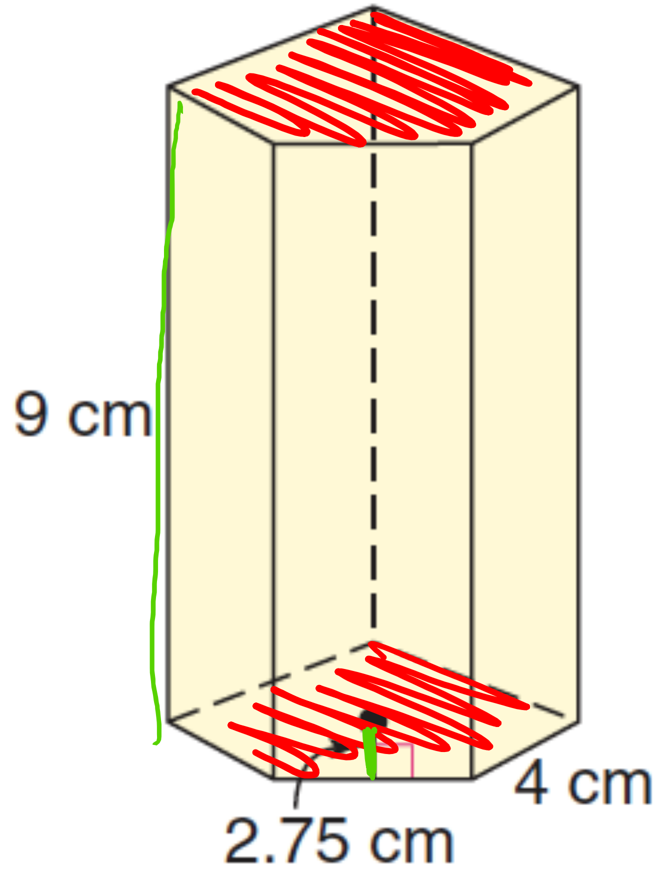
n = # of sides
s = side length
a = apothem

$n = 5$
$s = 4$
$a = 2.75$
$h = 9$

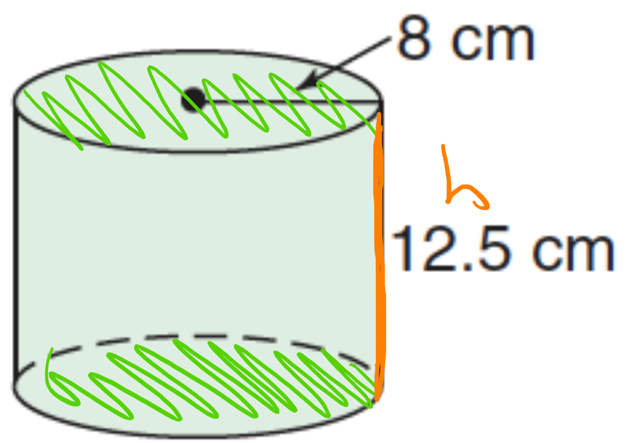
$A_b = \dfrac{5 \times 4 \times 2.75}{2}$

$A_b = 27.5\ cm^2$

$V = 27.5 \times 9 = 247.5\ cm^3$



Find the volume of the cylinder to the nearest hundredth.



$r = 8\,cm$

$h = 12.5\,cm$

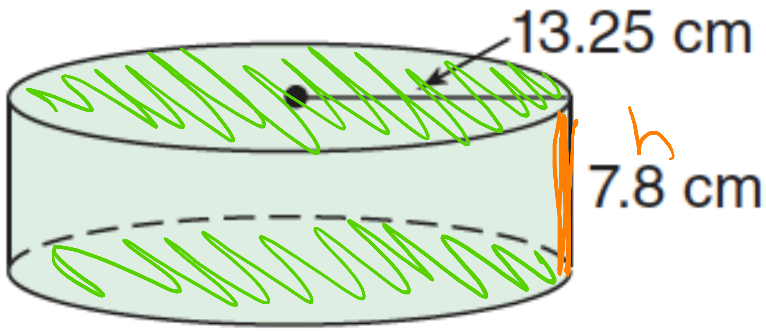
$V = A_b \cdot h$

$A = \pi r^2$

$A_b = \pi(8)^2 = 201.06\,cm^2$

$V = 201.06 \times 12.5 = 2513.25\,cm^3$

Find the volume of the cylinder to the nearest hundredth.



$$A_b = \pi r^2 = \pi(13.25)^2 = 551.55\ cm^2$$

$$V = A_b \cdot h = 551.55 \times 7.8 = \cancel{4333.29}\ cm^3$$

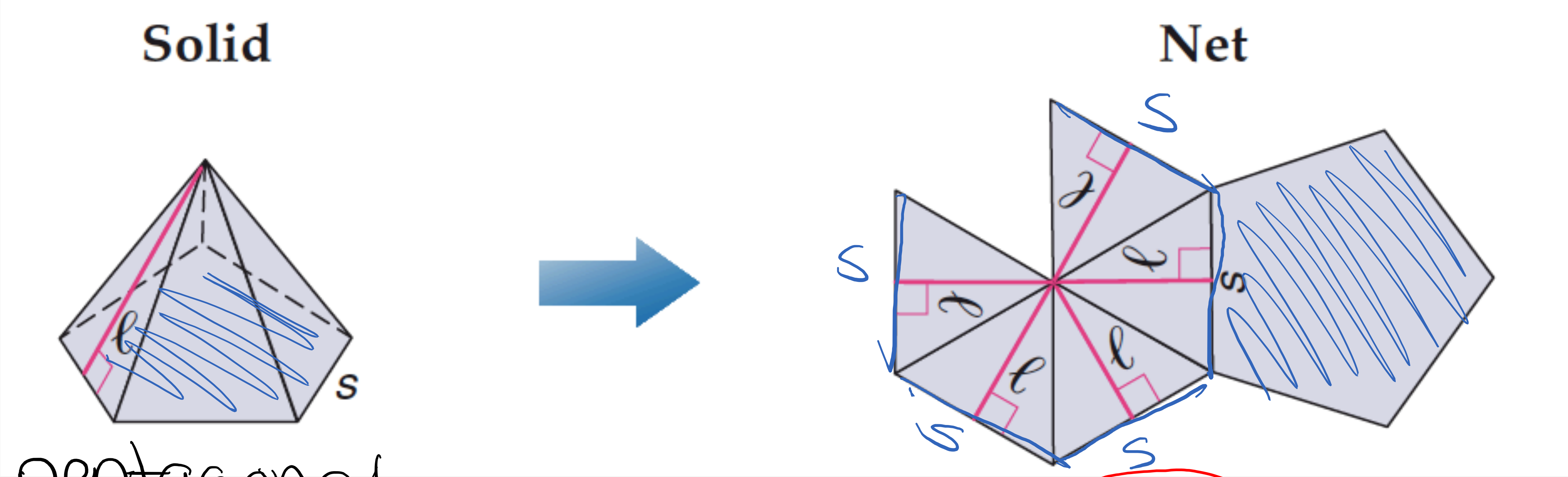
$$4302.06\ cm^3$$

this was a calculator typo.

12.4 – SURFACE AREA OF PYRAMIDS AND CONES

- 1 base
- pyramids triangular lateral faces
- lateral faces meet at the apex.

Area using nets



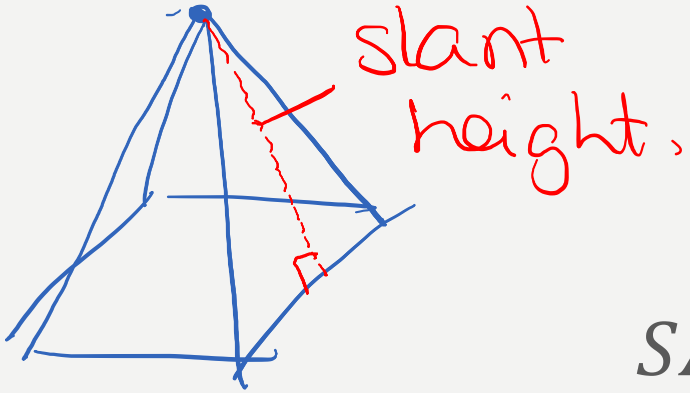
pentagonal pyramid.

$LA = \frac{1}{2} \ell \cdot s \cdot 5$ Perimeter of the pentagon

$SA = LA + A_{base}$

$LA = \frac{P_{base} \cdot \ell}{2}$

Area using perimeter



l = slant height

$$LA = \frac{1}{2}P_{base} \cdot l = \frac{P_b \cdot l}{2}$$

$$SA = \frac{1}{2}P_{base}l + A_{base}$$

For a cone, you can replace perimeter and area of circle by their formula.

$$LA = \frac{\cancel{2}\pi r \cdot l}{\cancel{2}} = \pi r l$$

$$LA = \pi r l$$

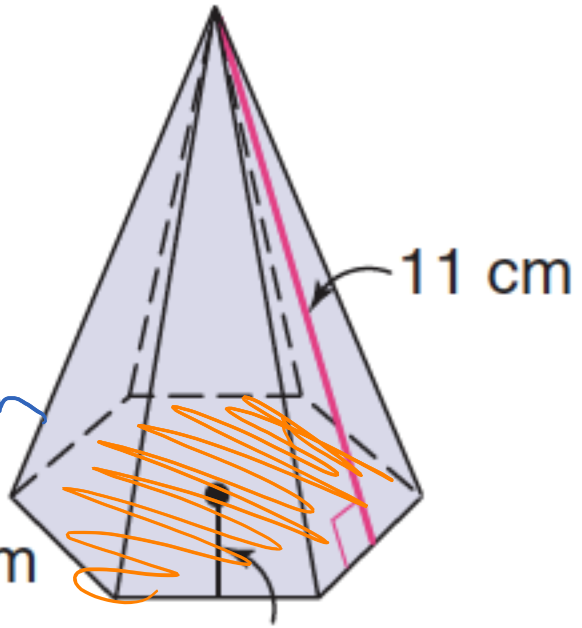
$$SA = \pi r l + \pi r^2$$

Find the lateral area and the surface area of the regular hexagonal pyramid.

Polygon:

$$A = \frac{n \times s \times a}{2}$$

n = # sides
s = side length
a = $\frac{1}{2}$ height.



n = 6

$$P_b = 6 \times 6 = 36 \text{ cm}$$

$$l = 11 \text{ cm}$$

$$A_b = \frac{6 \times 6 \times 5.2}{2} = 93.6 \text{ cm}^2$$

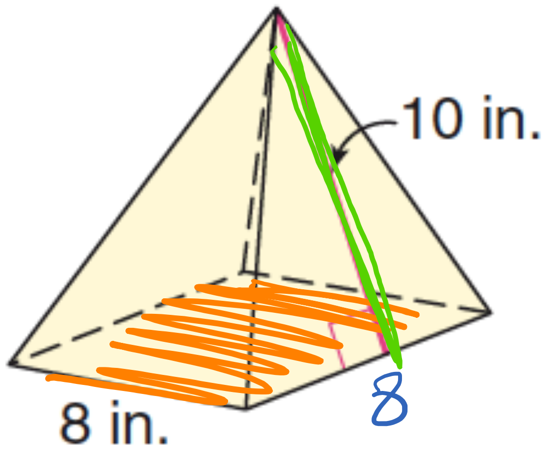
$$LA = P_b \cdot l = 36 \times 11 = 396 \text{ cm}^2$$

$$SA = 396 + 93.6 = 489.6 \text{ cm}^2$$

P_b = perimeter of base.
l = slant height
A_b = area of base
LA = lateral area
SA = surface area.

Find the lateral area and the surface area of each regular pyramid.

a.



$P_b = 4 \times 8 = 32$ in

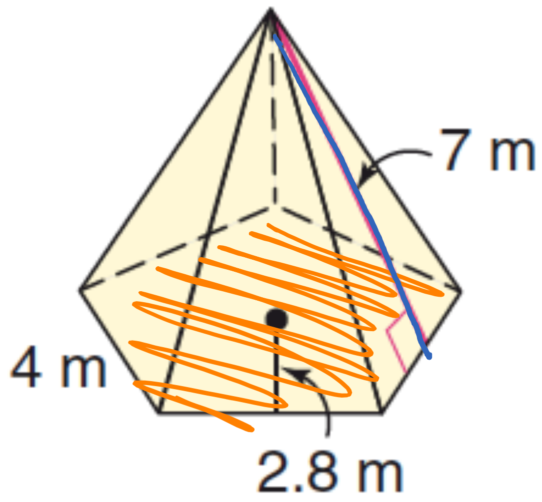
$l = 10$ in

$A_b = 8 \times 8 = 64 \text{ in}^2$

$LA = 32 \times 10 = 320 \text{ in}^2$

$SA = 320 + 64 = 384 \text{ in}^2$

b.



$$A = \frac{n \times s \times a}{2}$$

$P_b = 5 \times 4 = 20$ m

$l = 7$ m

$A_b = \dfrac{5 \times 4 \times 2.8}{2} = 28 \text{ m}^2$

$LA = 20 \times 7 = 140 \text{ m}^2$

$SA = 140 + 28 = 168 \text{ m}^2$

Find the lateral area and the surface area of the cone to the nearest hundredth.

$C = 2\pi r = \pi d$

$A = \pi r^2$

$P_b = \pi(10) = 31.42 \text{ ft}$

$\ell^2 = 5^2 + 12^2$

$\ell^2 = 25 + 144$

$\ell^2 = 169$

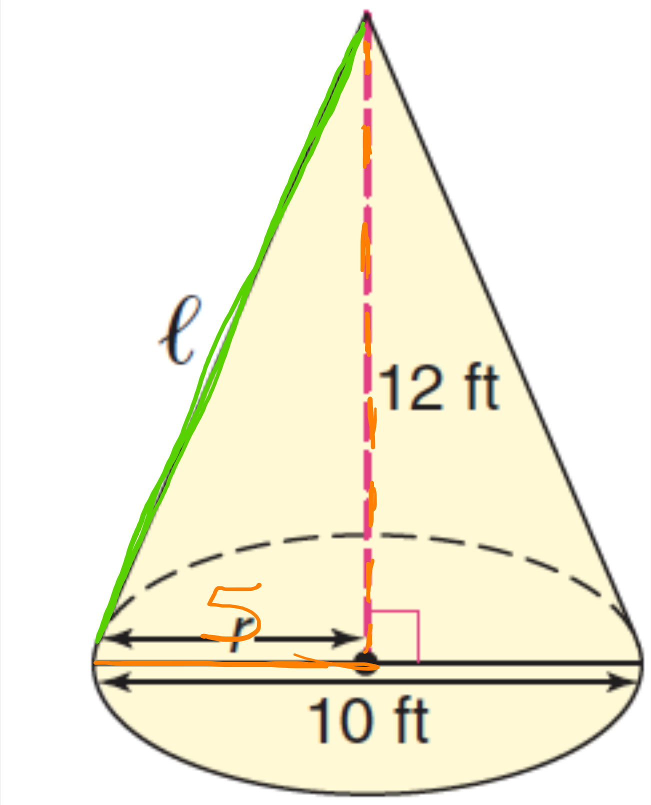
$\ell = 13$

$\ell = 13 \text{ ft}$

$A_b = \pi(5)^2 = 78.54 \text{ ft}^2$

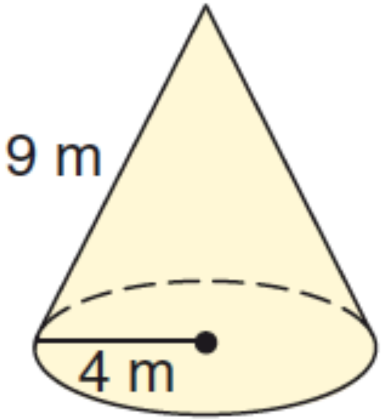
$LA = 31.42 \times 13 = 408.46 \text{ ft}^2$

$SA = 408.48 + 78.54 = 487.02 \text{ ft}^2$



Find the lateral area and the surface area of each cone. Round to the nearest hundredth.

c.



$P_b = 2\pi(4) = 25.13\,m$

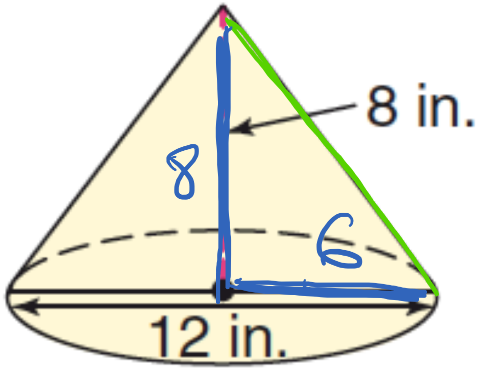
$l = 9\,m$

$A_b = \pi(4)^2 = 50.27\,m^2$

$LA = 25.13 \times 9 = 226.17\,m^2$

$SA = 226.17 + 50.27 = 276.44\,m^2$

d.



$l^2 = 8^2 + 6^2$

$l^2 = 64 + 36$

$l^2 = 100$

$l = 10$

$P_b = \pi(12) = 37.70\,in$

$l = 10$

$A_b = \pi(6)^2 = 113.10\,in^2$

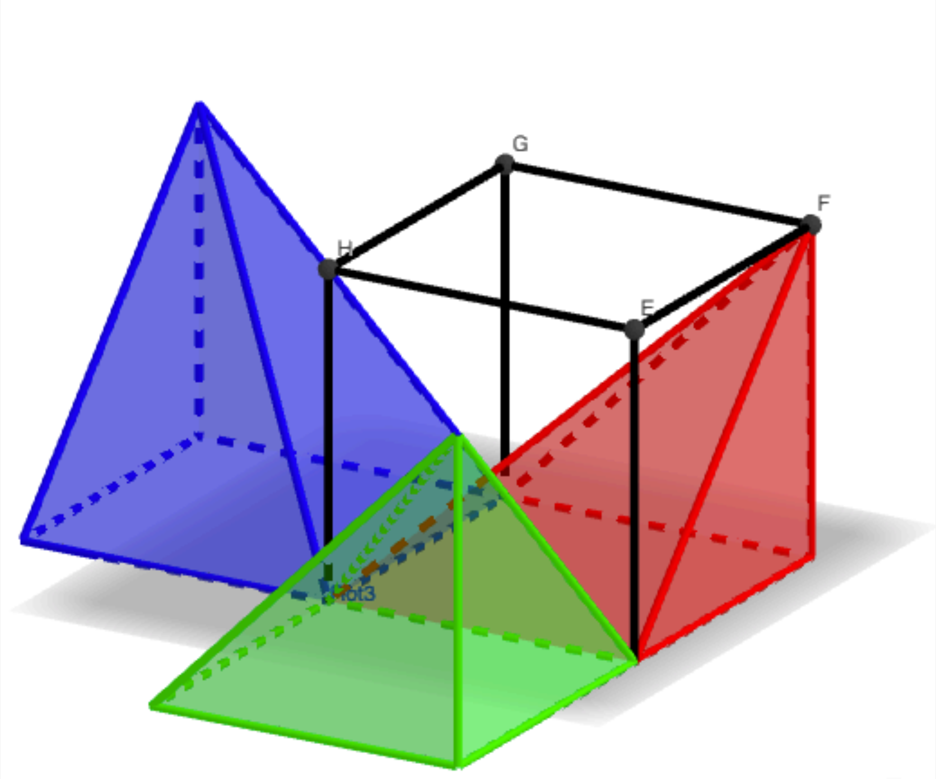
$LA = 37.70 \times 10 = 377\,in^2$

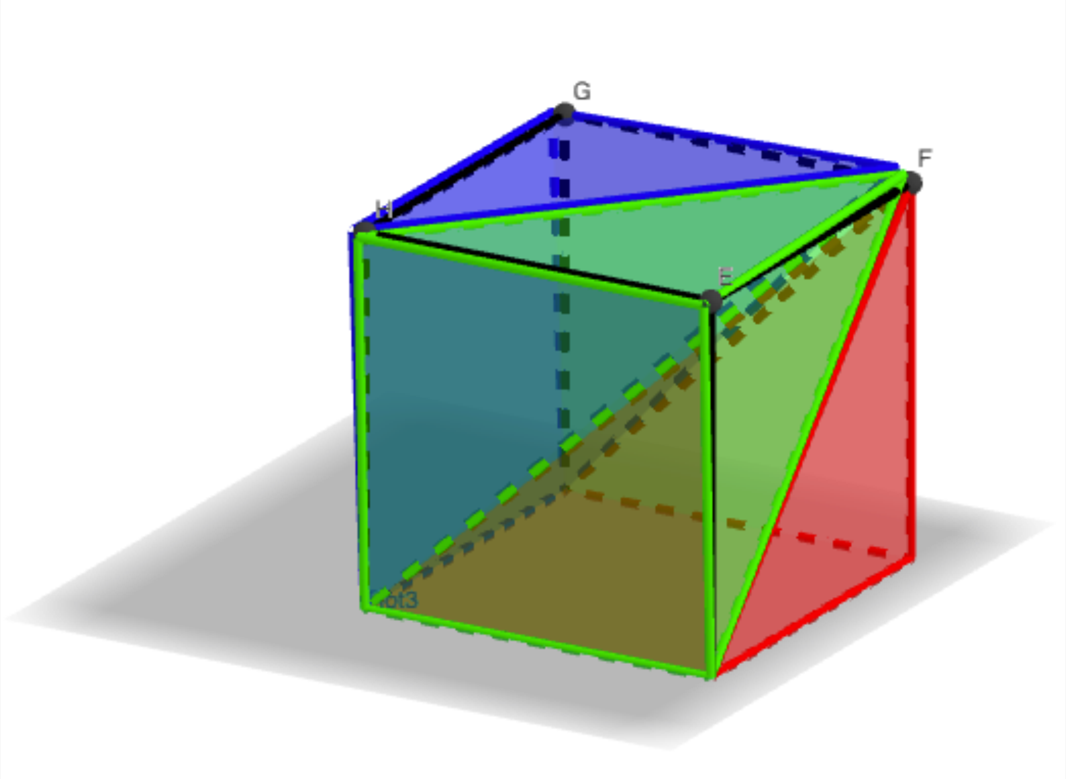
$SA = 377 + 113.1 = 490.1\,in^2$

12.5 – VOLUME OF PYRAMIDS AND CONES

Volume of a pyramid demo

https://www.geogebra.org/m/jwf5y73q





Volume of pyramids and cones

Volume of prism = $A_{base} \cdot h$

$$V = \frac{1}{3} A_{base} \cdot h = \frac{A_b \cdot h}{3}$$

For a cone, replace with formula for area of a circle:

$$V = \frac{1}{3} \pi r^2 h$$

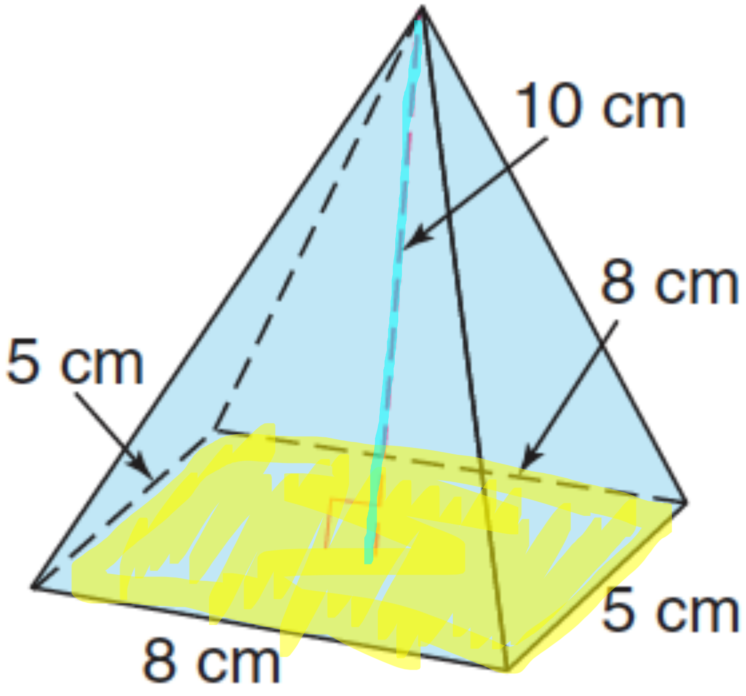
Find the volume of the rectangular pyramid to the nearest hundredth.

$V = \frac{A_b \cdot h}{3}$

$A_b = 8 \times 5 = 40\ cm^2$

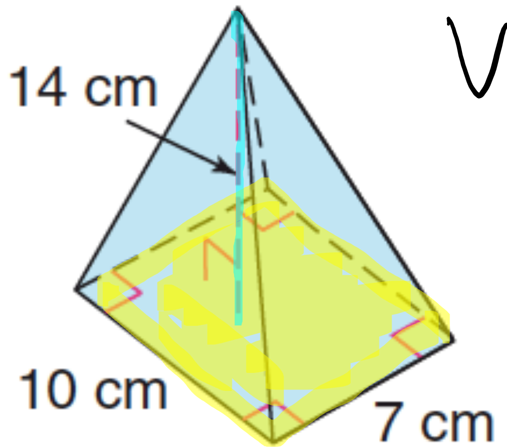
$h = 10\ cm$

$V = \frac{40 \times 10}{3} = 133.33\ cm^3$



Find the volume of each pyramid. Round to the nearest hundredth.

a.



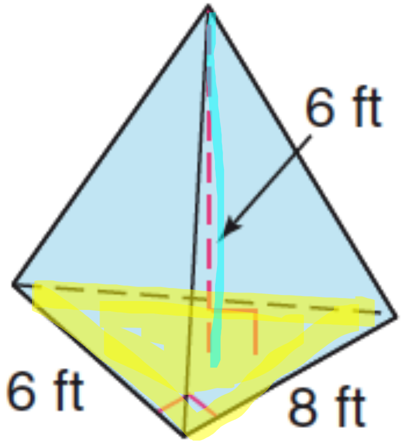
$V = \frac{A_b \cdot h}{3}$

$A_b = 10 \times 7 = 70\,cm^2$

$h = 14\ cm$

$V = \frac{70 \times 14}{3} = 326.66\,cm^3$

b.



A of triangle

$\frac{b \times h}{2}$

$A_b = \frac{6 \times 8}{2} = 24\ ft^2$

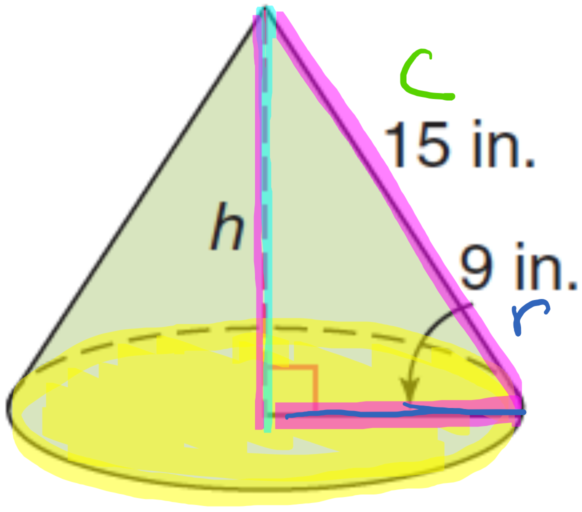
$h = 6\ ft$

$V = \frac{24 \times 6}{3} = 48\ cm^3$

Find the volume of the cone to the nearest hundredth.

$V = \frac{A_b \cdot h}{3}$

$A_O = \pi r^2$



$A_b = \pi(9)^2 = 254.47 \text{ in}^2$

$h = 12 \text{ in}$

$15^2 = h^2 + 9^2$

$225 = h^2 + 81$

$144 = h^2$

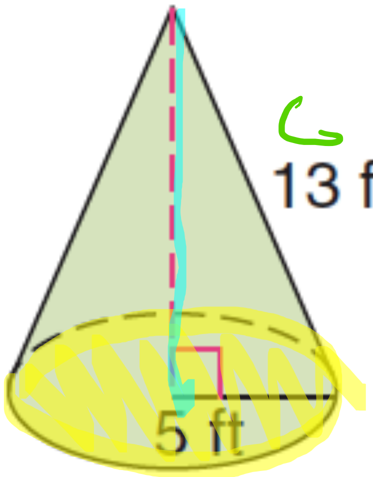
$12 = h$

$V = \frac{254.47 \times 12}{3} = 1017.88 \text{ in}^3$

Find the volume of each cone to the nearest hundredth.

$$V = \frac{A_b \cdot h}{3}$$

c.



$A_b = \pi(5)^2 = 78.54 \text{ ft}^2$

$h = 12 \text{ ft}$

$13^2 = h^2 + 5^2$

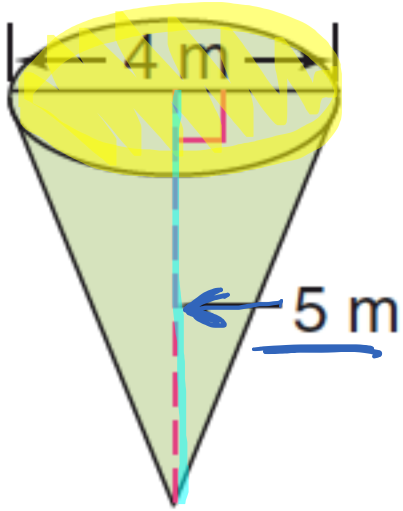
$169 = h^2 + 25$

$144 = h^2$

$12 = h$

$V = \frac{78.54 \times 12}{3} = 314.16 \text{ ft}^3$

d.



$d = 4 m$

$r = 2 m$

$A_b = \pi(2)^2 = 12.57 m^2$

$h = 5$

$V = \frac{12.57 \times 5}{3} = 20.95 in^2$

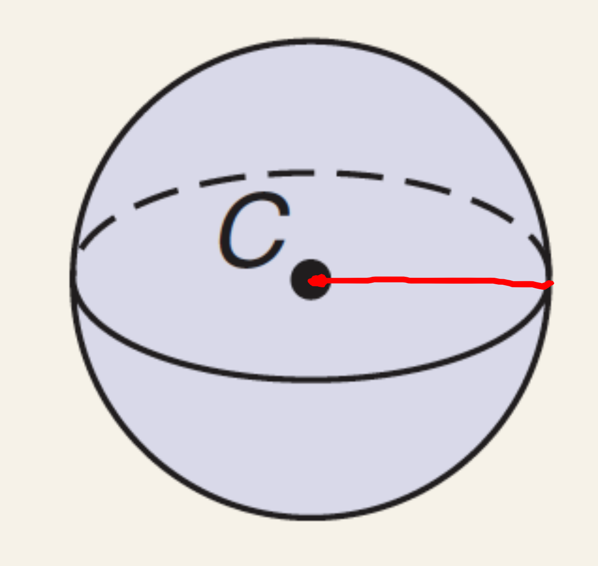
12.6 – SPHERES

Area and Volume Formulas

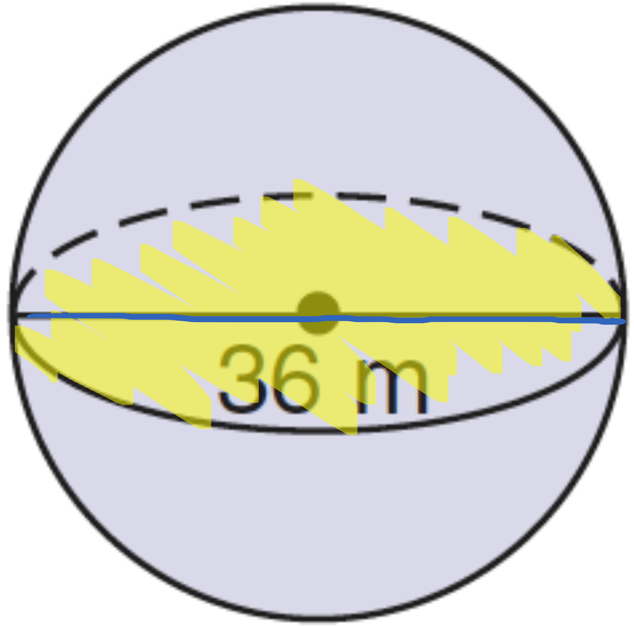
Spheres have no base, so there is only one area (no distinction between lateral and surface areas.)

Surface Area: $SA = 4\pi r^2$ → only one measure for area.

Volume: $V = \frac{4}{3}\pi r^3$



Find the surface area and volume of the sphere.



$d = 36m$

$r = 36 \div 2 = 18m$

$SA = 4\pi(18)^2 = 4071.5\ m^2$

$V = \frac{4}{3}\pi(18)^3 = 24429.02m^3$

$SA = 4\pi r^2$

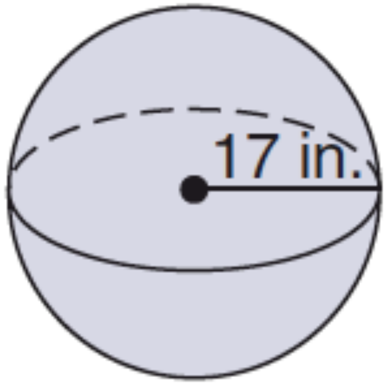
$V = \frac{4}{3}\pi r^3$

Find the surface area and volume of each sphere. Round to the nearest hundredth.

$SA = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

a.

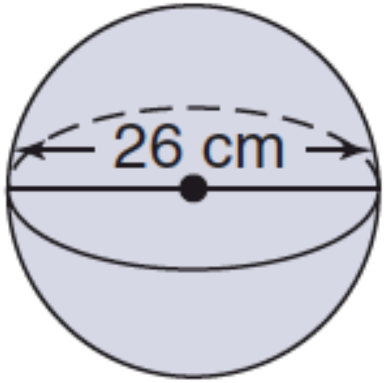


$r = 17$

$SA = 4\pi(17)^2 = 3\,631.68\ in^2$

$V = \frac{4}{3}\pi(17)^3 = 20\,579.53\ in^3$

b.



$d = 26\ cm$

$r = 26 \div 2$

$= 13\ cm$

$SA = 4\pi(13)^2 = 2\,123.72\ cm^2$

$V = \frac{4}{3}\pi(13)^3 = 9\,202.77\ cm^3$

Area and volume of composite solids

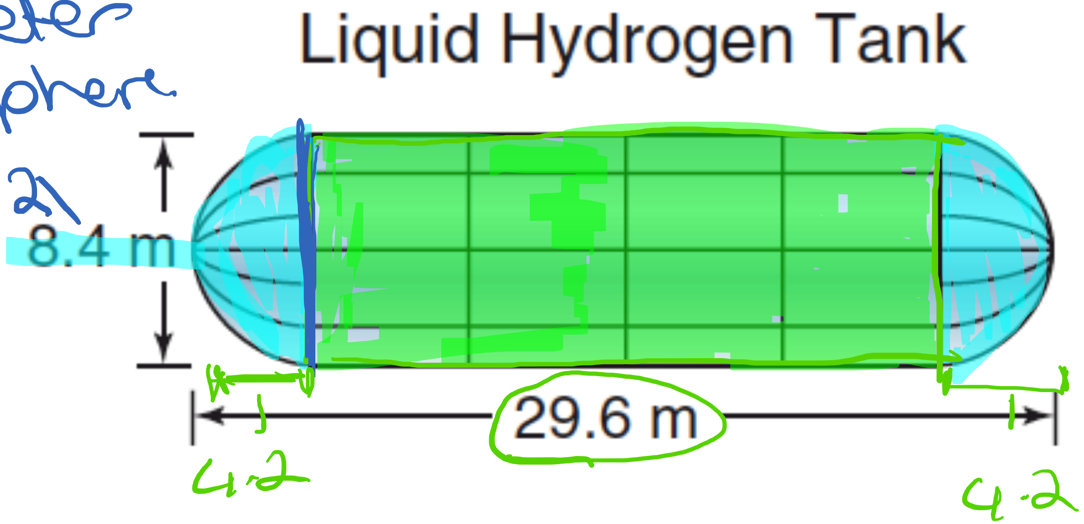
- To find the area or volume of composite solids, calculate the area or volume of the individual solids they are made up of and add them together.

Total volume $= 310.34 + 1174.9$
$= 1485.24\ m^3$

The large external tank attached to the space shuttle at the time of launch contains the propellants for takeoff. It holds three tanks, including the liquid hydrogen tank. If the ends of the liquid hydrogen tank are hemispheres, find the volume of this tank to the nearest hundredth.

diameter of sphere

$r = 8.4 \div 2$



$h = 29.6 - 4.2 - 4.2$
$= 21.2\ m$

Sphere

$V = \frac{4}{3}\pi(4.2)^3$

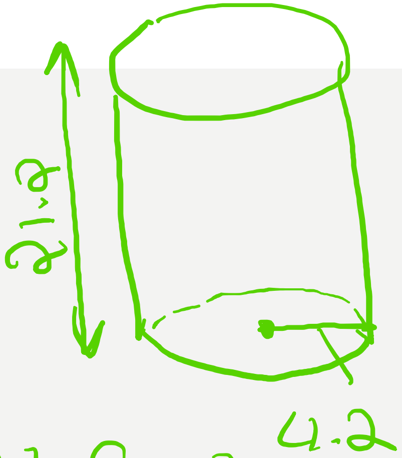
$= 310.34\ m^3$

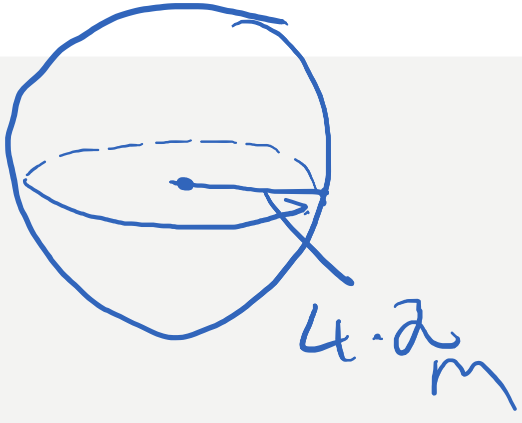
Cylinder

$A_b = \pi(4.2)^2 = 55.42$

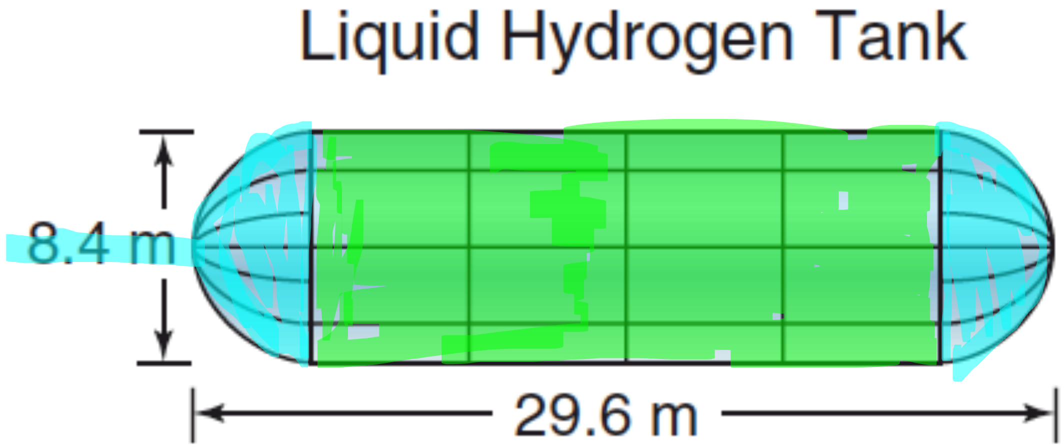
$h = 21.2$

$V = 55.42 \times 21.2 = 1174.9\ m^3$





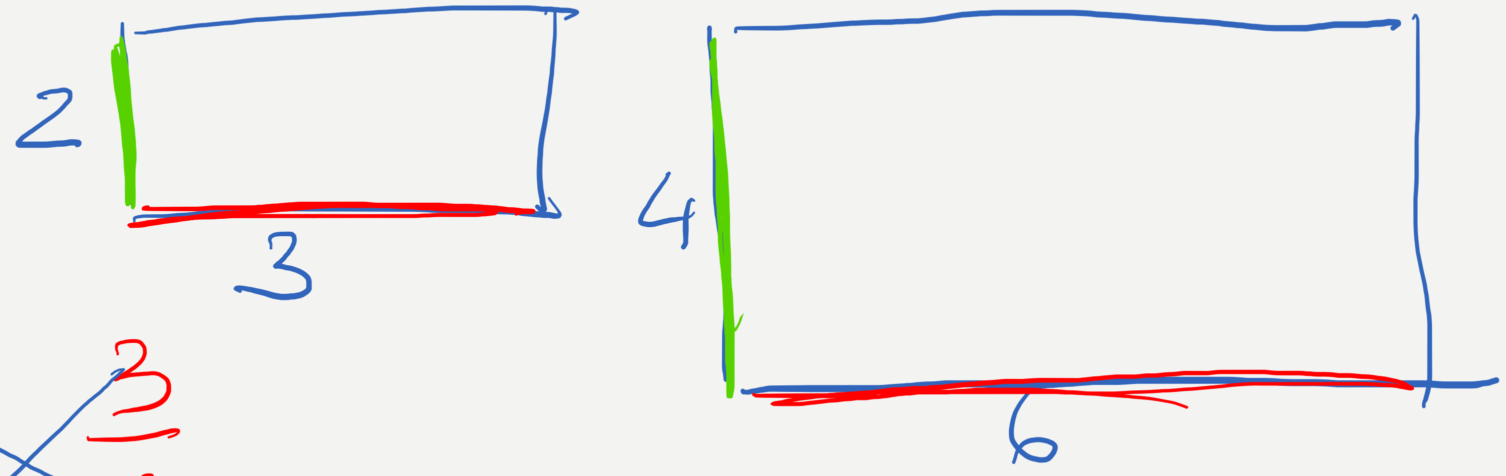
The large external tank attached to the space shuttle at the time of launch contains the propellants for takeoff. It holds three tanks, including the liquid hydrogen tank. If the ends of the liquid hydrogen tank are hemispheres, find the volume of this tank to the nearest hundredth.



12.7 – SIMILARITY OF SOLID FIGURES

Similar solids

Just like similar figures, **similar solids** have the same shape, but not the same size. All their measures are proportional.

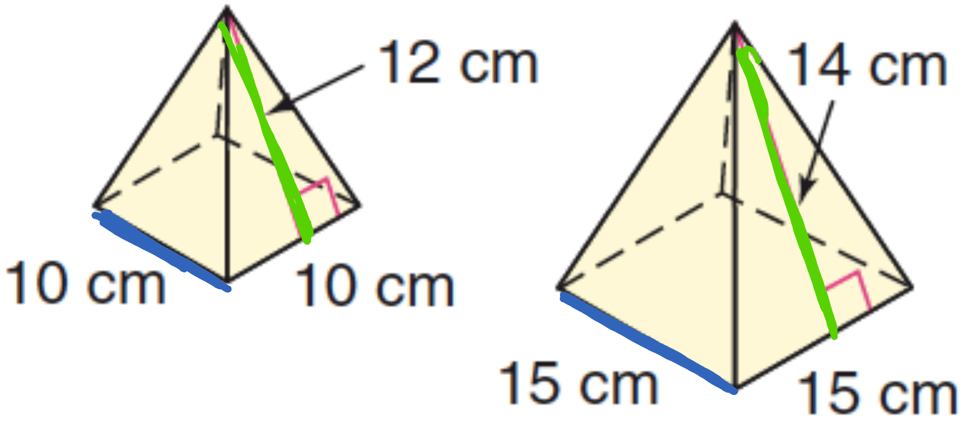


$$\frac{2}{4} = \frac{3}{6}$$

$$12 = 12$$

Determine whether each pair of solids is similar.

1

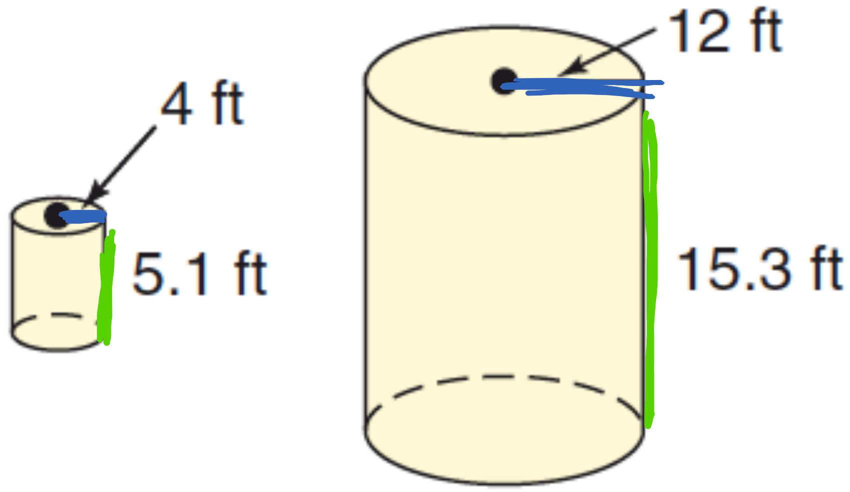


$$\frac{10}{15} \quad \frac{12}{14}$$

$180 \neq 140$

Not similar.

2



$$\frac{4}{12} \quad \frac{5.1}{15.3}$$

$61.2 = 61.2$

Similar.

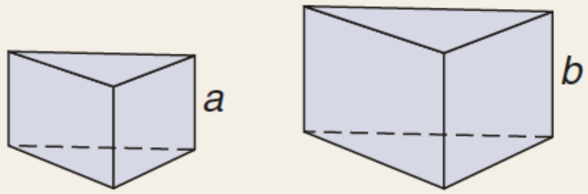
Scale factor relationships

In similar solids, the areas and volumes are also proportional, but their scale factors are squares for area and cubed for volume.

Theorem 12–15

Words: If two solids are similar with a scale factor of $a:b$, then the surface areas have a ratio of $a^2:b^2$ and the volumes have a ratio of $a^3:b^3$.

Model:



Solid A Solid B

Symbols: scale factor of solid A to solid B $= \frac{a}{b}$

$$\frac{\text{surface area of solid A}}{\text{surface area of solid B}} = \frac{a^2}{b^2}$$

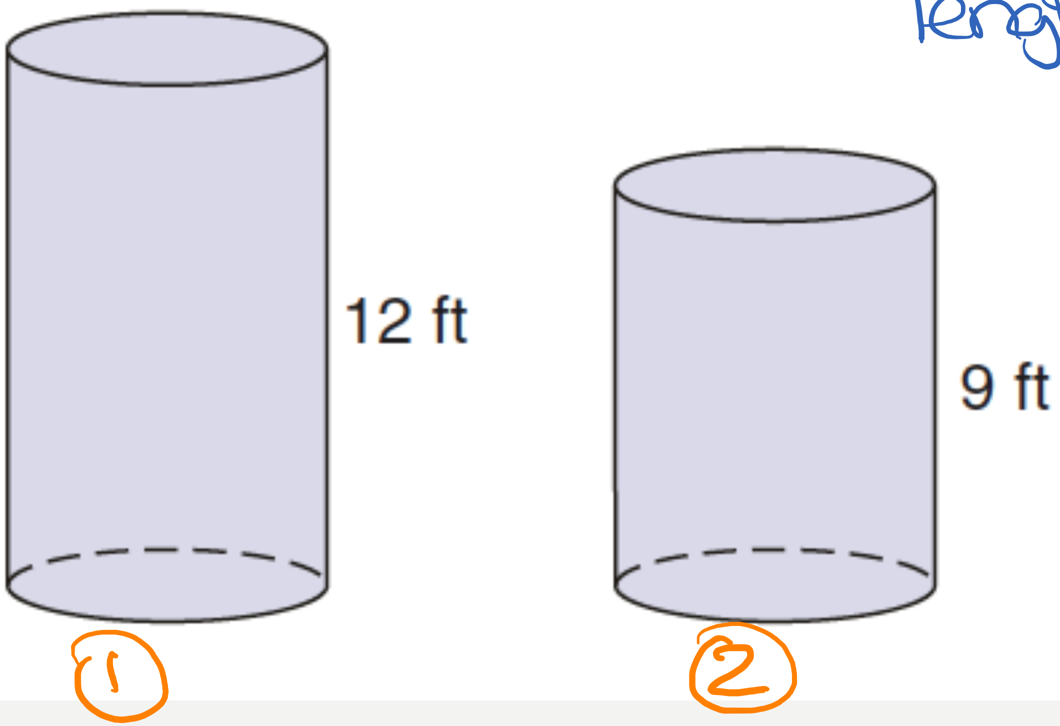
$$\frac{\text{volume of solid A}}{\text{volume of solid B}} = \frac{a^3}{b^3}$$

length scale factor $= k$

area scale factor $= k^2$

volume scale factor $= k^3$

For the similar cylinders, find the scale factor of the cylinder on the left to the cylinder on the right. Then find the ratios of the surface areas and the volumes.



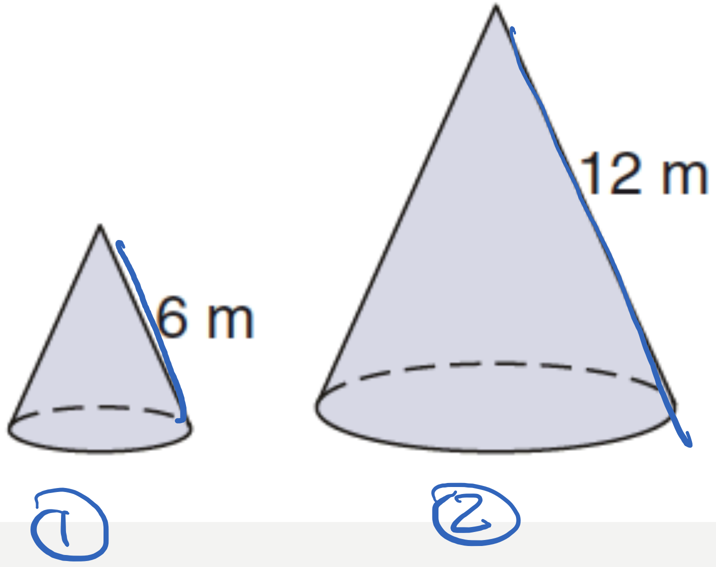
lengths: $k = \frac{12}{9} = \frac{4}{3}$

area: $k^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$

volume: $k^3 = \left(\frac{4}{3}\right)^3 = \frac{64}{27}$

For each pair of similar solids, find the scale factor of the solid on the left to the solid on the right. Then find the ratios of the surface areas and the volumes.

c.

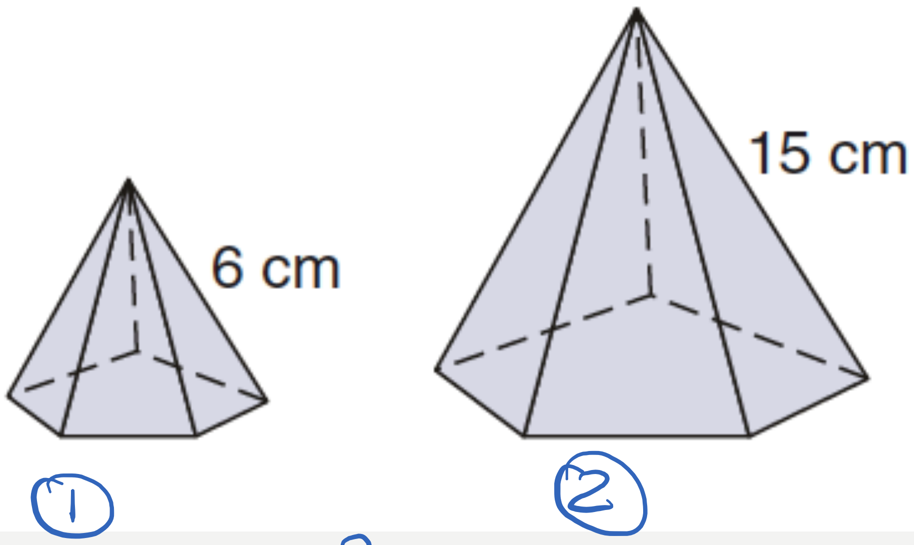


$k = \frac{6}{12} = \frac{1}{2}$

$k^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$k^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

d.



$k = \frac{6}{15} = \frac{2}{5}$

$k^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$

$k^3 = \left(\frac{2}{5}\right)^3 = \frac{8}{125}$